Multiple Contracting in Insurance Markets*

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Abstract

We study an adverse-selection insurance economy in which consumers can purchase coverage from several insurers. We show that a single budget-balanced allocation is implementable by an entry-proof tariff. In this allocation, different layers of coverage are fairly priced according to the types of consumers who purchase them, giving rise to cross-subsidies between types. This allocation can, under certain conditions, be uniquely implemented as the equilibrium outcome of a game as long as cross-subsidies between contracts are prohibited. In equilibrium, riskier consumers demand greater aggregate coverage at an increasing unit price, but the contracts offered by firms exhibit quantity discounts. We emphasize the need to regulate the supply side of insurance markets, while consumers can be left free to choose their preferred amount of coverage.

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1 Introduction

Multiple contracting, whereby individuals consumers purchase several policies from different insurers to cover the same risk, is a widespread phenomenon in insurance markets. A case in point is the US life-insurance market, in which around 25 percent of consumers hold more than one term policy.[1] A similar phenomenon arises in annuity markets: as an example, the six million annuities in payment in the UK in 2013 were owned by about five millions individuals.[2] Most health-insurance markets also exhibit multiple contracting, in forms that depend on the relative importance of the public and private insurance sectors. First, private health insurance can be used as a source of basic coverage for individuals who do not, or choose not to, obtain public health insurance. In this case, which prevails in Germany, Netherlands, and Switzerland, at least half of the insured households typically hold more than one policy.[3] Second, private insurance can be used to cover healthcare needs that are already partially covered by public funds. This role is prominent in Australia, Denmark, and, in particular, France, where about 92 percent of the population complement the public mandatory coverage with some private insurance.[4] In the US, the Medicare supplementary market performs a similar role, with 10 million out of the 42 million individuals covered by Medicare opting for Medigap plans issued by private insurers. The healthcare services of retirees who supplement Medicare beneficiaries with employer-sponsored retiree health insurance represent an additional source of multiple contracting.[5]

Since the early works of Arrow (1963), Akerlof (1970), Pauly (1974), and Rothschild and Stiglitz (1976), there has been a presumption that these insurance markets may be exposed to adverse selection. More recently, a large body of empirical work has attempted at providing a quantitative assessment of the extent to which this is the case, along two main lines. First, the very existence of adverse selection has been investigated, leading to mixed results.[6] Second, several measures of the potential welfare costs of adverse selection have

[1]A term life policy provides coverage for a limited period of time, which makes it a pure insurance product. Information about the buyers of such policies is provided by He (2009) on the basis of the Health and Retirement Study (HRS) panel.

[2]See the 2014 UK Insurance Key Facts document issued by the Association of British Insurers, available at https://www.abi.org.uk/∼/media/Files/Documents/Publications/Public/2014/Key%20Facts/ABI%20Key%20Facts%202014.pdf.

[3]See Paccagnella, Rebba, and Weber (2013).

[4]See Thomson, Osborne, Squires, and Jun (2013).

[5]The income from employment-based pension schemes is currently relevant for about half of the US retirees, see Poterba (2014).

[6]Cawley and Phillipson (1999) and Finkelstein and McGarry (2006) find no evidence of adverse selection in life and long-term-care insurance markets. Finkelstein and Poterba (2004) find some evidence of adverse selection using claims data from the UK annuity market. Hendren (2013) finds strong evidence of adverse selection in all these markets by looking at the entire set of potential customers, including rejected ones. See

been proposed.[7] Yet a common feature of these empirical studies is the limited attention devoted to the organization of markets and the nature of competition between insurers. This is particularly true for multiple contracting, despite its being, as argued above, a key feature of the markets under scrutiny. Indeed, standard tests for adverse selection, such as the positive-correlation property—individuals facing a higher probability of a loss should receive higher coverage—and the convex-pricing property—the unit price of coverage should increase in the total amount of coverage purchased—are derived with reference to the Rothschild and Stiglitz (1976) model, in which each individual can purchase insurance from at most one insurer.[8] Accordingly, the standard empirical strategy relies on the insurers' ability to induce different individuals to self-select into different contracts.

This strategy may, however, lead to a fundamental misspecification problem. When an individual engages in multiple contracting, each insurer has a limited basis for inferring her total amount of coverage. This makes it more difficult for insurers to screen individuals according to how much coverage they purchase, as this information is not directly available to them. In addition, the impossibility of fully controlling individual transactions may constitute an important source of welfare losses under adverse selection. These difficulties point to the need for a new theoretical framework, both for testing for the presence of adverse selection in markets where multiple contracting is prevalent and for assessing the combined welfare impact of adverse selection and multiple contracting.

We address these issues in a generalized version of the Rothschild and Stiglitz (1976) economy, in which a risk-averse consumer trades insurance contracts simultaneously issued by competing insurers. As in Rothschild and Stiglitz (1976), the consumer's risk is her private information; an insurance contract stipulates a coverage or, in the case of coinsurance, a coverage rate, in exchange for a premium. Trade is threatened by adverse selection: insurers prefer to trade smaller amounts of coverage with riskier consumer types, who, however, are willing to purchase a larger amount of coverage. Unlike in Rothschild and Stiglitz (1976), the consumer is free to purchase coverage from any number of insurers.

Central to our approach is to identify an appropriate notion of feasibility. To capture the restrictions that informational and contracting frictions impose on feasible trades, we characterize the set of allocations that can be achieved by a planner who observes neither the consumer's risk nor her trades with private insurers. Whereas the constraints induced

Cohen and Siegelman (2010) and Chiappori and Salanié (2013) for extensive surveys.

[7] See Einav, Finkelstein, and Levin (2010) for a survey.

[8] This approach is explicitly followed in the empirical analyses of Cawley and Phillipson (1999) and Finkelstein and Poterba (2004), among many others.

by private information on insurance provision are by now well understood,[9] little is known about how the opportunity for consumers to secretly sign bilateral agreements with private insurers further restricts the set of feasible allocations.

To capture these additional constraints, we require feasible allocations to be not only incentive-compatible, but also robust to further trading opportunities provided by private insurers. That is, any price-quantity scheme, or tariff, posted by the planner must be entry-proof. We show in Theorem 1 that a single budget-balanced allocation satisfies this robust incentive-compatibility requirement. In that allocation, both low- and high-risk consumers purchase the same basic amount of coverage, which the high-risk consumer complements by purchasing some additional coverage. Each marginal amount of coverage, or layer, is fairly priced given the consumer types who purchase it, which corresponds to a marginal version of Akerlof (1970) pricing. This allocation, which was first described by Jaynes (1978), Hellwig (1988), and Glosten (1994), cannot be improved in the Pareto sense without making entry profitable for a private insurer. Overall, the existence of private insurance markets dramatically constrains the planner, making redistribution among different types of consumers impossible.

It remains to understand to which extent private markets can perform their allocative role in these circumstances. In principle, multiple contracting affects the behavior of private insurers along two main dimensions. On the one hand, each insurer can exploit the offers of his competitors by proposing additional, possibly small, trades that are attractive to the consumer. From a strategic viewpoint, this corresponds to enlarging the set of available deviations with respect to the benchmark case in which exclusivity clauses are enforced from the outset, as in Rothschild and Stiglitz (1976); this makes undercutting an easier task for insurers. On the other hand, cream-skimming deviations may be blocked by additional threats which take the form of ad-hoc, latent contracts in one's competitors' offers; such contracts may be thought of as playing an anti-competitive role. Equilibrium must hence strike a delicate balance between these two forces, the interplay of which determines the effective supply of insurance under multiple contracting.

A natural benchmark for equilibrium analysis is the situation in which competition is fully nonexclusive, that is, insurers post arbitrary menu offers which the consumer is free to combine. In this scenario, however, the set of trading opportunities available "in the dark" is very large, and markets fail to be an effective device to allocate resources. Despite being the only equilibrium candidate, the Jaynes–Hellwig–Glosten (JHG) allocation described above

[9] See, for instance, Prescott and Townsend (1984) and Crocker and Snow (1985).

can only be decentralized in the degenerate case in which only the riskiest consumer is willing to trade at the prevailing price; market equilibria otherwise fail to exist. The logic underlying this result, extensively analyzed in Attar, Mariotti, and Salanié (2014, 2015), is that, in a given candidate equilibrium, any of the incumbent insurers can make a profit by selling basic coverage only to the low-risk consumer, while making a small loss by trading with the high-risk consumer some complementary coverage priced at slightly better terms than by his competitors. This sophisticated deviation involves cross-subsidization between contracts and crucially exploits the nonexclusive nature of competition. The deviating insurer minimizes his losses by sharing with his competitors the cost of providing a high coverage to the high-risk consumer ("lemon dropping"); this in turn enables him to profitably attract the low-risk consumer ("cherry picking").

This fully nonexclusive scenario, however, does not provide an accurate description of competition in modern insurance markets: the size of regulatory interventions in OECD countries is sufficiently large to affect the conduct of these markets, the relevant degree of information sharing, and the set of services available to consumers.[10] Instead of providing a detailed assessment of the impact of different waves of insurance regulation, let us stress that the joint issuance of loss-making contracts and of contracts designed to make profits on basic insurance may be particularly costly in several instances. 1) In private insurance markets, a loss-making contract is often identified as onerous, which forces insurers to recognize the net obligation associated with it as an accrued liability and offsetting expense in their financial statements. If the losses of such a contract are not simultaneously offset by the gains on some other assets, onerous contracts may be a source of operational restructuring charges.[11] 2) In health insurance, several European countries, notably Germany and Switzerland, rely on a central fund to redistribute costs among insurers according to a risk-equalization scheme.[12] These cost-sharing mechanisms, by pooling and redistributing costs among sellers of a basic standardized coverage contract, prevent insurers from earning abnormal profits on basic insurance.[13] This, again, casts some doubt on the feasibility of the deviation specific to the

[10] In 2010, the US State Insurance Departments employed about 12,000 regulatory workers, collecting around $19 billion in revenues from insurance sources.

[11] At the beginning of the 21st century, no International Financial Reporting Standard (IFRS) for insurance contracts existed. Since then, the International Accounting Standards Board (IASB) has introduced several measures to provide a unified treatment of the principles that an entity should apply to report information to users of its financial statements about the nature, amount, timing, and uncertainty of cash flows from insurance contracts (IASB (2013)). In particular, since 2011, insurers have to perform an onerous-contract test when facts and circumstances indicate that the contract might be onerous, with the unavoidable cost required to fulfill the agreement being higher than its economic benefit.

[12] See Thomson and Mossialos (2009, page 84). Besides Germany and Switzerland, other countries using such schemes include Australia, Ireland, Netherlands, and Slovenia.

[13] In Switzerland, the basic coverage contract is defined at the national level; then insurers compete over

fully nonexclusive scenario.

In light of these remarks, it is important to derive predictions for competitive insurance markets in which insurers cannot engage into cross-subsidization between different contracts. A parsimonious way to achieve this goal is to consider a game in which insurers can only make simple take-it-or-leave-it offers, so that regulation prevents from the outset firms from destabilizing the market through dumping practices. This restriction does not undermine the power of competition under multiple contracting. Indeed, we show in Theorem 2 that the JHG allocation remains the only candidate equilibrium allocation even in this single-contract game: multiple contracting allows firms to Bertrand-compete over take-it-or-leave-it contracts on each layer of coverage, in a way that would not be feasible under exclusivity. We moreover identify necessary conditions for equilibrium existence. First, high-risk consumers must not be willing to purchase twice the basic coverage. This reflects the idea that no private insurer is indispensable in a market equilibrium. Second, the amount of complementary coverage purchased by a high-risk consumer should not exceed that of the basic coverage. This prevents private insurers from profitably conducting additional trades with high-risk consumers on top of any available amount of basic coverage. Last, we provide in Theorem 3 a condition on consumers' preferences that ensure that these necessary conditions are also sufficient to guarantee the existence of an equilibrium. Together with Theorem 1, Theorems 2 and 3 provide weak versions of the First and Second Welfare Theorems for our economy: every equilibrium implements the only budget-balanced allocation that is robustly incentive-compatible, and this allocation can be implemented in a market equilibrium under additional conditions on preferences.

Overall, our equilibrium analysis suggests new avenues for empirical research on adverse selection. When markets successfully allocate resources, consumers end up in equilibrium trading with several insurers, as data shows. Empirical exercises may therefore be performed by considering consumer surveys or, alternatively, by looking at insurer-level data. These two approaches are treated as equivalent in recent empirical work, reflecting the fact that, under exclusive contracting, any consumer's demand for coverage must be met by a single contract. But this equivalence collapses as soon as consumers trade several contracts: while

prices to provide the corresponding amount of coverage. Yet, an additional rule specifies that costs are pooled and redistributed among insurers. In Germany, the basic coverage contract is also defined at the national level and is offered by 134 not-for-profit, nongovernmental "sickness funds." Insurees contribute a fixed fraction of their wealth; these contributions are then centrally pooled and redistributed to sickness funds according to a rather precise risk-adjusted capitation formula. More generally, risk equalization involves transfer payments between health insurers so as to spread some of the claims cost of the high-risk, older, and less healthy members amongst all the private health insurers in the market, in proportion to their respective market shares.

the positive-correlation and the convex-pricing properties hold at the consumer level, the contracts offered by insurers exhibit a negative correlation between risk and coverage, and display quantity discounts. In this respect, our results challenge the negative conclusions derived by Cawley and Philipson (1999), Finkelstein and Poterba (2006), and Finkelstein and McGarry (2006) when testing for adverse selection: by putting at the center stage of our analysis the actual organization of insurance markets, we can reconcile the predictions of the theory with the broad features of the data on insurance markets in which multiple contracting is a prominent feature.

Yet, our analysis also shows that market equilibria fail to exist in many circumstances. That is, insurers' behavior may have a destabilizing effect even when cross-subsidization between contracts is not feasible. As multiple contracting raises a fundamental obstacle against redistribution, a failure in the market allocative mechanism calls for nonstandard normative implications: although the state has no redistributive role, it may play an active role in the allocation process.

We discuss several instances of such interventions in the context of health insurance. We first argue that multiple contracting is compatible with mandatory systems in which the state provides basic insurance. However, to the extent that enforcing such programs may end up being particularly expensive, we next show how the simple threat of the state standing ready to complement private insurance is sufficient to implement the JHG allocation in a market equilibrium. That is, basic coverage can be provided taking into account both the incentives of private insurers and the consumers' freedom to choose among them.

More generally, our results suggest novel insights for the design of public interventions in financial markets plagued by adverse selection. A prominent example is the interbank market: we argue that, under the threat of multiple contracting, an optimal lending program may need to pool all types of each borrower. This contrasts with the received view that such programs should target the least profitable borrowers, so as to unfreeze the market faced by private banks.

Contributions to the Literature Starting with the early contributions of Hammond (1979, 1987), Allen (1985), and Jacklin (1987), several authors have attempted at identifying the constraints on risk sharing that arise when agents are free to engage in side trades in financial markets. Such trades are typically formalized by letting privately informed agents free to exchange commodities in Walrasian markets, so that they can complement their trades with the planner by trading at linear prices. This exacerbates the tension between incentive compatibility and optimal insurance. In private-value environments, von Thadden

(1999), Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), and Farhi, Golosov and Tsyvinski (2009) show how, given the threat of hidden borrowing and saving, optimality may involve agents receiving no additional insurance beyond self-insurance. The same allocation may also be supported in an equilibrium of a strategic game featuring competition among private insurers (Ales and Maziero (2016)). This approach, however, is hard to reconcile with adverse selection settings, in which the identity of one's trading partner matters over and above the terms of trade that are settled on.

The present paper extends these analyses by considering a common-value environment in which a privately informed consumer can purchase coverage from competing private insurers. The unique robustly incentive-compatible allocation features cross-subsidization between different consumer types, so that additional trading opportunities are effectively exploited despite adverse selection and multiple contracting. In particular, this allocation does not coincide with the one arising in a competitive market in which the consumer can only self-insure through savings.

Despite the practical relevance of financial markets in which consumers' aggregate trades cannot be fully monitored, few attempts have been made to incorporate multiple contracting in a theoretical analysis of decentralized markets under adverse selection. Indeed, adverse-selection extensions of the Walrasian paradigm (Prescott and Townsend (1984), Dubey and Geanakoplos (2002), Bisin and Gottardi (2006)) and of the competitive-search approach (Gale (1996), Guerrieri, Shimer and Wright (2010)) postulate exclusive contracting from the outset. An alternative route has been suggested by Bisin and Gottardi (1999), who study private information economies in which, due to nonexclusivity, prices are restricted to be linear with respect to aggregate trades.[14] Yet, under adverse selection, linear pricing turns out not to be robust to sellers' strategic manipulations whenever multiple contracting is taken into account. Indeed, Attar, Mariotti and Salanié (2015) show that, in such circumstances, it is always profitable for at least one seller to reduce the riskiness of his portfolio by restricting the maximal quantity that he stands ready to sell at a given price.

Modeling nonexclusive competition under adverse selection has been at the center stage of several recent reformulations of the competitive-screening literature initiated by Rothschild and Stiglitz (1976), Miyazaki (1977), Spence (1977), and Wilson (1977). Attar, Mariotti and Salanié (2011, 2014) show that nonexclusivity worsens the impact of adverse selection. Pure-strategy equilibria may fail to exist, and when they exist they necessarily feature the market breakdown emphasized by Akerlof (1970). Besides, although individuals can enter several

[14] Bisin and Gottardi (2003) argue that a "minimal" degree of nonlinearity, in the form of a bid-ask spread, may be needed to guarantee equilibrium existence.

trading agreements, equilibrium need not feature multiple contracting. Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2012) analyze the strategic interaction between liquidity suppliers who post price-quantity schedules to match the market order of a privately informed trader. When there are finitely many market makers, each of them earns a strictly positive profit; the JHG allocation only obtains in the limit when the number of market makers goes to infinity. As each type of the trader symmetrically splits his market order between all market makers, the equilibrium does not require the trade of qualitatively different contracts, unlike our basic and complementary coverage contracts.

We contribute to this strategic approach by building a competitive-screening game in which the JHG allocation is the only allocation that can be sustained in a pure-strategy equilibrium, and individual trades feature multiple contracting. To implement this outcome, we do not rely on alternative extensive forms which allow sellers to condition their behaviors on their competitors' offers, as in Hellwig (1988), or let the buyer solicit additional proposals from an infinite sequence of sellers' cohorts, as in Beaudry and Poitevin (1995). Rather, we only require sellers to be prevented from exploiting cross-subsidization between contracts.

The paper is organized as follows. Section 2 describes the model. Section 3 shows that the JHG allocation is the only budget-balanced allocation that is robust to side trades with private insurers. Section 4 discusses how to implement the JHG allocation in a competitive economy. Section 5 draws the implications of our analysis for public intervention in insurance and financial markets. Section 6 concludes. Proofs not given in the text can be found in Appendices A and B.

2 The Economy

In this section, we describe an adverse-selection insurance economy in which a risk-averse buyer can purchase coverage from several risk-neutral sellers. We allow for a large class of convex preferences for the buyer, only assumed to be ordered by a single-crossing property. As a result, our framework is quite general and can be used to model other financial markets.

2.1 The Buyer

The buyer is privately informed of her preferences. She may be of two types, $i = 1, 2$, with positive probabilities m_1 and m_2 such that $m_1 + m_2 = 1$. Type i's preferences over aggregate coverage-premium pairs $(Q, T) \in \mathbb{R}_+ \times \mathbb{R}$ are represented by a utility function U_i. We assume that U_i is twice continuously differentiable, with $\partial U_i/\partial T < 0$, and that U_i is strictly quasiconcave.[15] Hence type i's marginal rate of substitution of coverage for premium,

$$\tau_i \equiv -\frac{\partial U_i/\partial Q}{\partial U_i/\partial T}, \tag{1}$$

is everywhere well defined and strictly decreasing along her indifference curves. The following single-crossing (SC) assumption is key to our results.

Assumption SC *For each* $(Q,T) \in \mathbb{R}_+ \times \mathbb{R}$, $\tau_2(Q,T) > \tau_1(Q,T)$.

Geometrically, in the (Q,T) plane, an indifference curve for type 2 crosses an indifference curve for type 1 only once, from below. As a result, type 2 is more eager to increase her purchase of coverage than type 1 is.

2.2 The Sellers

There are $n \geq 3$ identical sellers. If a seller provides type i with coverage q for a premium t, he earns a profit $t - v_i q$, where the cost v_i of serving type i is her risk. The following common-value (CV) assumption is maintained throughout the analysis.

Assumption CV $v_2 > v_1$.

Thus type 2 represents a greater risk for sellers than type 1 does. Along with Assumption SC, Assumption CV generates adverse selection: type 2 is more willing to trade at the margin than type 1 is, but she faces sellers who are less willing to trade with her than with type 1. We let $v \equiv m_1 v_1 + m_2 v_2$ be the average risk of the buyer, so that $v_2 > v > v_1$.

2.3 Contracts and Trades

The first step of our analysis does not require that we describe in detail the precise structure of the sellers' offers; this will be done in Section 4. Rather, following Rothschild and Stiglitz (1976), we represent supply by a set of contracts simultaneously made available to the buyer, and which she is free to combine at will.

Contracts are bilateral: each seller monitors the amount of coverage the buyer purchases from him, but not the amounts of coverage the buyer purchases from his competitors. As a result, a contract between the buyer and a seller is just a coverage-premium pair $(q,t) \in \mathbb{R}_+ \times \mathbb{R}$. The no-trade contract is $(0,0)$ and a contract (q,t) with positive coverage has unit price t/q.

[15] One can dispense with the differentiability requirement, but this generalization comes at the price of more cumbersome statements and proofs.

After privately learning her type, the buyer chooses from the set of offered contracts. In this respect, the only difference with Rothschild and Stiglitz (1976) is that she can choose to trade multiple contracts offered by different sellers. Thus, letting K be the set of sellers with whom a given type of the buyer chooses to trade, and (q^k, t^k) be the contract she trades with seller $k \in K$, her aggregate trade is $(Q, T) \equiv (\sum_{k \in K} q^k, \sum_{k \in K} t^k)$.

2.4 Examples

2.4.1 The Rothschild–Stiglitz Economy

Our first example is the Rothschild and Stiglitz (1976) economy, only modified to allow for multiple contracting. The buyer has initial wealth W_0 and faces the risk of a loss L with a probability $v_i \in (0,1)$ that defines her type. Type i's preferences over aggregate coverage-premium pairs have an expected-utility representation

$$U_i(Q,T) \equiv v_i u(W_0 - L + Q - T) + (1 - v_i)u(W_0 - T), \tag{2}$$

where u is a twice continuously differentiable, strictly increasing, and strictly concave von Neumann–Morgenstern utility function. Assumption CV states that type 2 has a higher probability of incurring a loss than type 1. This implies that her willingness to substitute coverage for premium is everywhere higher than type 1's, which is Assumption SC.

2.4.2 Coinsurance

Our next example allows for multiple loss levels, while focusing on coinsurance contracts. Thus a contract (q, t) specifies that a fraction q of the loss is covered for a premium t, and multiple contracts can be additively aggregated in a natural way. The buyer has initial wealth W_0 and faces the risk of a loss L distributed according to a density f_i that defines her type. Type i's preferences over aggregate coverage-premium pairs have an expected-utility representation

$$U_i(Q,T) \equiv \int u(W_0 - (1-Q)L - T) f_i(L)\,\mathrm{d}L, \tag{3}$$

where u is a twice continuously differentiable, strictly increasing, and strictly concave von Neumann–Morgenstern utility function. Assumption SC is satisfied if f_2 dominates f_1 in the monotone-likelihood-ratio order, that is, if type 2 is relatively more likely to incur large losses than type 1 is. This, in turn, implies that she is more costly to serve than type 1 is, $v_2 = \int L f_2(L)\,\mathrm{d}L > \int L f_1(L)\,\mathrm{d}L = v_1$, which is Assumption CV.

2.4.3 Financial Markets

Our last example is a market-microstructure model inspired by Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2013). The buyer is an insider who can purchase shares of a risky asset from sellers acting as market makers. The buyer maximizes expected utility with constant absolute risk aversion α and faces residual Gaussian noise with variance σ^2. Type i's preferences over aggregate share-money pairs are thus represented by

$$U_i(Q,T) \equiv \theta_i Q - \frac{\alpha\sigma^2}{2} Q^2 - T, \tag{4}$$

where type i's marginal valuation θ_i reflects her informational and risk-sharing motivations to trade the asset. Assumption SC requires $\theta_2 > \theta_1$. The market makers are risk-neutral and the cost v_i of selling a share of the asset to type i is its expected value conditional on the insider's being of type i. Assumption CV requires that this expected value be higher for type 2 than for type 1.

2.4.4 On the Assumptions of the Model

Our focus on general preferences allows us to avoid relying on particular properties of, for instance, the expected-utility model. In fact, we can handle non-expected utilities in our framework, provided these preferences are: (i) consequentialist, in the sense that the agent only cares about the distribution of final outcomes, and: (ii) sufficiently regular, such as, for instance, the Fréchet-differentiable preferences introduced by Machina (1989). As shown by the coinsurance example, we can also handle multiple loss levels, as long as the additive aggregation of insurance contracts is consistent; as a counter-example, contracts with deductibles do not aggregate in this way. Overall, what really matters is that insurance coverage can be summarized by a one-dimensional additive index, and that preferences satisfy a single-crossing property along that dimension.

3 Robust Incentive Compatibility

In this section, we introduce the notion of incentive compatibility that is relevant for our multiple-contracting setting and we show that a unique budget-balanced allocation satisfies this property.

3.1 Definition

In the benchmark situation where a planner can design an incentive-compatible trading mechanism while perfectly monitoring trades (Myerson (1979, 1982), Harris and Townsend

(1981)), the Taxation Principle tells us that he can with no loss of generality offer a tariff specifying a transfer $T^P(Q)$ to be paid as a function of the aggregate coverage Q demanded by the buyer (Hammond (1979), Guesnerie (1981), Rochet (1985)).[16]

Definition 1 *The tariff T^P implements the allocation $((Q_1, T_1), (Q_2, T_2))$ if, for each i,*

$$Q_i \in \arg\max\{U_i(Q, T^P(Q)) : Q \geq 0\},$$
$$T_i = T^P(Q_i).$$

Incentive compatibility is the relevant notion of feasibility when the planner can perfectly monitor trades. As such, it is key to the characterization of the second-best efficiency frontier obtained by Prescott and Townsend (1984) and Crocker and Snow (1985a) in the Rothschild–Stiglitz economy, which provides a benchmark for assessing insurance-market outcomes when each seller is able to enforce exclusivity.

In our multiple-contracting setting, however, no outside party can monitor the trades between the buyer and any seller. To incorporate the additional constraints that this imposes on the planner, we require that the tariff T^P be robust to entry: that is, no seller can profitably offer additional contracts that the buyer can trade along with some amount of coverage offered by the planner. To precisely model this, observe that the Taxation Principle again implies that, once the planner has offered his tariff, the best an entrant can do is also to offer a tariff T^E. This motivates the following definition.

Definition 2 *The tariff T^P is entry-proof if, for any tariff T^E offered by an entrant, there exists for each i a solution (q_i^P, q_i^E) to type i's problem*

$$\max\{U_i(q^P + q^E, T^P(q^P) + T^E(q^E)) : q^P \geq 0 \text{ and } q^E \geq 0\} \tag{5}$$

such that the expected profit of the entrant is at most zero,

$$m_1[T^E(q_1^E) - v_1 q_1^E] + m_2[T^E(q_2^E) - v_2 q_2^E] \leq 0. \tag{6}$$

That is, the tariff offered by the planner is entry-proof if, no matter the tariff subsequently offered by an entrant, there is an optimal way for the buyer to combine these offers that prevents the entrant from making a profit. Notice that we confer the entrant a lot of power by allowing him to offer an arbitrary tariff. This contrasts with models of optimal allocation under private information in the presence of hidden trades, in which it is typically

[16]For any tariff T, we set $T(0) \equiv 0$ to deal with participation in a simple way, and we set $T(Q) \equiv \infty$ if the tariff does not allow the buyer to trade the coverage Q.

assumed that such trades take place on Walrasian markets (Hammond (1979, 1987), Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007), Farhi, Golosov, and Tsyvinski (2009)). In the present context, this assumption would amount to restrict the entrant to offer a linear tariff. Whereas this assumption is perhaps relatively innocuous in the private-value environments that have been studied in the literature, it is much less natural in our adverse-selection environment, if only because sellers have an incentive to restrict the maximum amount of coverage they offer at any given price.

Our concept of robust incentive compatibility then naturally follows from Definitions 1–2.

Definition 3 *An allocation is robustly incentive-compatible if there exists an entry-proof tariff that implements it.*

Remark The requirement that a robustly incentive-compatible allocation be implementable by an entry-proof tariff is reminiscent of the definition by Kahn and Mookherjee (1998) or Bisin and Guaitoli (2004) of third-best allocations in moral-hazard environments; in such an allocation, the incentive contract offered by the planner must deter sellers from offering additional contracts. This requirement is also reminiscent of Laffont and Martimort's (1997) collusion-proofness principle.

3.2 The JHG Allocation

We now describe an allocation, introduced by Jaynes (1978), Hellwig (1988), and Glosten (1994), that plays a central role in our analysis. In this allocation, both type 1 and type 2 purchase the same basic coverage, which type 2 complements by purchasing additional coverage. A marginal version of Akerlof (1970) pricing holds: each layer of coverage is fairly priced given the types who purchase it, and the size of each layer is optimally chosen subject to this constraint. This calls for a recursive definition. The first layer Q_1^* is optimal for type 1 at unit price v,

$$Q_1^* \equiv \arg\max\{U_1(Q, vQ) : Q \geq 0\}, \tag{7}$$

$$T_1^* \equiv vQ_1^*. \tag{8}$$

Then the second layer $Q_2^* - Q_1^*$ is optimal for type 2 at unit price v_2, given that she already purchases the first layer Q_1^* at unit price v,

$$Q_2^* - Q_1^* \equiv \arg\max\{U_2(Q_1^* + Q, T_1^* + v_2 Q) : Q \geq 0\}, \tag{9}$$

$$T_2^* - T_1^* \equiv v_2(Q_2^* - Q_1^*). \tag{10}$$

Because each layer of coverage is fairly priced given the types who purchase it, the JHG allocation $((Q_1^*, T_1^*), (Q_2^*, T_2^*))$ makes zero expected profit. However, if $Q_1^* > 0$, the aggregate coverages Q_1^* and Q_2^* of types 1 and 2 are not fairly priced: because the coverage Q_1^* is sold at the average premium rate $v > v_1$, type 1 subsidizes type 2. Figure 1 depicts the JHG allocation in the case where both layers Q_1^* and $Q_2^* - Q_1^*$ are positive.

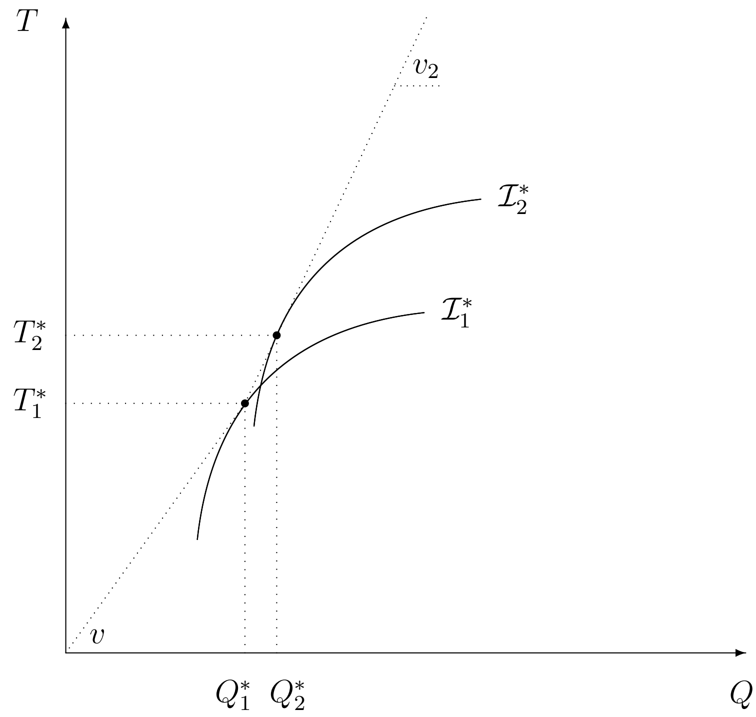

Figure 1 The JHG allocation.

Remark Jaynes (1978) and Hellwig (1988) assume as we do that there are finitely many buyer types, so that their definition coincides with the above one. This contrasts with Glosten (1994), who assumes that demand is continuously distributed. Despite this difference, the key feature shared by the allocations described by these authors is that the marginal price of any amount of coverage is the upper-tail conditional expectation of the cost of serving the types who buy at least that amount. It is interesting to compare this price structure to that arising under monopoly. According to Wilson's (1993) demand-profile interpretation, the layers Q_1^m and $Q_2^m - Q_1^m$ sold by a monopolist are priced in a profit-maximizing way, given the types who purchase them. By contrast, in the JHG allocation, the layers Q_1^* and $Q_2^* - Q_1^*$ are priced in a competitive way, given the types who purchase them.

In the Rothschild–Stiglitz economy, type 2 obtains full coverage in the JHG allocation, $Q_2^* = L$, while type 1 obtains less than full coverage, $Q_1^* < L$. In the coinsurance example, we similarly have $Q_2^* = 1$ and $Q_1^* < 1$. In both cases, type 2's optimal complementary layer of coverage $Q_2^* - Q_1^*$ is strictly positive at the fair premium rate. From (9), this implies that her incentive-compatibility constraint is slack, $U_2(Q_2^*, T_2^*) > U_2(Q_1^*, T_1^*)$.

It follows from this observation that, in these two key examples, the JHG allocation does not belong to the second-best efficiency frontier. Indeed, because type 1 obtains less than full coverage, she is willing to purchase additional coverage at the fair premium rate, that is, $\tau_1(Q_1^*, T_1^*) > v_1$. A planner with the ability to perfectly control trades can then slightly perturb the JHG allocation by complementing type 1's aggregate trade (Q_1^*, T_1^*) with additional coverage at a premium rate between v_1 and $\tau_1(Q_1^*, T_1^*)$, thereby increasing her utility. As long as the additional amount of coverage involved is small enough, this does not cause type 2 to deviate from her aggregate trade (Q_2^*, T_2^*), and the planner even achieves a small positive expected budget surplus.

This reasoning fails when the planner cannot perfectly control trades. Indeed, increasing the coverage sold to type 1 beyond Q_1^* at a unit price less than $\tau_1(Q_1^*, T_1^*)$ now makes it feasible for an entrant to attract type 2 with complementary coverage at a premium rate slightly below the fair premium rate; the reason is that type 2 can now combine such coverage with the coverage intended by the planner for type 1. Thus an entrant can exploit the trades offered by the planner to make a profit, leading to a deficit for the planner. This reflects that, under multiple contracting, the relevant analogue of a binding incentive-compatibility constraint for type 2 is (9) or, equivalently,

$$U_2(Q_2^*, T_2^*) = \max\{U_2(Q_1^* + Q, T_1^* + v_2 Q) : Q \geq 0\},$$

which states that she is indifferent between trading (Q_2^*, T_2^*) and trading (Q_1^*, T_1^*) along with contracts issued by an entrant at the fair premium rate v_2.

3.3 Characterization

The above remarks suggest that the threat of entry severely limits the scope for improving on the JHG allocation. Our first result confirms this intuition by showing that our robustness criterion drastically reduces the set of feasible allocations.

Theorem 1 *The JHG allocation is the unique budget-balanced allocation that is robustly incentive-compatible.*

Because the argument is simple and instructive, we give its main structure in the body of the paper, leaving a few technical details for Appendix A.

We first show uniqueness. A robustly incentive-compatible allocation $((Q_1, T_1), (Q_2, T_2))$ must satisfy $Q_2 \geq Q_1$ by Assumption SC and, additionally,

$$U_1(Q_1, T_1) \geq \max\{U_1(Q, vQ) : Q \geq 0\}, \tag{11}$$

because, otherwise, an entrant can offer a contract with a unit price slightly above v that profitably attracts type 1 and that is profitable even if it also attracts type 2. Similarly, $((Q_1, T_1), (Q_2, T_2))$ must also satisfy

$$U_2(Q_2, T_2) \geq \max\{U_2(Q_1 + Q, T_1 + v_2 Q) : Q \geq 0\}, \tag{12}$$

because, otherwise, an entrant can offer a contract with a unit price slightly above v_2 that profitably attracts type 2 along with the aggregate trade (Q_1, T_1) and that is even more profitable if it also attracts type 1. These two inequalities imply

$$T_1 \leq vQ_1 \tag{13}$$

and

$$T_2 \leq T_1 + v_2(Q_2 - Q_1), \tag{14}$$

so that the budget-balance constraint, which can be rewritten as

$$T_1 - vQ_1 + m_2[T_2 - T_1 - v_2(Q_2 - Q_1)] \geq 0,$$

is satisfied if and only if all inequalities (11)–(14) are in fact equalities. Together with the recursive definition (7)–(10), this implies that the robustly incentive-compatible allocation $((Q_1, T_1), (Q_2, T_2))$ must coincide with the JHG allocation.

To show existence, we only need to exhibit a tariff for the planner that implements the JHG allocation and that is entry-proof. For this purpose, consider the piecewise-linear convex tariff

$$T^P(q) \equiv 1_{\{q \leq Q_1^*\}} vq + 1_{\{q > Q_1^*\}} [vQ_1^* + v_2(q - Q_1^*)], \tag{15}$$

which is the analogue in our two-type setting of the tariff constructed by Glosten (1994) when demand is continuously distributed. According to (7)–(10), T^P implements the JHG allocation. Consider now what happens when an entrant offers a tariff T^E. As shown in Appendix A, the convexity of T^P implies that there exist solutions (q_1^P, q_1^E) and (q_2^P, q_2^E) to (5) for $i = 1, 2$ such that $q_2^E \geq q_1^E$. Now, as T^P allows type 1 to buy her optimal coverage Q_1^* at unit price v, one must have

$$T^E(q_1^E) \leq vq_1^E. \tag{16}$$

Next, because $q_2^E \geq q_1^E$, type 2 can purchase the same aggregate coverage $q_2^P + q_2^E$ by trading (q_1^E, t_1^E) with the entrant and purchasing complementary coverage $q_2^P + q_2^E - q_1^E$ from the planner, paying overall $T^P(q_2^P + q_2^E - q_1^E) + T^E(q_1^E)$. As she pays $T^P(q_2^P) + T^E(q_2^E)$ instead, one must have

$$T^E(q_2^E) - T^E(q_1^E) \leq T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P). \tag{17}$$

Because T^P is convex with slope at most v_2 and $q_2^E \geq q_1^E$,

$$T^P(q_2^P + q_2^E - q_1^E) - T^P(q_2^P) \leq v_2(q_2^E - q_1^E). \tag{18}$$

Collecting (16) and (17)–(18) leads to

$$T^E(q_1^E) - vq_1^E + m_2[T^E(q_2^E) - T^E(q_1^E) - v_2(q_2^E - q_1^E)] \leq 0,$$

so that the expected profit of the entrant is at most zero, as requested by (6). This concludes the proof of Theorem 1.

Remark Note that the JHG allocation emerges as the unique candidate for a robustly incentive-compatible and budget-balanced allocation even when the entrant is restricted to offer a single contract, whereas the tariff (15) is robust to entry even when the entrant can offer an arbitrary tariff. This contrasts with Glosten's (1994) characterization of an entry-proof tariff when demand is continuously distributed, which crucially relies on the entrant offering a tariff satisfying a property he dubs "single crossing" and that generalizes convexity. Another difference with Glosten (1994) is that we do not request the buyer's preferences to be quasilinear. In fact, our characterization is very general and does not even rely on an expected-utility representation for the buyer's preferences such as (2), (3), or (4).

3.4 Discussion

3.4.1 Sorting and Tie Breaking

Our definition of an entry-proof tariff only requires that, given any tariff of the entrant, the buyer has a best response such that the entrant does not make a profit. Hence we do not require this no-profit property to be satisfied for all best responses of the buyer. This rules out positive sorting, whereby the entrant would be able to break the buyer's ties in his favor. The existence part in the proof of Theorem 1 exploited this degree of freedom by considering a best response of the buyer in which type 2 purchases at least as much coverage from the entrant as type 1 does. By contrast, giving the entrant full powers to select the buyer's best response, in line with Peters' (2001) and Han's (2007) strongly robust equilibrium refinement, would undermine the very existence of an entry-proof tariff.

3.4.2 Uniqueness and Existence

According to Theorem 1, robustness to entry pins down a unique budget-balanced allocation. This shows that the planner is severely constrained by his inability to control the buyer's trades with a potential entrant, as the threat of such trades effectively deprives him of any possibility to transfer utility between the two types. This contrasts with the multiplicity of second-best allocations, which, in the Rothschild–Stiglitz economy, for instance, form a nondegenerate frontier. The possibility of multiple contracting is key to this result, as exclusive contracting would allow the planner to prevent entry.

Another important insight of Theorem 1 is that, no matter the distribution of types, there always exists a budget-balanced allocation that is implementable by an entry-proof tariff. This contrasts with the exclusive case, in which an entry-proof allocation can robustly fail to exist: for instance, under exclusivity, the only candidate for an entry-proof allocation in the Rothschild–Stiglitz economy is not robust to entry with a pooling contract if the proportion of type-1 buyers is too high. The difference is that an entrant in the exclusive case can fully control the buyer's trades, whereas, in our setting, any contract he may offer can be combined by the buyer with the contracts offered by the planner. Although this enlarges the set of contracts an entrant can offer to attract the buyer, this also gives the planner more instruments to deter entry, as we shall now see.

3.4.3 Allocations, Tariffs, and Latent Contracts

An important property of the tariff (15) we use to implement the JHG allocation is that it allows the buyer to trade other amounts of coverage than Q_1^* and Q_2^*. This contrasts with the case in which the planner can fully monitor trades: according to the Revelation Principle (Myerson (1979, 1982)), tariffs then need not include trades other than the ones the planner wishes to implement and, therefore, in our two-type setting, involve at most two positive amounts of coverage. That is, tariffs need not be distinguished from allocations in that case. This distinction, however, is crucial under multiple contracting.

To clarify this point, consider the configuration illustrated in Figure 2, where it is assumed that the JHG allocation is such that $Q_1^* > Q_2^* - Q_1^* > 0$ and $U_2(Q_2^*, T_2^*) > U_2(2Q_1^*, 2T_1^*)$. Suppose the only nonzero trades made available by the planner are (Q_1^*, T_1^*) and (Q_2^*, T_2^*). Now, consider the contract (q, t) as shown. This contract allows the buyer to purchase an amount of coverage less than but close to Q_1^* at a unit price lower than v. It certainly attracts type 1 and yields a strictly positive profit to an entrant offering it if it does not attract type 2. To see that this is indeed the case, observe that combining (q, t) with (Q_1^*, T_1^*)

or, a fortiori, (Q_2^*, T_2^*), leaves type 2 with a strictly lower utility than just trading (Q_2^*, T_2^*) with the planner.[17] An entrant offering the contract (q, t) can thus cream skim type 1 and make a profit. Hence, in this configuration, the JHG allocation is not, per se, entry-proof.

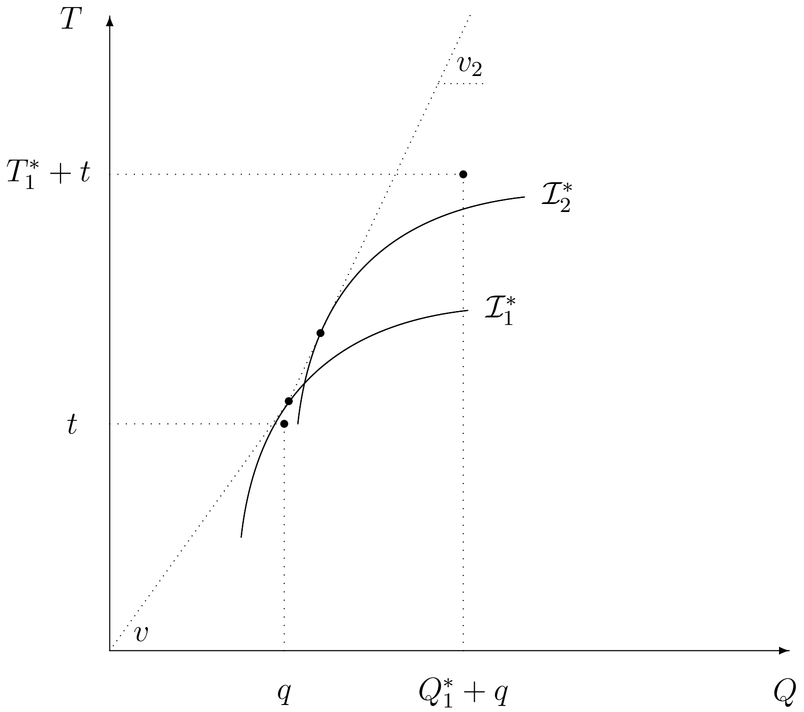

Figure 2 Cream skimming type 1.

This example shows that, to implement the JHG allocation in an entry-proof way, the planner may have to issue additional, latent contracts, which are not traded by the buyer but are only meant to block entry. For instance, in the above example, the attempt at cream skimming type 1 with contract (q, t) is defeated if the buyer can trade, in addition to it, any amount of coverage up to Q_1^* at unit price v, for then type 2 is also attracted by (q, t). Observe that, whereas such contracts may be necessary, the planner must make sure that, by merely offering them, he does not create further profitable entry opportunities. The tariff (15) strikes a balance between these two requirements.

4 Implementation

Whether incentive-compatible allocations can be implemented as equilibrium outcomes is a fundamental question for evaluating how markets perform under adverse selection. Bisin

[17]This is because $(Q_1^* + q, T_1^* + t)$ is close to $(2Q_1^*, 2T_1^*)$ and thus is, by assumption, strictly less preferred by type 2 than (Q_2^*, T_2^*), and because $(Q_2^* + q, T_2^* + t)$ is even less preferred by type 2 than $(Q_1^* + q, T_1^* + t)$ as $\tau_2(Q_1^* + q, T_1^* + t) < v_2$ and the unit price of the layer $Q_2^* - Q_1^*$ in the JHG allocation is v_2.

and Gottardi (2006) provide a positive answer in the case of fully observable trades: for any second-best allocation, there exists a system of transfers ensuring that this allocation obtains in a competitive equilibrium of an economy in which sellers offer exclusive contracts. When sellers cannot enforce exclusivity, however, the decentralization of the JHG allocation raises the issue of specifying a competitive environment in which sellers do not observe the buyer's aggregate trade. In this section, we address this issue by motivating and studying a parsimonious model of trade under adverse selection in which the JHG allocation emerges, under circumstances we characterize, as the unique equilibrium allocation.

4.1 The Nonexclusive Benchmark

As a benchmark, it is useful to start with the fully nonexclusive situation where no restrictions are imposed on the set of contractual instruments available to any seller, subject to the constraint that he cannot monitor the trades that the buyer makes with his competitors. According to the Delegation Principle (Peters (2001), Martimort and Stole (2002)), there is no loss of generality in assuming that sellers compete by offering arbitrary menus of bilateral contracts or, equivalently, arbitrary tariffs, from which the buyer is then free to choose according to her information. Attar, Mariotti, and Salanié (2014) provide a general analysis of this arbitrary-menu game for the economy studied in this paper.

The main insight of their analysis is that a positive level of trade for type 2 can be sustained in equilibrium only if type 1 is left out of the market. As a result, the only allocation that can be supported in equilibrium is a degenerate JHG allocation in which $(Q_1^*, T_1^*) = (0, 0)$. That is, type 1 purchases no coverage at the average premium rate v, while type 2 obtains full coverage at the actuarially fair rate v_2. A necessary and sufficient condition for implementation is $\tau_1(0, 0) \leq v$, that is, Akerlof's (1970) condition for a market breakdown in which only the worse-quality goods are traded. When this condition is not met, the JHG allocation satisfies $Q_1^* > 0$ and at least one seller can profitably destabilize this allocation by exploiting the buyer's ability to engage in multiple trades.

To clarify this point, consider a candidate equilibrium in which the aggregate trades for the buyer are described by a JHG allocation satisfying $Q_1^* > 0$. Each seller then earns zero expected profit. A Bertrand-like argument shows that no seller is indispensable to provide the first layer Q_1^* at unit price v. That is, the aggregate trade (Q_1^*, T_1^*) remains available if any seller unilaterally withdraws his menu offer. This observation, along with the fact that type 1 subsidizes type 2 at (Q_1^*, T_1^*), implies that any seller who is actively trading with type 1 has a profitable menu deviation consisting of two nonzero contracts. The first contract, targeted at type 1, is approximatively the same as the one the seller trades with type 1 on the candidate equilibrium path, and makes a profit when traded by type 1 only. The second contract, targeted at type 2, allows the buyer to purchase the second layer $Q_2^* - Q_1^*$ at a unit price slightly less than v_2, and makes a small loss when traded by type 2. Because the seller now offers the second layer at slightly better terms than his competitors, it is optimal for type 2 to trade it with him on top of the first layer Q_1^* provided by the other sellers at unit price v. By deviating in this way, the seller almost neutralizes his loss with type 2, while securing a profit with type 1. This amounts to dumping bad risks on one's competitors by selling complementary coverage to type 2 slightly below the fair premium rate, and basic coverage to type 1 significantly above the fair premium rate.

4.2 The Single-Contract Game

The above argument is very general and only relies on the sellers' ability to cross-subsidize between different contracts at the deviation stage. Yet, as pointed out in the Introduction, we do in practice observe insurance markets in which multiple contracting is prevalent and features the basic- versus complementary-coverage distinction. The nonexclusive benchmark thus does not seem to provide an adequate description of their working. Nevertheless, an important lesson to be drawn is that cross-subsidies between contracts must somehow be prevented for these markets to run efficiently. As a matter of fact, these markets are subject to regulation, a key aspect of which is the redistribution of costs between insurers providing basic coverage. In the light of our analysis, this can be interpreted as a way to prevent insurers from dumping bad risks on their competitors so as to boost the profits they make on good risks.

To capture in a parsimonious way the need to prevent cross-subsidies between contracts at the firm level, we follow Rothschild and Stiglitz (1976), Wilson (1977), and Hellwig (1987) in assuming that each seller offers at most one contract. The timing of the corresponding single-contract game is as follows:

1. Each seller $k \in \{1, \ldots, n\}$ offers a contract $(q^k, t^k) \in \mathbb{R}_+ \times \mathbb{R}$.

2. After privately learning her type, the buyer selects which contracts to trade with the sellers, if any.

Given a vector of contract offers $((q^1, t^1), \ldots, (q^n, t^n))$, type i's problem is then

$$\max \left\{ U_i \left(\sum_{k \in K} q^k, \sum_{k \in K} t^k \right) : K \subset \{1, \ldots, n\} \right\}, \tag{19}$$

with $\sum_{\emptyset} = 0$ by convention. We use perfect Bayesian equilibrium as our equilibrium concept. Throughout the analysis, we focus on pure-strategy equilibria.

4.3 Equilibrium: Necessary Conditions

Our first implementation result can be stated as follows.

Theorem 2 *Any equilibrium of the single-contract game implements the JHG allocation.*

In combination with Theorem 1, Theorem 2 provides a version of the First Welfare Theorem for our economy: if the single-contract game has an equilibrium, this equilibrium implements the unique robustly incentive-compatible allocation.

Remark The characterization provided in Theorem 2 differs from the earlier contributions of Jaynes (1978), Hellwig (1988), and Glosten (1994) in three crucial ways. First, sellers in our trading game cannot exchange information about the buyer. This contrasts with Jaynes (1978) and Hellwig (1988), in which such communication is explicitly allowed for and indeed plays a key role in the characterization of equilibrium. Second, sellers in our trading game cannot react to the offers of their competitors. This contrasts with Hellwig (1988), who considers a specific sequential timing for the sellers' offers. Third, our analysis is fully strategic. This contrasts with Glosten (1994), whose analysis is entirely based on free-entry arguments.

A direct implication of Theorem 2 is that any equilibrium of the single-contract game involves zero expected profit for the sellers. An easy corollary is that any traded contract is issued at unit price v or v_2: that is, each active seller either provides basic coverage to both types at the average premium rate, or complementary coverage to type 2 only at the fair premium rate.

An important insight of our analysis is that, as in standard Bertrand competition, no seller is indispensable in providing types 1 and 2 with their equilibrium utilities: otherwise, he could earn a strictly positive expected profit by slightly increasing his price. Specifically, we show in Appendix A that, if any seller withdraws his contract offer, type 1 can still trade (Q_1^*, T_1^*), while type 2 can still obtain her equilibrium utility by purchasing an amount of coverage at least equal to Q_2^*. Thus any equilibrium of the single-contract game features free entry, in the sense that at least one seller is inactive on the equilibrium path.

To examine the implications of this dispensability property, and to focus on the most relevant scenario for applications, we hereafter restrict attention to the case where the two layers Q_1^* and $Q_2^* - Q_1^*$ are positive. That is, the JHG allocation is such that both types are actively trading and there is no pooling. The following assumption on gains from trade (GT) is accordingly maintained in the remainder of the analysis.

Assumption GT *$\tau_1(0,0) > v$ and $\tau_2(Q_1^*, vQ_1^*) > v_2$.*

Assumption GT is implicit in the insurance models of Jaynes (1978) and Hellwig (1988). In fact, in these models where the loss size is the same for both types, the second half of Assumption GT automatically holds as the first layer Q_1^* entails less than full coverage. The same property is satisfied in the coinsurance example. In the case where the first half of Assumption GT does not hold, the JHG allocation is degenerate with $(Q_1^*, T_1^*) = (0,0)$ and is, as shown by Attar, Mariotti, and Salanié (2014), the unique equilibrium allocation of the arbitrary-menu game.

Under Assumption GT, multiple contracting must take place in equilibrium. Indeed, as any traded contract is issued at unit price v or v_2, type 2 must buy the first layer Q_1^* from a first group of sellers, and the second layer $Q_2^* - Q_1^*$ from a second group of sellers. Interestingly, the existence of an equilibrium imposes additional restrictions on the relative size of these layers. To see why, note from the dispensability property that, if $Q_1^* > 0$, the sellers' aggregate supply at the relatively low price v must exceed Q_1^*. This excess supply has no value for type 1, as Q_1^* is her demand at price v. However, it could be attractive for type 2, who may be interested in purchasing more basic coverage at the average premium rate. A necessary condition for the existence of equilibrium is thus that type 2 be not willing to exploit these additional trading opportunities. A second necessary condition is that it be impossible for a deviating buyer to profitably take advantage of them. These two conditions can be formulated as follows.

Corollary 1 *If the single-contract game has an equilibrium, then the JHG allocation satisfies*

$$U_2(Q_2^*, T_2^*) \geq U_2(2Q_1^*, 2T_1^*), \tag{20}$$

$$Q_1^* > Q_2^* - Q_1^*. \tag{21}$$

Conditions (20)–(21) are most easily understood when only two sellers issue contracts at unit price v. Because none of them is indispensable, they must each offer a contract equal to type 1's equilibrium aggregate trade (Q_1^*, T_1^*). Condition (20) then simply expresses that type 2 is not willing to trade (Q_1^*, T_1^*) twice on the equilibrium path. Consider now what happens if condition (21) does not hold. Then some other seller can attract type 2 by offering her to purchase an amount of coverage $Q_2^* - 2Q_1^*$ at a unit price slightly above v_2. Indeed, combined with the trade $(2Q_1^*, 2T_1^*)$ made available by the two sellers issuing contracts at unit price v, this offer allows type 2 to pay less than T_2^* for her equilibrium coverage Q_2^*. As this deviation is clearly profitable, condition (21) must hold for an equilibrium to exist.[18] This logic easily extends when more than two sellers issue contracts at unit price v.

Geometrically, conditions (20)–(21) hold when the aggregate trade $(2Q_1^*, 2T_1^*)$ is located in the lower contour set of (Q_2^*, T_2^*) for type 2, to the right of (Q_2^*, T_2^*), as illustrated in Figure 2. This requires that the second layer $Q_2^* - Q_1^*$ that type 2 wants to trade at unit price v_2 be sufficiently small relative to the first layer Q_1^* that both types want to trade at unit price v. In the Rothschild–Stiglitz economy or in the coinsurance example, this is the case whenever the loss distributions of type 1 and type 2 are not too different. The testable implications of these conditions are discussed at greater length in Section 4.5.

4.4 Equilibrium: Sufficient Conditions

Theorem 2 singles out the JHG allocation as the unique candidate equilibrium allocation of the single-contract game. We now investigate how to construct an equilibrium that indeed implements this allocation. It is throughout assumed that Assumption GT is satisfied, as well as the necessary conditions (20)–(21).

We pointed out in Section 3.4.3 that, in these circumstances, cream skimming is possible if the only feasible trades are (Q_1^*, T_1^*) and (Q_2^*, T_2^*). This more generally holds if all contracts are issued at unit price v or v_2. As this holds for all traded contracts in any equilibrium of the single-contract game, these contracts must be complemented by latent contracts. Our goal in this section is to characterize a single latent contract that allows for a minimal implementation of the JHG allocation. The characterizing property is as follows.

Definition 4 *A contract (q^ℓ, t^ℓ) deters cream skimming if, for any contract (q, t),*

$$U_1(q,t) \geq U_1(Q_1^*, T_1^*) \quad \textit{implies} \quad U_2(q+q^\ell, t+t^\ell) \geq U_2(Q_2^*, T_2^*). \tag{22}$$

Geometrically, (22) states that the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector (q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. This implies that any contract (q, t) that attracts type 1 also attracts type 2 in combination with (q^ℓ, t^ℓ) when the latter contract is offered by some other seller. Then (q, t) cannot be a profitable

[18] Note that $Q_2^* \neq 2Q_1^*$ if an equilibrium exists. Otherwise, $2T_1^* = 2vQ_1^* < vQ_1^* + v_2(Q_2^* - Q_1^*) = T_2^*$ and type 2 is better off trading $(2Q_1^*, 2T_1^*)$ instead of (Q_2^*, T_2^*).

deviation, as it must have a unit price at most equal to v to attract type 1. Our first result shows that there is a single candidate for (q^ℓ, t^ℓ), characterized by

$$U_2(Q_1^* + q^\ell, T_1^* + t^\ell) = U_2(Q_2^*, T_2^*), \tag{23}$$

$$\tau_2(Q_1^* + q^\ell, T_1^* + t^\ell) = v. \tag{24}$$

Specifically, the following result holds.

Lemma 1 *The contract (q^ℓ, t^ℓ) characterized by (23)–(24) is, among the contracts issued but not traded in equilibrium, the only one susceptible to deter cream skimming.*

We now provide a sufficient condition for (22), stated in terms of the Gaussian curvatures

$$\kappa_i \equiv \frac{1}{\|\partial U_i\|^3} \begin{vmatrix} -\partial^2 U_i & \partial U_i \\ -\partial U_i^\top & 0 \end{vmatrix}$$

of type 1's and type 2's indifference curves (Debreu (1972)).

Assumption C *If $\tau_1(Q_1, T_1) = \tau_2(Q_2, T_2)$, then $\kappa_1(Q_1, T_1) > \kappa_2(Q_2, T_2)$.*

Assumption C can also be phrased in terms of the Hicksian demand functions $H_i(p, \mathbf{u}) \equiv \min\{pQ - T : U_i(Q, T) \geq \mathbf{u}\}$ associated to each type's preferences. Indeed, it is equivalent to the property that $H_2(p, \mathbf{u_2}) - H_1(p, \mathbf{u_1})$ is strictly decreasing in p for all $\mathbf{u_1}$ and $\mathbf{u_2}$; that is, type 2's Hicksian demand is more sensitive than type 1's to changes in the price of insurance, whatever utility levels are used as references. This occurs whenever type 2's indifference curves are flatter than type 1's, once these curves are translated so as to make them tangent at the point under study.[19]

The following result then holds.

Lemma 2 *If the buyer's preferences satisfy Assumption C, the contract (q^ℓ, t^ℓ) characterized by (23)–(24) deters cream skimming.*

Given the role it plays in our construction, it is natural to ask how restrictive Assumption C is. A limiting case arises in the Rothschild–Stiglitz economy if the utility function in (2) has constant absolute risk aversion, so that the buyer's Hicksian and Marshallian demand functions coincide.[20] One can check that any pair of indifference curves for types 1 and 2 are then translates of each other, which implies that the contract (q^ℓ, t^ℓ) deters cream

[19]Observe the difference with the single-crossing condition, which does not allow for translations.

[20]Einav, Finkelstein, and Cullen (2010) similarly abstract from income effects in their estimation of the welfare cost of adverse selection.

skimming.[21] More generally, Assumption C is satisfied if type 1 is uniformly more risk-averse than type 2 in the sense of Aumann and Serrano (2008). This, however, creates a tension with Assumption SC, which requires that type 2 be more eager to buy insurance than type 1. If each type i has constant absolute risk aversion α_i, these assumptions are jointly satisfied if

$$0 > \alpha_2 - \alpha_1 > \frac{1}{L} \ln\left(\frac{(1 - v_2)/v_2}{(1 - v_1)/v_1}\right),$$

that is, type 2 is less risk-averse but sufficiently riskier than type 1. We are now ready to state our minimal-implementation result.

Theorem 3 *Suppose the buyer's preferences satisfy Assumptions GT and C, as well as conditions (20)–(21). Then, if there are sufficiently many sellers, the single-contract game has an equilibrium.*

Our equilibrium construction relies on three contracts: a basic-coverage contract, (Q_1^*, T_1^*), a complementary-coverage contract, $(Q_2^* - Q_1^*, T_2^* - T_1^*)$, and the latent contract (q^ℓ, t^ℓ). The requirement that there be sufficiently many sellers reflects that no seller can be indispensable in offering basic or complementary coverage, and that a large enough number of copies of the latent contract must be available.

In combination with Theorems 1–2, Theorem 3 provides a version of the Second Welfare Theorem for our economy: under additional conditions on the buyer's preferences, the single-contract game has an equilibrium, and this equilibrium implements the unique robustly incentive-compatible allocation.

4.5 Testable Implications of Equilibrium

Taking into account multiple contracting yields new testable implications, compared to those of the exclusive-contracting benchmark. The key difference is that in the latter case, whether data are collected from consumers' surveys or from the trade records of a single insurer makes no difference, because each consumer's demand for coverage must be met by a single contract, sold by a single insurer. However, when multiple contracting is allowed, the second approach yields strikingly different results, as we now argue.

Since Chiappori and Salanié (2000), many empirical studies have tested the validity of the positive-correlation property, which states that, under asymmetric information, there

[21] The same property of indifference curves is satisfied when the buyer's preferences have the quadratic representation (4).

should be a positive correlation between the coverage purchased by a consumer and her risk. Due to the single-crossing assumption, this property still holds in our setting when one considers the aggregate coverage bought by a consumer: indeed, riskier consumers are also those that are more eager to buy more insurance.[22] The Rothschild and Stiglitz (1976) model also yields the prediction that, in a separating equilibrium, the unit price of coverage should increase with coverage, as each consumer pays the fair price, v_1 or v_2, associated to his type. Under multiple contracting, this property also holds for the JHG allocation, though in a less clear-cut manner. Indeed, the unit price paid by the low-risk consumer is v, while the unit price paid by the high-risk consumer lies between v and v_2, so that the difference is bound to be lower than $v_2 - v_1$. In other words, multiple contracting reduces the convexity of the tariff for aggregate coverage, as observed on consumers' surveys.

However most studies do not rely on consumers' surveys that are often imprecise and limited in size, but instead use records from a subset of insurance companies. We now examine what difference it makes under multiple contracting, based on our analysis of the single-contract game. Recall that our equilibrium construction relies on the basic-coverage contract (Q_1^*, T_1^*), with unit price v, the complementary-coverage contract $(Q_2^* - Q_1^*, T_2^* - T_1^*)$, with unit price v_2, and the latent contract (q^ℓ, t^ℓ). Now, under conditions (20)–(21), which are necessary for an equilibrium to exist, the following inequalities hold:

$$Q_1^* > q^\ell > Q_2^* - Q_1^* \quad \text{and} \quad \frac{T_1^*}{Q_1^*} < \frac{t^\ell}{q^\ell} < \frac{T_2^* - T_1^*}{Q_2^* - Q_1^*}.$$

That is, *contracts that offer higher amounts of coverage have a lower unit price.* Therefore, a testable implication of equilibrium is that, although consumers end up paying quantity premia for their aggregate coverage, the individual contracts offered by insurers exhibit quantity discounts.

This striking result stands in stark contrast with the natural intuition that allowing for multiple contracting should push consumers towards splitting their demands between insurers.[23] The reason why, in our competitive setting, insurers end up proposing quantity discounts is that the basic layer of coverage must be larger than the complementary one to prevent high-risk consumers from trading several basic policies with different insurers.[24] Each consumer then finds in her own interest to concentrate her trades on a minimum

[22]Chiappori, Jullien, Salanié and Salanié (2006) show that this property, and similar ones, can also be derived in much more general settings from a simple inequality on equilibrium profits, even when single crossing is not postulated.

[23]See, for instance, Chiappori (2000) for an articulation of this view.

[24]This explanation for quantity discounts differs from that proposed by Biais, Martimort, and Rochet (2000) and Chade and Schlee (2012) when they study the case of a monopolistic insurer as in Stiglitz (1977). In these papers, the key role is played by properties of the hazard rate of the distribution of types.

number of contracts. The key is that insurers together only offer a few contracts, which the consumer can combine. Low-risk consumers then end up trading a single contract, while high-risk consumers end up trading two different contracts.

This contrast, under multiple contracting, between the implications of equilibrium for the demand and supply sides of the market is also relevant for the positive-correlation property. Indeed, a novel testable implication of equilibrium is that, with data originating from a single insurer, *one should now observe a negative correlation between risk and coverage*, because the relatively small complementary layer of coverage is only purchased by high-risk consumers. Finally, a robust prediction of our analysis is that, conditionally on buying basic coverage, a consumer should on average appear as riskier if she also buys further coverage from another insurer: consumers holding more than one insurance policy are on average more likely to experience a greater level of loss.

These observations are useful when considering the empirical evidence, as exemplified by the work of Cawley and Philipson (1999) on life insurance, that of Finkelstein and Poterba (2004) on annuities, or that of Finkelstein and McGarry (2006) on long-term care. Because the reference model in those papers is the exclusive-competition model of Rothschild and Stiglitz (1976), the distinction between demand- and supply-side approaches stressed above is overlooked. As a result, the absence of quantity premia or the failure of the positive-correlation property are interpreted as rejecting the presence of adverse selection on life-insurance, annuity, and long-term care markets. However, because multiple contracting is allowed and even prevalent on these markets, one must be very careful when testing for the existence of quantity premia or for the positive-correlation property: in principle, one would need to observe, for each consumer, her aggregate coverage and her aggregate premium. In particular, checking only the contracts offered by insurers or the contracts sold by a given insurer can be insufficient and even misleading. A careful empirical analysis is beyond the scope of the present paper, but it would certainly be worth proceeding to this task while taking all precautions to ensure that data are comprehensive.[25]

5 Public Intervention

In this section, we discuss why, in the light of our analysis, public intervention in insurance markets may be needed and which forms it can take. We then argue that our results are

[25] At least one of the econometric treatments performed in Cawley and Philipson (1999) seems to escape this criticism, as it is based on a consumer survey (AHEAD) that includes information on aggregate demand. For a recent and positive test for adverse selection using the same data, see He (2009).

more generally relevant for public intervention in financial markets.

5.1 Why Markets May Fail

Our implementation of the JHG allocation suggests that the observed prevalence of multiple contracting in insurance markets can in principle be reconciled with the existence of adverse selection. Yet competition under adverse selection may fail to lead to an equilibrium in a relevant set of circumstances. First, the equilibrium construction provided in Section 4.4 crucially exploits the properties of a specific class of consumers' preferences. Second, and more fundamentally, the necessary conditions for equilibrium derived in Section 4.3 impose severe restrictions on each consumer type's willingness to trade. In particular, high-risk consumers should find it optimal to trade only a relatively small layer of complementary coverage on top of the layer of basic coverage.

Overall, the impossibility to prevent consumers from simultaneously trading with several insurers fundamentally alters the standard view of decentralization. On the one hand, as discussed in Section 3.4, redistribution between different consumer types is made impossible: the JHG allocation is the only budget-balanced allocation implementable by an entry-proof tariff. On the other hand, the market mechanism, based on competition among private insurers, may fail to perform its allocative role. Thus public intervention may be needed to implement the only allocation robust to competition from the private sector.

Theoretically, a straightforward way to implement the JHG allocation is for the state to post the tariff (15). The supply of public insurance then effectively dissuades any private insurer from proposing additional trades. However, public intervention need not involve such a complete crowding out of the private sector, and can instead help stabilize the market. Indeed, as we now discuss, our multiple-contracting framework can accommodate several forms of coexistence between public and private insurance. This may, in particular, shed light on several existing health-insurance systems in which private coverage complements public insurance.[26]

5.2 Mandatory Insurance

Different forms of mandatory health insurance, whereby consumers are not allowed to remain uninsured, are in place in France, Germany, Japan, Netherlands, and Switzerland. The

[26] In this context, private coverage is said to complement public insurance when it covers all or part of the residual nonreimbursed costs in the form of copayment or cost sharing. We refer to the surveys of Thomson and Mossialos (2009) and Thomson, Osborne, Squires, and Jun (2013) for institutional details and cross-country evidence.

modalities of mandatory insurance vary from country to country. It can be publicly provided, as in France; privately provided, as in Japan, Netherlands, or Switzerland; or consumers can have the choice to opt out from the public-insurance system to buy basic coverage designed and priced by private insurers, as in Germany. Mandatory insurance can be complemented by additional, privately-provided coverage, such as *mutuelles* in France.

In the context of our model, one can think of mandatory insurance as a situation in which it is compulsory for each consumer to purchase the layer of basic coverage Q_1^* at price T_1^*, independently of any additional coverage that she may privately purchase. The contract (Q_1^*, T_1^*) can be provided by the state or by private insurers in a cost-effective way. Given this obligation, private insurers then engage in Bertrand competition for complementary insurance services.

In equilibrium, at least two insurers stand ready to sell any amount of complementary coverage at the fair premium rate v_2, allowing high-risk consumers to trade according to the JHG allocation. Mandatory insurance acts as a threat against deviations and entry. Indeed, to profitably attract low-risk consumers, an insurer should issue a contract that, in combination with (Q_1^*, T_1^*), yields them at least utility $U_1(Q_1^*, T_1^*)$. In this case, however, the deviating contract is also traded by high-risk consumers, who can complement it with additional coverage provided by some other insurer. Equilibrium existence is thus restored and the market fulfills its allocative role: basic mandatory coverage is either provided by the state or the private sector, while complementary coverage is provided by the private sector.[27]

5.3 Public Versus Private Insurance

Mandatory health insurance schemes require to identify and penalize both the consumers who choose to remain uninsured, and the insurers who deny coverage to some consumers. In practice, this raises the question of their enforceability.[28] Moreover, allowing consumers to opt out from the public-insurance system and turn instead to private insurers for basic coverage, as in Germany, creates an incentive to cream skim low-risk consumers.[29]

[27]The role of mandatory insurance under adverse selection has been so far only analyzed under exclusive contracting. Mandatory insurance is evoked in Akerlof (1970), and has been the focus of much empirical work (Finkelstein (2004), Einav, Finkelstein, and Cullen (2010), Einav and Finkelstein (2011)). Wilson (1977), Dahlby (1981), and Crocker and Snow (1985a) show that making basic coverage mandatory and simultaneously allowing private insurers to compete on an extended coverage allows one to reach a second-best outcome. Villeneuve (2003) performs a similar analysis under nonexclusive contracting, but in a model that assumes linear pricing.

[28]A key reference is provided by the recent discussion of the system of penalties associated to the Affordable Care Act (see https://www.treasury.gov/tigta/auditreports/2015reports/201543030fr.pdf).

[29]In line with this point, Thomson, Osborne, Squires, and Jun (2013, page 57) note that "Especially for young people with a good income, [...] [privately offered basic coverage] [...] is attractive, as the insurance

Our analysis, however, suggests an alternative policy proposal that does not require such a legal requirement. Indeed, a less intrusive form of public intervention has the state offering any amount of basic coverage up to Q_1^* at the average premium rate v. As any private insurer is ready to sell any amount of coverage at the high premium rate v_2, the state together with any of them make available the entry-proof tariff (15). Thus no private insurer has an incentive to deviate and entry is impossible. The JHG allocation is thereby implemented by a mix of public and private insurance, while letting consumers free to choose their preferred level of coverage.

Public intervention thus need not interfere with the choices of consumers, who can remain sovereign in their decisions to purchase insurance. Neither are taxes or subsidies needed.[30] This contrasts with policy recommendations from exclusive models of competitive insurance markets under adverse selection: in our setting, competition is powerful enough to select a unique equilibrium in which prices efficiently reflect costs—though this rule applies to successive layers of insurance and not to the aggregate coverage bought by each type of consumer.

5.4 Financial Markets

In the aftermath of the recent crisis, the design of public intervention under the threat of adverse selection has become a central issue for the regulation of credit and interbank markets. In particular, the opportunity for agents to opt out of a public program and trade in private markets has been acknowledged as a key constraint for the design of financial institutions. In this respect, the recent works of Philippon and Skreta (2012) and Tirole (2012) suggest a foundation for liquidity-injection programs that provide a credible signal to uninformed lenders by rejuvenating the relevant markets. The logic underlying their results can be summarized as follows. First, assuming that lenders and informed borrowers have linear preferences over the traded assets, market equilibria in the absence of any intervention feature the market unraveling originally described by Akerlof (1970). Second, assuming that public and private liquidity are mutually exclusive, an optimal intervention consists in the state attracting only the least profitable borrowers, either through direct lending (Philippon and Skreta (2012)), or by repurchasing low-quality assets (Tirole (2012)). By participating in a bailout program, a borrower may however end up signalling her financial weakness to the market, crucially affecting her reputation. A large literature has analyzed the potentially

may offer contracts with more extensive ranges of services and lower premiums."

[30] See Crocker and Snow (1985b) for a study of taxes and subsidies under exclusive contracting.

perverse implications of such a stigma effect.[31]

In modern financial markets, however, borrowers' choices are not limited to opting out of a public program or exclusively participating to it, because they have the opportunity to complement such a program with additional funds raised on private markets. Focusing on the US interbank market over the 2007–2010 period, Armantier, Ghysels, Sarkar, and Shrader (2015) document how banks combine their loans at the Fed Discount Window with additional funding raised on ABCP and Repo markets, which exhibit similar lending terms with respect to eligibility, collateral and maturity.[32] To the extent that these practices create an effective threat for the state, our results offer novel insights for the design of public intervention under adverse selection. In these contexts, as long as borrowers are risk-averse, the absence of any intervention may imply nonexistence of a market equilibrium.[33] Thus, public intervention is not needed to unfreeze the market, but rather to guarantee its functioning. To achieve this task, a program must successfully discipline lenders' strategic behavior, preventing them from engaging in dumping practices. This would require public liquidity provision to involve a price sufficiently low, v, so as to attract the most profitable borrowers, corresponding to type-1 buyers in our model, and a maximum available quantity Q_1^* such that no overborrowing by the least profitable ones is possible. Overall, such an intervention would achieve budget balance, unlike those proposed by Philippon and Skreta (2012) and Tirole (2012), and induce all types of borrowers to participate. This in turn would make it harder to infer their individual financial conditions, mitigating the impact of the stigma effect.

6 Conclusion

In modern economies, the insurance sector plays a key role by allowing agents to share risk. Because those risks are often private information, the properties of equilibrium allocations, and in fact the very existence of equilibrium, are still the subject of a lively debate among academics. The absence of consensus on the justifications and on the right design of public intervention may also be related to the fact that different countries display strikingly different regulatory systems for, in particular, health insurance. In this paper, we have proposed to put multiple contracting at the center stage of the analysis. We have characterized a robustly

[31] See Gorton (2015) for a survey.

[32] Relatedly, Berger, Black, Bouwman, and Dlugosz (2016) show how, in the same period, the banks' relying on both the Fed's Discount Window and Term Auction Facility liquidity programs significantly increased their aggregate lending.

[33] In the terminology of Hendren (2014), this is an instance of unraveling of market equilibrium.

incentive-compatible allocation, and we have analyzed whether and how competition between insurers allows to implement it. Our equilibrium construction opens a new and rich avenue for empirical research. We also hope that our policy proposals may renew the existing policy debates about health insurance, and more generally about the management of financial markets that operate under adverse selection.

Appendix A: Proofs of the Main Results

Proof of Theorem 1. The only thing that remains to prove is that, given the tariff T^P offered by the planner and the tariff T^E offered by the entrant, there exist a pair of solutions (q_1^P, q_1^E) and (q_2^P, q_2^E) to (5) for $i = 1, 2$ such that $q_2^E \geq q_1^E$. In line with Attar, Mariotti, and Salanié (2015), notice that each type i evaluates any pair (q^E, t^E) she may trade with the entrant through the indirect utility function

$$z_i(q^E, t^E) \equiv \max\{U_i(q^P + q^E, T^P(q^P) + t^E) : q^P \geq 0\}. \tag{25}$$

Observe that the maximum in (25) is always attained and that, if (q_i^P, q_i^E) is a solution to (5), then q_i^E maximizes $z_i(q^E, T^E(q^E))$ with respect to q^E. As shown in Attar, Mariotti, and Salanié (2015), the convexity of the tariff T^P and Assumption SC together imply that the functions z_i satisfy the following single-crossing property: for all $\underline{q}^E \leq \overline{q}^E$, $\underline{t}^E$, and $\overline{t}^E$,

$$z_1(\underline{q}^E, \underline{t}^E) < z_1(\overline{q}^E, \overline{t}^E) \quad \text{implies} \quad z_2(\underline{q}^E, \underline{t}^E) < z_2(\overline{q}^E, \overline{t}^E). \tag{26}$$

We then obtain the desired result by a standard monotone-comparative-statics argument: indeed, suppose, by way of contradiction, that $q_2^E < q_1^E$ at any pair of solutions (q_1^P, q_1^E) and (q_2^P, q_2^E) to (5) for $i = 1, 2$. Therefore, at any such pair, $z_1(q_2^E, T^E(q_2^E)) < z_1(q_1^E, T^E(q_1^E))$, so that, according to (26), $z_2(q_2^E, T^E(q_2^E)) < z_2(q_1^E, T^E(q_1^E))$. But then, according to the above observation, trading q_2^E with the entrant cannot be part of a solution to (5) for type 2, a contradiction. Hence the result. ■

Proof of Theorem 2. Suppose an equilibrium exists, and let $((Q_1, T_1), (Q_2, T_2))$ be the equilibrium allocation. On the equilibrium path, we can partition the set of sellers into $K_{\emptyset\emptyset}$, the subset of sellers who trade with neither type 1 nor type 2, $K_{1\emptyset}$, the subset of sellers who trade with type 1 only, $K_{\emptyset 2}$, the subset of sellers who trade with type 2 only, and K_{12}, the subset of sellers who trade with both type 1 and type 2. If the subset $K_{\emptyset\emptyset}$ of inactive sellers is nonempty, then any such seller can behave as an entrant, and the first step of the proof of Theorem 1 implies that the equilibrium allocation $((Q_1, T_1), (Q_2, T_2))$ coincides with the JHG allocation $((Q_1^*, T_1^*), (Q_2^*, T_2^*))$. The bulk of the argument consists in showing that any candidate equilibrium in which $K_{\emptyset\emptyset}$ is empty and hence all sellers are active also implements the JHG allocation. We later show that this property actually implies that such an equilibrium cannot exist, and thus that any equilibrium involves some inactive sellers. The proof consists of three steps.

Step 1 Suppose first that, in the candidate equilibrium under consideration, $K_{1\emptyset}$ is empty and there are at least two sellers in K_{12}. Then the buyer's aggregate trade (Q_{12}, T_{12}) with the sellers in K_{12} coincides with (Q_1, T_1). Now, each seller $k \in K_{12}$ can claim the aggregate expected profit $T_1 - vQ_1$ by deviating to $(Q_1, T_1 - \varepsilon)$, for some positive and small enough ε. But then, denoting by b^k seller k's equilibrium expected profit, it follows that $b^k \geq T_1 - vQ_1 = \sum_{k' \in K_{12}} b^{k'}$ for all $k \in K_{12}$, so that any seller $k \in K_{12}$ earns zero expected profit, $b^k = t^k - vq^k = 0$, and $T_1 = vQ_1$.

This observation implies that, in analogy with (11),

$$U_1(Q_1, T_1) \geq \max\{U_1(Q, vQ) : Q \geq 0\}. \tag{27}$$

Otherwise, any seller in K_{12} can deviate to $(Q_1^*, vQ_1^* + \varepsilon)$, for some positive and small enough ε, a contract that attracts type 1 and is profitable even if it also attracts type 2, a contradiction. Hence (27) holds. Additionally, we have $T_1 = vQ_1$, so that (27) holds with equality. This uniquely pins down Q_1, which must coincide with Q_1^* by (7).

Now, because Q_1^* is the demand of type 1 at price v and the sellers in K_{12} trade (Q_1^*, vQ_1^*) in the aggregate, any seller $k \in K_{12}$ is indispensable for type 1 to reach her equilibrium utility. Indeed, if some seller $k \in K_{12}$ withdraws his contract offer (q^k, vq^k), type 1 can only trade with the other sellers in K_{12}, who sell at unit price v but whose aggregate supply is strictly less than Q_1^*, or with the sellers in K_2, who sell at a unit price no less than $v_2 > v$. Therefore, if seller k deviates to $(q^k, vq^k + \varepsilon)$, for some positive and small enough ε, he still attracts type 1. Yet this contract is profitable even if it also attracts type 2, a contradiction. This shows that, in any candidate equilibrium in which all sellers are active, one cannot simultaneously have $K_{1\emptyset}$ empty and at least two sellers in K_{12}.

Step 2 A direct implication of Step 1 is that $K_{\emptyset 2}$ is nonempty in any candidate equilibrium in which all sellers are active. Otherwise $Q_2 = Q_{12}$, which, as $Q_2 \geq Q_1$ by Assumption SC and $Q_1 \geq Q_{12}$ by construction, implies that $Q_1 = Q_2 = Q_{12}$; but then all sellers belong to K_{12}, which contradicts Step 1. We now discuss the possible cases for $K_{\emptyset 2}$, starting in this step with the case in which it contains a single seller k.

In this case, seller k behaves as a monopolist for the layer $Q_2 - Q_{12}$. In particular, he cannot increase t^k, so that the sellers other than k must offer some aggregate trade (Q^{-k}, T^{-k}) such that $U_2(Q^{-k}, T^{-k}) = U_2(Q_2, T_2)$. As these sellers are exactly those who sell to type 1, one must have $Q^{-k} \leq Q_1$. Moreover, Assumption SC applied to the case where the buyer only faces the sellers other than k implies that one must have $Q^{-k} \geq Q_1$. It follows that $Q^{-k} = Q_1 \leq Q_2$ and thus that $U_2(Q_1, T_1) = U_2(Q_2, T_2)$.

We next prove that $Q_2 > Q_1 > Q_{12}$. The proof is by contradiction, starting from the observation that $Q_2 \geq Q_1 \geq Q_{12}$. First, if $Q_1 = Q_2$, then the layer $Q_1 - Q_{12} = Q_2 - Q_{12}$ is traded both by seller(s) in $K_{1\emptyset}$ and by the single seller k in $K_{\emptyset 2}$, so that one must have $T_1 = T_2$. But then seller k can slightly reduce t^k in order to profitably attract both types, a contradiction. Hence $Q_2 > Q_1$. Second, if $Q_{12} = Q_1$, then $K_{1\emptyset}$ is empty. However, by assumption, all $n \geq 3$ sellers are active and there is a single seller in $K_{\emptyset 2}$. Thus there must be at least two sellers in K_{12} if $K_{1\emptyset}$ is empty, which contradicts Step 1. Hence, overall, $Q_2 > Q_1 > Q_{12}$, as claimed.

It follows from this observation that the single seller k in $K_{\emptyset 2}$ can, instead of offering $(Q_2 - Q_{12}, T_2 - T_{12})$, deviate to $(Q_2 - Q_1, T_2 - T_1 - \varepsilon)$, for some positive ε. This contract attracts type 2, along with the contracts proposed by the sellers in $K_{1\emptyset} \cup K_{12}$. Hence one must have, letting ε go to zero,

$$T_2 - T_{12} - v_2(Q_2 - Q_{12}) \geq T_2 - T_1 - v_2(Q_2 - Q_1)$$

or, equivalently,

$$T_1 - T_{12} \geq v_2(Q_1 - Q_{12}). \tag{28}$$

Thus sellers in $K_{1\emptyset}$ sell at a unit price at least equal to v_2. This, in turn, implies that K_{12} is empty. Otherwise, any seller in $k' \in K_{12}$ can deviate to $(q^{k'} + Q_1 - Q_{12}, t^{k'} + T_1 - T_{12} - \varepsilon)$, for some positive ε. This contract attracts type 1, along with the contracts proposed by the sellers in K_{12} other than k'. Moreover, because $U_2(Q_2, T_2) = U_2(Q_1, T_1)$, it also attracts type 2, along with the same contracts. As the layer $Q_1 - Q_{12}$ is sold at a unit price at least equal to v_2, this deviation is profitable for ε small enough, a contradiction. Hence K_{12} is empty, as claimed.

Because, by assumption, all $n \geq 3$ sellers are active and there is a single seller in $K_{\emptyset 2}$, and because, as just shown, K_{12} is empty, there must be at least two sellers in $K_{1\emptyset}$. Any such seller can deviate to $(Q_1, T_1 - \varepsilon)$, for some positive ε, thus attracting both types as $U_2(Q_2, T_2) = U_2(Q_1, T_1)$. Therefore, one must have, for each $k' \in K_{1\emptyset}$,

$$m_1(t^{k'} - v_1 q^{k'}) \geq T_1 - vQ_1.$$

Summing over $k' \in K_1$ yields

$$m_1(T_1 - v_1 Q_1) \geq |K_1|(T_1 - vQ_1)$$

or, equivalently,

$$0 \geq (|K_1| - 1)(T_1 - vQ_1) + m_2(T_1 - v_2 Q_1).$$

But, as $|K_1| \geq 2$ and $T_1 \geq v_2 Q_1$ by (28) along with the fact that K_{12} is empty, the first term on the right-hand side of this inequality is strictly positive and the second term is nonnegative, a contradiction. Hence there exists no equilibrium in which all sellers are active and $K_{\emptyset 2}$ contains a single seller.

Step 3 Suppose finally that there are at least two sellers in $K_{\emptyset 2}$. These sellers can undercut each other to attract type 2, so that they earn zero profit. Hence any of them can behave as an entrant to attract type 1, from which (27) follows. Moreover, in analogy with (9),

$$U_2(Q_2, T_2) \geq \max\{U_2(Q_1 + Q, T_1 + v_2 Q) : Q \geq 0\}. \tag{29}$$

Otherwise, letting Q be the solution to the problem on the right-hand side of (29), any seller in $K_{\emptyset 2}$ can deviate to $(Q, v_2 Q + \varepsilon)$, for some positive and small enough ε, a contract that attracts type 1 and is profitable even if it also attracts type 2, a contradiction. Thus both (27) and (29) hold, and one can conclude as in the proof of Theorem 1 that the equilibrium allocation coincides with the JHG allocation. Hence the result.

We provide below additional properties that are satisfied by any equilibrium, and that are mentioned in the discussion of Theorem 2.

First, because the equilibrium allocation coincides with the JHG allocation, which makes zero expected profit, each seller earns zero expected profit in equilibrium. Hence the sellers in K_{12} sell at unit price v and the sellers in $K_{\emptyset 2}$ sell at unit price v_2. If there are some sellers in $K_{1\emptyset}$, then they must sell at unit price v_1. But, as the coverage Q_1^* for type 1 is jointly provided at unit price v by the sellers in $K_{1\emptyset} \cup K_{12}$, it follows that $K_{1\emptyset}$ is empty. As a result, any traded contract is issued at unit price v or v_2. Hence, in any equilibrium, the situation is the following:

1. If $Q_1^* > 0$, then sellers in K_{12} together supply Q_1^* at unit price v. None of these sellers can be indispensable for type 1 to reach her equilibrium utility, as characterized by (7). Therefore, for each $k \in K_{12}$, the sellers other than k must offer some aggregate trade (Q^{-k}, T^{-k}) such that $U_1(Q^{-k}, T^{-k}) = U_1(Q_1^*, T_1^*)$. We prove below that $Q^{-k} = Q_1^*$.[34] Thus, if any seller in K_{12} withdraws his contract offer, type 1 can still trade (Q_1^*, T_1^*). As the other active sellers cannot together propose (Q_1^*, T_1^*), this shows that $K_{\emptyset\emptyset}$ is nonempty: any equilibrium involves inactive players and thus features free entry.

To prove the claim, suppose, by way of contradiction, that $Q^{-k} \neq Q_1^*$. Then, from (7)–(8) and the strict quasiconcavity of U_1, one must have $T^{-k} - T_1^* < v(Q^{-k} - Q_1^*)$ or, equivalently,

[34] A similar proof of this claim appears in Attar, Mariotti, and Salanié (2014).

$T^{-k} < vQ^{-k}$. We distinguish three cases.

(i) If $Q^{-k} < Q_1^*$, then seller k can deviate to $(Q_1^* - Q^{-k}, T_1^* - T^{-k} - \varepsilon)$, for some positive and small enough ε. This contract attracts type 1 along with the aggregate trade (Q^{-k}, T^{-k}) and, because $T^{-k} - T_1^* < v(Q^{-k} - Q_1^*)$, it is profitable even if it also attracts type 2, a contradiction.

(ii) If $Q_1^* < Q^{-k} < Q_2^*$, then seller k can deviate to $(Q_2^* - Q^{-k}, T_2^* - T^{-k} - \varepsilon)$, for some positive and small enough ε. This contract attracts type 2 along with the aggregate trade (Q^{-k}, T^{-k}) and, because

$$T_2^* - T^{-k} - v_2(Q_2^* - Q^{-k}) > T_2^* - v_2 Q_2^* + (v_2 - v)Q^{-k} = (v_2 - v)(Q^{-k} - Q_1^*) > 0,$$

it is profitable even if it does not attract type 1, a contradiction.

(iii) Suppose finally that $Q^{-k} \geq Q_2^*$. Because type 2 is not attracted by (Q^{-k}, T^{-k}), one must have $U_2(Q^{-k}, T^{-k}) \leq U_2(Q_2^*, T_2^*)$, and because $T^{-k} < vQ^{-k}$, one must have $Q^{-k} > Q_2^*$. By Assumption SC, we get that $U_1(Q^{-k}, T^{-k}) < U_1(Q_2^*, T_2^*)$, which, as $U_1(Q_2^*, T_2^*) \leq U_1(Q_1^*, T_1^*)$ by revealed preference on the equilibrium path, contradicts the assumption that $U_1(Q^{-k}, T^{-k}) = U_1(Q_1^*, T_1^*)$.

This proves that $Q^{-k} = Q_1^*$, as claimed. Note that the reasoning in (i) more generally shows that no contract $(q^{k'}, t^{k'})$ such that $q^{k'} \leq Q_1^*$ and $t^{k'} < vq^{k'}$ can be issued in equilibrium. Thus (Q_1^*, T_1^*) can only be obtained through contracts with unit price v.

2. If $Q_2^* > Q_1^*$, then sellers in $K_{\emptyset 2}$ together supply $Q_2^* - Q_1^*$ at unit price v_2. None of these sellers can be indispensable for type 2 to reach her equilibrium utility, as characterized by (9). Therefore, for each $k \in K_{\emptyset 2}$, the sellers other than k must offer some aggregate trade (Q^{-k}, T^{-k}) such that $U_2(Q^{-k}, T^{-k}) = U_2(Q_2^*, T_2^*)$. We prove below that $Q^{-k} \geq Q_2^*$. Notice that, unlike for type 1, the equilibrium aggregate trade (Q_2^*, T_2^*) of type 2 need not remain available if any of the sellers who trade with her withdraws his contract offer.[35] As the other active sellers' together supply less than Q_2^*, this shows again that $K_{\emptyset\emptyset}$ is nonempty.

To prove the claim, suppose, by way of contradiction, that $Q^{-k} < Q_2^*$. Then, from (9)–(10) and the strict quasiconcavity of U_2, one must have $T^{-k} - T_2^* < v_2(Q^{-k} - Q_2^*)$. Then seller k can deviate to $(Q_2^* - Q^{-k}, T_2^* - T^{-k} - \varepsilon)$, for some positive and small enough ε. This contract attracts type 2 along with the aggregate trade (Q^{-k}, T^{-k}) and is profitable even if it does not attract type 1, a contradiction. ■

[35] In Attar, Mariotti, and Salanié (2014), this property was satisfied because trades of negative quantities were allowed, which is not the case in our setting.

Proof of Corollary 1. Fix an equilibrium of the single-contract game, assuming that such an equilibrium exists. Let K_v be the set of sellers issuing contracts at unit price v and, for each $k \in K_v$, let $\alpha^k \equiv q/Q_1^*$. Fix some $k \in K_v \cap K_{12}$, so that, in particular, $0 < \alpha^k \leq 1$. Because, as shown in the proof of Theorem 2, type 1 can still trade (Q_1^*, T_1^*) if any seller in K_{12} withdraws his contract offer, and no contract $(q^{k'}, t^{k'})$ such that $q^{k'} \leq Q_1^*$ and $t^{k'} < vt^{k'}$ can be issued in equilibrium, there must exist $K_v^{-k} \subset K_v \setminus \{k\}$ such that $\sum_{k' \in K_v^{-k}} \alpha^{k'} = 1$. Therefore, we have

$$1 < \sum_{k' \in K_v^{-k}} \alpha^{k'} + \alpha^k = 1 + \alpha^k \leq 2.$$

Note that the aggregate trade $((1+\alpha^k)Q_1^*, v(1+\alpha^k)Q_1^*)$ is available on the equilibrium path, so that

$$U_2(Q_2^*, T_2^*) \geq U_2((1+\alpha^k)Q_1^*, (1+\alpha^k)vQ_1^*). \tag{30}$$

Moreover, because some sellers trade contracts with unit price v_2, there exists some seller $k'' \notin K_v^{-k} \cup \{k\}$. To conclude the proof, we only need to show that

$$(1+\alpha^k)Q_1^* > Q_2^*. \tag{31}$$

Indeed, along with (30), (31) implies that $2Q_1^* > Q_2^*$, which is (21), and, in turn, that $U_2(Q_2^*, T_2^*) \geq U_2(2Q_1^*, 2vQ_1^*)$, which is (20). To establish (31), observe first that, because $T_2^* > vQ_2^*$, $(1+\alpha^k)Q_1^* \neq Q_2^*$. Let us suppose, by way of contradiction, that $(1+\alpha^k)Q_1^* < Q_2^*$. Then any seller $k'' \notin \{k\} \cup K_v^{-k}$ can deviate to $(Q_2^* - (1+\alpha^k)Q_1, T_2^* - v(1+\alpha^k)Q_1 - \varepsilon)$, for some positive and small enough ε. This contract attracts type 2 along with the aggregate trade $((1+\alpha^k)Q_1^*, (1+\alpha^k)vQ_1^*)$ and, because

$$\begin{aligned} T_2^* - v(1+\alpha^k)Q_1^* - v_2[Q_2^* - (1+\alpha^k)Q_1^*] &> T_2^* - v_2Q_2^* + (v_2 - v)(1+\alpha^k)Q_1^* \\ &= (v_2 - v)\alpha^k Q_1^* \\ &> 0, \end{aligned}$$

it is profitable even if it does not attract type 1, a contradiction. Hence the result. ■

Proof of Lemma 1. If a contract (q^ℓ, t^ℓ) deters cream skimming, then, by (22) applied to $(q, t) = (Q_1^*, T_1^*)$, we have $U_2(Q_1^* + q^\ell, T_1^* + t^\ell) \geq U_2(Q_2^*, T_2^*)$. But this inequality cannot be strict if the contract (q^ℓ, t^ℓ) is issued but not traded in equilibrium, because, otherwise, type 2 would be better off trading it on top of type 1's aggregate trade (Q_1^*, T_1^*). Hence (23). Next, by (22), the translate of the upper contour set of (Q_1^*, T_1^*) for type 1 along the vector

(q^ℓ, t^ℓ) lies in the upper contour set of (Q_2^*, T_2^*) for type 2. As these two sets intersect at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ by (23), we obtain along the lines of Benveniste and Scheinkman (1979, Lemma 1) that the slope of type 2's equilibrium indifference curve at $(Q_1^* + q^\ell, T_1^* + t^\ell)$ must equal the slope of type 1's equilibrium indifference curve at (Q_1^*, T_1^*), that is, v. Hence (24). The result follows.

For future reference, note that Assumption GT and condition (20) ensure the existence and uniqueness of the contract (q^ℓ, t^ℓ) and also imply $Q_1^* > q^\ell$. Moreover, because the slope of type 2's equilibrium indifference curve is higher at (Q_2^*, T_2^*) than at $(Q_1^* + q^\ell, T_1^* + t^\ell)$, one must have $q^\ell > Q_2^* - Q_1^*$. The unit price t^ℓ / q^ℓ is therefore above v and below v_2. ■

Proof of Lemma 2. We shall hereafter slightly abuse notation by indentifying each type's equilibrium indifference curve with its functional expression $T = \mathcal{I}_i^*(Q)$. Recalling that $\mathcal{I}_2^*(Q_1^* + q^\ell) = \mathcal{I}_1^*(Q_1^*) + t^\ell$, we only need to prove that the translate of $\mathcal{I}_1^*$ along the vector (q^ℓ, t^ℓ) lies below $\mathcal{I}_2^*$. For this, it is enough to show that

$$\partial \mathcal{I}_1^*(Q) \gtrless \partial \mathcal{I}_2^*(Q + q^\ell) \quad \text{if} \quad Q \lesseqgtr Q_1^*.$$

A sufficient condition for this is the following single-crossing property:

$$\partial \mathcal{I}_1^*(Q) = \partial \mathcal{I}_2^*(Q + q^\ell) \quad \text{implies} \quad \partial^2 \mathcal{I}_1^*(Q) < \partial^2 \mathcal{I}_2^*(Q + q^\ell).$$

which, under Assumption C, is a direct implication of the identities

$$\begin{aligned} \tau_i &= \partial \mathcal{I}_i^*, \\ \kappa_i &= -\frac{\partial^2 \mathcal{I}_i^*}{[1 + (\partial \mathcal{I}_i^*)^2]^{3/2}}. \end{aligned}$$

The result follows. ■

Proof of Theorem 3. Suppose that two sellers offer the contract $c \equiv (Q_1^*, T_1^*)$, two sellers offer the contract $c' \equiv (Q_2^* - Q_1^*, T_2^* - T_1^*)$, and sufficiently many sellers offer the contract $c^\ell \equiv (q^\ell, t^\ell)$ characterized by (23)–(24). We derive below a bound for this number beyond which existence of an equilibrium is guaranteed. The proof consists of three steps.

Step 1 The first thing we have to check is that each buyer type chooses to trade according to the JHG allocation, given the supply of contracts just defined. Consider first type 1. Because $\tau_1(c_1) = v$, c is her most preferred contract with unit price v. As all offered contracts have unit prices at least equal to v, trading a single contract c is thus optimal for type 1.

Consider next type 2. Because $\tau_2(c+c')=v_2$, and the unit price v_2 of c' is strictly higher than the unit price v of c, she is strictly worse off trading only contracts c' than trading a contract c along with a contract c'. Thus type 2 optimally trades at least one contract c. Moreover, if she trades exactly one contract c, it is optimal for her to trade in addition exactly one contract c'. To prove that she cannot be strictly better off trading c twice, note that, because, by (20)–(21), $2c$ is located in the lower contour set of $c+c'$ for type 2, to the right of the line with slope v_2 going through $c+c'$, and because $\tau_2(c+c')=v_2$, $\tau_2(2c)$ must be lower than the unit price v_2 of c'. This, together with (20), implies that trading c twice, possibly along with one or two contracts c', cannot yield type 2 a higher utility than trading a contract c along with a contract c'. Finally, as $U_2(c+c^\ell)=U_2(c+c')$, we only need to check that, if type 2 trades c^ℓ once, the optimal thing for her to do is to combine this contract c^ℓ with exactly one contract c. Indeed, because $\tau_2(c+c^\ell)=v$, c is, among all contracts with unit price v, the best that type 2 can combine with c^ℓ. As all offered contracts have unit prices at least equal to v, trading a single contract c is, therefore, the unique optimal choice for type 2 once she has traded c^ℓ. Thus, given the contracts offered, each buyer type chooses to trade according to the JHG allocation, as claimed.

Step 2 We next check that any deviation that attracts type 2 is unprofitable.

First, we show that there is no profitable deviation for a seller that attracts both types. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v. However, recall that $\tau_1(c)=v$ and that all offered contracts have unit prices at least equal to v. Trading $\tilde{c}$ would then yield type 1 a strictly lower utility than trading a single contract c, which remains feasible following any seller's unilateral deviation. Such a deviation is thus not possible.

Second, we show that there is no profitable deviation for a seller that only attracts type 2. Indeed, to be profitable, the corresponding contract $\tilde{c}$ would need to have a unit price strictly higher than v_2. However, recall that $\tau_2(c+c')=v_2$ and that, as shown in Step 1, type 2 cannot gain from combining any contract with unit price strictly higher than v_2 with $2c$. Trading $\tilde{c}$, possibly along with some contracts c or c', would then yield type 2 a strictly lower utility than trading a contract c along with a contract c', which remains feasible following any seller's unilateral deviation. The possibility remains that type 2 combines c^ℓ with $\tilde{c}$. However, as all offered contracts have unit prices at least equal to v, and strictly so for $\tilde{c}$, trading a single contract c is the unique optimal choice for type 2 once she has traded c^ℓ, as shown in Step 1. Such a deviation is thus not possible.

Step 3 We finally derive an upper bound on the required number of sellers offering the contract c^ℓ, which in turn gives us a bound for the number of sellers beyond which existence of an equilibrium is guaranteed. Specifically, define

$$\begin{aligned} A_1 &\equiv \{(q,t) \in \mathbb{R}_+ \times \mathbb{R} : q \leq Q_2 \text{ and } vq \geq t \geq v_1 q\}, \\ A_2 &\equiv \{Nc + N'c' : (N, N') \in \{0,1,2\} \times \{0,1,2\}\}, \\ A_3 &\equiv \{(Q,T) \in \mathbb{R}_+ \times \mathbb{R} : U_1(Q,T) \geq U_1(c)\}. \end{aligned}$$

To interpret A_1, observe that if type 1 were attracted by a contract (q,t) with $q \geq Q_2$ issued by a deviating seller, then this would mean that by trading (q,t), possibly along with other available contracts, she could reach an aggregate trade (Q,T) with $Q \geq Q_2$, which she would weakly prefer to the aggregate trade $c + c'$ that remains available following any unilateral deviation. Because $c + c'$ is the equilibrium aggregate trade of type 2 and involves an aggregate coverage Q_2, it would follow from Assumption SC that type 2 would strictly prefer (Q,T) to her equilibrium aggregate trade $c+c'$, and thus would be strictly attracted by the contract (q,t). Therefore, we can safely restrict our quest for potential cream-skimming deviation to the set of contracts (q,t) such that $q \leq Q_2$. In addition, the contracts in A_1 imply no loss for the sellers when only traded by type 1 and have a unit price lower than v, so that they are potentially attractive for type 1, either per se or combined with other available contracts. Next, A_2 is the set of aggregate trades that can be made with four sellers, two of whom offer the contract c and two of whom offer the contract c'. Last, A_3 is the upper contour set of c for type 1. Then

$$N^\ell \equiv \max\{N \in \mathbb{N} : (A_1 + A_2 + Nc^\ell) \cap A_3 \neq \emptyset\} \tag{32}$$

is the maximum number of contracts c^ℓ type 1 may ever want to trade, if she were offered a contract in A_1, which she could complement by aggregate trades in A_2 and as many contracts c^ℓ as she wishes. Because A_1 is compact, $c \in A_1 \cap A_3$, $(0,0) \in A_2$, and $\tau_1(c) = v$ is strictly lower than v_2 and the unit price of c^ℓ, N^ℓ is well defined and finite. Suppose now that two sellers offer the contract c, two sellers offer the contract c', and $\max\{N^\ell + 1, 2\}$ sellers offer the contract c^ℓ. Consider now a deviation that attracts type 1. Trading the corresponding contract $\tilde{c}$, possibly along with contracts c, c', and c^ℓ, must yield type 1 at least her equilibrium utility. However, because the contract c^ℓ deters cream skimming under Assumption C, type 2 could also weakly increase her utility by trading the same contracts as type 1, plus one additional contract c^ℓ. The definition (32) of N^ℓ ensures that type 1 will never trade more than N^ℓ contracts c^ℓ following the deviation. If, as postulated,

$\max\{N^\ell+1,2\}$ sellers offer the contract c^ℓ, a contract c^ℓ remains available for type 2 to trade even after mimicking type 1. As a result, one can construct the buyer's best response in such a way that both types trade $\tilde{c}$ with the deviating seller, which, as shown in Step 2, cannot be profitable for him. Overall, existence of an equilibrium is guaranteed as soon as there are at least $\max\{N^\ell+1,2\}+4$ sellers. Hence the result. ■

Appendix B: Additional Proofs and Calculations

On Single Crossing and Coinsurance. We show that Assumption SC is satisfied if f_2 dominates f_1 in the monotone-likelihood-ratio order. We have, for all (Q, T) and i,

$$\tau_i(Q,T) = \frac{\int L u'(W_0 - (1-Q)L - T) f_i(L)\,\mathrm{d}L}{\int u'(W_0 - (1-Q)L - T) f_i(L)\,\mathrm{d}L} = \int L\,\mathrm{d}G_i(L), \tag{33}$$

where G_i is a distribution with density

$$g_i(L) = \frac{u'(W_0 - (1-Q)L - T) f_i(L)}{\int u'(W_0 - (1-Q)L - T) f_i(L)\,\mathrm{d}L} \tag{34}$$

with respect to Lebesgue measure. If f_2 dominates f_1 in the monotone-likelihood-ratio order, then, by (34), g_2 also dominates g_1 in the monotone-likelihood-ratio order, and so G_2 a fortiori first-order stochastically dominates G_1. It then follows from (33) that $\tau_2(Q,T) > \tau_1(Q,T)$, which is precisely Assumption SC. ■

On Assumption C. We first show that Assumption C is equivalent to the property that $H_2(p, \mathbf{u_2}) - H_1(p, \mathbf{u_1})$ is strictly decreasing in p for all $\mathbf{u_1}$ and $\mathbf{u_2}$. We shall hereafter slightly abuse notation by identifying each type's indifference curve associated to utility level $\mathbf{u}$ with its functional expression $T = \mathcal{I}_i(Q, \mathbf{u})$. The strict quasiconcavity of U_i implies that, for each $\mathbf{u}$, $\mathcal{I}_i(Q, \mathbf{u})$ is strictly concave with respect to Q. By construction, the slope of $\mathcal{I}_i(\cdot, \mathbf{u})$ is type i's marginal rate of substitution,

$$\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u}) = \tau_i(Q, \mathcal{I}_i(Q, \mathbf{u})),$$

and $\partial \mathcal{I}_i / \partial Q$ is the inverse of the Hicksian demand function,

$$\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u}) = p \quad \text{if and only if} \quad Q = H_i(p, \mathbf{u}).$$

Assumption C states that, if

$$\frac{\partial \mathcal{I}_1}{\partial Q}(Q_1, \mathbf{u_1}) = \frac{\partial \mathcal{I}_2}{\partial Q}(Q_2, \mathbf{u_2}),$$

then

$$-\frac{(\partial^2 \mathcal{I}_1/\partial Q^2)(Q_1, \mathbf{u_1})}{\{1 + [(\partial \mathcal{I}_1/\partial Q)(Q_1, \mathbf{u_1})]^2\}^{\frac{3}{2}}} > -\frac{(\partial^2 \mathcal{I}_2/\partial Q^2)(Q_2, \mathbf{u_2})}{\{1 + [(\partial \mathcal{I}_2/\partial Q)(Q_2, \mathbf{u_2})]^2\}^{\frac{3}{2}}},$$

so that

$$\frac{\partial^2 \mathcal{I}_1}{\partial Q^2}(Q_1, \mathbf{u_1}) < \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}(Q_2, \mathbf{u_2}).$$

Call $p \equiv (\partial \mathcal{I}_1/\partial Q)(Q_1, \mathbf{u_1}) = (\partial \mathcal{I}_2/\partial Q)(Q_2, \mathbf{u_2})$. Overall, Assumption C reduces to the following condition:

$$\frac{\partial^2 \mathcal{I}_1}{\partial Q^2}\left(\left(\frac{\partial \mathcal{I}_1}{\partial Q}(\cdot, \mathbf{u_1})\right)^{-1}(p), \mathbf{u_1}\right) < \frac{\partial^2 \mathcal{I}_2}{\partial Q^2}\left(\left(\frac{\partial \mathcal{I}_2}{\partial Q}(\cdot, \mathbf{u_2})\right)^{-1}(p), \mathbf{u_2}\right),$$

that is, $((\partial \mathcal{I}_2/\partial Q)(\cdot, \mathbf{u_2}))^{-1}(p) - ((\partial \mathcal{I}_1/\partial Q)(\cdot, \mathbf{u_1}))^{-1}(p)$ is strictly decreasing in p, which is precisely the desired property of Hicksian demand functions.

Next, assuming that type i's preferences are represented by

$$U_i(Q, T) = v_i u_i(W_0 - L + Q - T) + (1 - v_i) u_i(W_0 - T),$$

we show that Assumption C is satisfied if type 1 is uniformly more risk-averse than type 2, that is, letting $\alpha_i(w) \equiv -u_i''(w)/u_i'(w)$ be type i's coefficient of absolute risk aversion at wealth w, if $\alpha_1(w_1) > \alpha_2(w_2)$ for any wealth levels w_1 and w_2. We have

$$\begin{aligned}\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u}) &= \tau_i(Q, \mathcal{I}_i(Q, \mathbf{u})) \\ &= \frac{1}{1 + [(1 - v_i)/v_i][u_i'(W_0 - \mathcal{I}_i(Q, \mathbf{u}))/u_i'(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u}))]}\end{aligned}$$

and hence

$$\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q, \mathbf{u}) = -\left(\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\right)^2 \left(\frac{1 - v_i}{v_i}\right) \frac{B(Q, \mathbf{u})}{[u_i'(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u}))]^2}, \tag{35}$$

where

$$\begin{aligned}B(Q, \mathbf{u}) \equiv & -u_i''(W_0 - \mathcal{I}_i(Q, \mathbf{u})) \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\, u_i'(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u})) \\ & - u_i'(W_0 - \mathcal{I}_i(Q, \mathbf{u}))\left[1 - \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\right] u_i''(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u})).\end{aligned}$$

Simple manipulations show that

$$\frac{B(Q, \mathbf{u})}{[u_i'(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u}))]^2} = \frac{u_i'(W_0 - \mathcal{I}_i(Q, \mathbf{u}))}{u_i'(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u}))}\, \overline{\alpha}_i(Q, \mathbf{u}),$$

where

$$\overline{\alpha}_i(Q, \mathbf{u}) \equiv \alpha_i(W_0 - \mathcal{I}_i(Q, \mathbf{u})) \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u}) + \alpha_i(W_0 - L + Q - \mathcal{I}_i(Q, \mathbf{u}))\left[1 - \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\right].$$

Substituting in (35) and simplifying yields

$$\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}(Q, \mathbf{u}) = -\frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\left[1 - \frac{\partial \mathcal{I}_i}{\partial Q}(Q, \mathbf{u})\right] \overline{\alpha}_i(Q, \mathbf{u}).$$

These two equalities imply that, if type 1 is uniformly more risk-averse than type 2, we have $\overline{\alpha}_1(Q, \mathbf{u}_1) > \overline{\alpha}_2(Q, \mathbf{u}_2)$ and thus $(\partial^2\mathcal{I}_1/\partial Q^2)(Q, \mathbf{u_1}) < (\partial^2\mathcal{I}_2/\partial Q^2)(Q, \mathbf{u_2})$ for all $\mathbf{u_1}$ and $\mathbf{u_2}$ such that $(\partial\mathcal{I}_1/\partial Q)(Q, \mathbf{u_1}) = (\partial\mathcal{I}_2/\partial Q)(Q, \mathbf{u_2})$, which is precisely Assumption C. ■

On the CARA Example. Preferences represented by (2) with $u(x) \equiv -\exp(-\alpha x)$ can alternatively be represented by

$$U_i(Q, T) = u_i(Q) - T \tag{36}$$

with

$$u_i(Q) \equiv -\frac{1}{\alpha}\ln(v_i \exp(-\alpha(W_0 - L + Q)) + (1 - v_i)\exp(-\alpha W_0)).$$

Moreover, we have

$$\partial u_2(Q + Q_0) = \partial u_1(Q) \tag{37}$$

with

$$Q_0 \equiv \frac{1}{\alpha}\ln\left(\frac{(1 - v_1)/v_1}{(1 - v_2)/v_2}\right) > 0.$$

Hence type 1 may be thought of having the same preferences as type 2, while having already traded a quantity Q_0. Geometrically, properties (36)–(37) imply that any pair of indifference curves for types 1 and 2 are, over the relevant domain, oblique or horizontal translates of each other. The translating vector is (q^ℓ, t^ℓ) and connects points of equal slopes on these indifference curves.

When each type i has constant absolute risk aversion α_i, one has

$$u_i(Q) \equiv -\frac{1}{\alpha_i}\ln(v_i \exp(-\alpha_i(W_0 - L + Q)) + (1 - v_i)\exp(-\alpha_i W_0)).$$

A direct calculation yields the following expression for type i's demand function:

$$H_i(p) = \max\left\{L + \frac{1}{\alpha_i}\ln\left(\frac{(1 - p)/p}{(1 - v_i)/v_i}\right), 0\right\}.$$

Assumption C is satisfied if $\partial H_2 < \partial H_1$, that is, if $\alpha_2 < \alpha_1$. Assumption SC is satisfied if $\partial u_2 > \partial u_1$, that is, if

$$\frac{1}{1 + [(1 - v_2)/v_2]\exp(-\alpha_2(L - Q))} > \frac{1}{1 + [(1 - v_1)/v_1]\exp(-\alpha_2(L - Q))},$$

or, equivalently,

$$(\alpha_2 - \alpha_1)(L - Q) > \ln\left(\frac{(1 - v_2)/v_2}{(1 - v_1)/v_1}\right)$$

for all Q. The left-hand side of this inequality is an increasing function of Q when $\alpha_2 < \alpha_1$, which leads to the condition given in the body of the paper. ■

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