The self-insurance clauses puzzle: risk *versus* ambiguity

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Abstract

In many insurance contracts, self-insurance clauses appeared. Our objective is to analyze if these self-insurance clauses are justified, function of the observability or not of the self-insurance by the insurer. For this purpose, we propose a theoretical model under risk and ambiguity jointly analysing insurance and self-insurance. Theoretical results show that self-insurance clauses are never justified under risk, and not justified under ambiguity when the self-insurance is observable by the insurer. Moreover, under ambiguity, when the self-insurance is unobservable by the insurer, we show that optimal self-insurance depends on ambiguity preferences. Our results also indicate that insurance and self-insurance are substitutes under risk and under ambiguity only when the self-insurance is observable by the insurer. Under ambiguity, when the self-insurance activity is unobservable, insurance and self-insurance may be or not substitutes when the decision maker has ambiguity aversion.

Keywords: risk, ambiguity, insurance, self-insurance

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1 Introduction

Since the seminal contribution of Ehrlich and Becker [8], two types of prevention activities have been identified: self-protection and self-insurance.

Self-protection expenditures are intended to reduce the probability of accident, while self-insurance ones are specifically devoted to the reduction of the size of loss. In many insurance contracts, self-insurance clauses appeared. For example, compulsory firewall systems in property insurance or the compulsory helmet use for the workers in the building's sector. In forest insurance, for instance, the insurer requires the SMP (Simple Management Plan) to be sure that the forest is well-managed in accordance to sustainable management. Contract theory seems unable to justify this type of self-insurance clauses under expected utility. Such a clause is used to face asymmetric information and may be justified for self-protection but not for self-insurance[1]. Thus, it seems puzzling to meet so many contract clauses involving self-insurance in insurance contracting.

We know since Ehrlich et Becker [8] that insurance and self-insurance are substitutable, in the sense that an increase in the insurance price raises the self-insurance. This result is true whatever the observability of self-insurance activities by the insurer. In addition, they show that self-protection and insurance are complement when self-protection is observable by the insurer, while they are substitutable when self-protection is unobservable. Few works deal with this substitutability between insurance and self-insurance. Bryis et al. [4] investigate whether this result is robust in case of non-reliability of self-insurance, *i.e.*, situation where the effectiveness of self-insurance is uncertain. They assume that the potential non-performance of self-insurance is known by the individual, who assigns a probability distribution to the effectiveness of the tool. In this context, they show that insurance and self-insurance may be complements. Courbage [6] proved that this substitutability between insurance and self-insurance is valid under the dual theory of choice. Finally, Pannequin and Corcos [11] analyze this substitutability under the Stiglitz insurance monopoly model (Stiglitz [16]). They show that when the monopoly fixes the price and quantity of insurance that maximize its profit, the existence of these self-insurance opportunities reduces the insurer market power on the residual insurance demand, despite a saturated participation constraint. The decision maker can thereby capture a rent not just on self-insurance activity, but also on a portion of the risk covered by insurance.

Few studies analyze empirically this substitutability. Carson [5] found an empirical evidence for this substitution between insurance and self-insurance in the case of homeowner insurance and catastrophic risks. Pannequin et al. [12] obtain an incomplete matching with respect to the theory.

[1] Indeed, asymmetric information is a major issue in the context of self-protection (see Shavell [14]).

When the unit price of insurance rises, the demand for insurance decreases and the demand for self-insurance increases. But, individuals do not choose their levels of coverage that equalizes the marginal benefits from both mechanisms. Instead, they seem to comply with a global accounting model: their sensitivity to price changes is strongly confined by the global amount of coverage (insurance and self-insurance) individuals wish to realize.

Under ambiguity, the literature on insurance economics is recent. Then, some papers analyzing the link between self-protection and insurance under ambiguity exist (Snow [15]; Treich [17]; Etner and Spaeter [9]), while those dealing with the link between insurance and self-insurance are non-existent, both theoretically and empirically. Brunette et al. [3] analyze the impact of governmental assistance on insurance demand under ambiguity from a theoretical point of view and test experimentally the results. They show that, while theoretically public help, contingent public help and insurance subsidy have an impact on insurance demand, empirically, only the public help has a negative effect on insurance demand. However, they only focus on insurance. Alary et al. [1] derive a set of simple conditions such that ambiguity aversion always raises the demand for self-insurance and the insurance coverage. Snow [15] showed that the level of self-insurance that are optimal for an ambiguity averse decision maker is higher in the presence of ambiguity than in its absence. However, these two last papers analyze insurance and self-insurance separately.

In such a context, we propose to analyze the relevance of self-insurance clauses in insurance contract both under risk and ambiguity. Indeed, perhaps ambiguity may justify such clauses. Agents can have different attitudes toward risk, so that they may also have different attitudes toward ambiguity. This can lead to individual choices in terms of insurance purchasing and/or prevention rather different from what is usually obtained when only well-known risks are considered. For instance, the presence of ambiguity can lead some individuals to invest more in prevention if they weigh heavily the worst possible states of nature, while others prefer to limit effort because of the lack of information about the effective distribution of risk (Etner and Spaeter [9]). For this purpose, we propose a theoretical model of insurance and self-insurance under risk and under ambiguity. Our ambiguity model is based on Klibanoff et al. [10]. Theoretical results show that self-insurance clauses are never justified under risk, and not justified under ambiguity when the self-insurance is observable by the insurer. Moreover, under ambiguity, when the self-insurance is unobservable by the insurer, we show that optimal self-insurance depends on ambiguity preferences. Our results also indicate that insurance and self-insurance are substitute under risk and under ambiguity only when the self-insurance is observable by the insurer. Under ambiguity, when the self-insurance activity is unobservable, insurance and self-insurance may be or not substitutes when the decision maker has ambiguity aversion.

The rest of the paper is structured as follows. Section 2 presents the theoretical model under risk, Section 3 the model of optimal coverage under ambiguity, Section 4 compares the two models and Section 5 concludes.

2 A model of optimal coverage under risk

Consider a decision maker facing a probability q of losing a portion x of the initial wealth W_0. The final wealth would thus be W_0 if no loss occurs, or $W_0 - x$ in the case of loss. In order to reduce the risk exposure, decision maker can invest in a self-insurance technology a. Assuming that the amount of the loss, x, is a decreasing function of the amount invested in self-insurance a, we have: $x = x(a)$, $x'(a) < 0$. Moreover, we assume that the returns on self-insurance are decreasing ($i.e.$, $x''(a) > 0$). The decision maker's preferences are characterized by a vNM utility function $U(W)$, which is strictly increasing and concave ($U'(W) > 0$, $U''(W) < 0$). In addition, the decision maker may purchase an insurance contract to a risk neutral insurer that specifies three factors: the insurance premium P, the compensation I and the deductible F. The unit price of insurance is denoted p, so that: $P = pI$.

Thus, the decision maker maximises the following expected utility:

$$Max_{(F,a)} EU(W) = (1-q)U(W_1) + qU(W_2) \tag{1}$$

where W_1 is the final wealth with no loss, and W_2 the final wealth when a loss occurs. In this context, we analyze the link between insurance and self-insurance under two conditions, first when the insurer observes the decision maker's self-insurance and second, when the insurer unobserves the decision maker's self-insurance.

2.1 Observability of self-insurance

When the insurer observes the decision maker's self-insurance, then the compensation is $I = x(\hat{a}) - \hat{F}$. In such a context, the wealth in the two states of the world is:

$$W_1 = W_0 - p(x(\hat{a}) - \hat{F}) - \hat{a} \tag{2}$$

$$W_2 = W_0 - p(x(\hat{a}) - \hat{F}) - \hat{a} - \hat{F} \tag{3}$$

Then, the first-order conditions are:

$$\frac{\partial U(W)}{\partial \hat{F}} = p(1-q)U'(\hat{W}_1^*) - (1-p)qU'(\hat{W}_2^*) = 0 \tag{4}$$

$$\frac{\partial U(W)}{\partial \hat{a}} = [(1-q)U'(\hat{W}_1^*) + qU'(\hat{W}_2^*)][-px'(\hat{a}^*) - 1] = 0 \tag{5}$$

Conditions (4) and (5) may be rewritten as follows:

$$(1-q)U'(\hat{W}_1^*) + qU'(\hat{W}_2^*) = qU'(\hat{W}_2^*) \times \frac{1}{p} \tag{6}$$

$$(1-q)U'(\hat{W}_1^*) + qU'(\hat{W}_2^*) = -x'(\hat{a}^*)p[(1-q)U'(\hat{W}_1^*) + qU'(\hat{W}_2^*)] \tag{7}$$

At the optimum, the marginal cost of each mechanism (LHS) is equal to its marginal benefit (RHS), in terms of expected utility.

Conditions (6) and (7) let appeared that the decision maker invests in self-insurance in order to equalize the marginal returns of each mechanism:

$$-x'(\hat{a}^*) = \frac{1}{p} \tag{8}$$

At the equilibrium, the marginal return of self-insurance equalizes the reverse of the marginal return of insurance. Then, the unit price of insurance indirectly settles the self-insurance investment chosen by the decision maker. She sets her level of self-insurance at the point that equalizes marginal returns, and complements it by buying some insurance coverage for the residual risk.

2.2 Unobservability of self-insurance

When the insurer unobserves the decision maker's self-insurance, she considers $a = 0$ when calculating the compensation. Then, the compensation is $I = x(0) - F$. In such a context, the wealth in the two states of the world is:

$$W_1 = W_0 - p(x(0) - F) - a \tag{9}$$

$$W_2 = W_0 - p(x(0) - F) - a - x(a) + x(0) - F \tag{10}$$

The optimal choices are described by the first-order conditions:

$$\frac{\partial U(W)}{\partial F} = p(1-q)U'(W_1^*) - (1-p)qU'(W_2^*) = 0 \tag{11}$$

$$\frac{\partial U(W)}{\partial a} = -(1-q)U'(W_1^*) - (1 + x'(a^*))qU'(W_2^*) = 0 \tag{12}$$

These conditions are rewritten below in order to compare, for each mechanism, its marginal cost to its marginal benefit:

$$(1-q)U'(W_1^*) + qU'(W_2^*) = qU'(W_2^*) \times \frac{1}{p} \tag{13}$$

$$(1-q)U'(W_1^*) + qU'(W2^*) = -x'(a^*)qU'(W_2^*) \tag{14}$$

Then, at the optimum, the marginal cost of each mechanism (LHS) is equal to its marginal benefit (RHS), in expected utility terms.

The LHS of conditions (13) and (14) are identical, then equalizing the RHS leads to a fundamental result. A rational expected utility agent invests in self-insurance in order to equalize the marginal returns of insurance and self-insurance as follows:

$$-x'(a^*) = \frac{1}{p} \tag{15}$$

This condition is the same as condition (8).

2.3 Observability, risk aversion and substitution

The impact of the observability or not of self-insurance on insurance decision may be summarized by the following proposition:

Proposition 1 *Under risk, the optimal level of self-insurance is the same whatever the observability of the self-insurance.*

The proof of this proposition is immediate due to the comparison of conditions (8) and (15). As the conditions are identical, this proposition suggests that the clauses included in some insurance contracts are not justified. Indeed, with or without these clauses, the decision maker chooses the same level of coverage.

An another interesting result emerges from the comparison of conditions (6) and (8) obtained under observable self-insurance. Indeed, we can observe that the optimal level of insurance, defined by condition (6) depends on risk aversion, while, on the contrary, condition (8) defining the optimal level of self-insurance is independent of risk aversion. The same result appeared from the comparison of conditions (13) and (15) obtained under unobservable self-insurance. This result is very interesting because, the literature shows that self-insurance raises with risk aversion (Dionne and Eeckhoudt [7]) and insurance raises with risk aversion (Pratt [13]; Arrow [2]). Our result suggests that, when considering both insurance and self-insurance, risk aversion only impacts the insurance and not the self-insurance, whatever the observability of self-insurance by the insurer. This result is in line with Pannequin and Corcos [11].

Finally, a well-established result obtained by Ehrlich and Becker [8] is that insurance and self-insurance are substitutes. In our model, this result is confirmed under observability or not of the self-insurance. Indeed, from conditions (8) and (15), we can easily observe the impact of an increase in the unit price of insurance p on self-insurance. Such an increase leads to an increase in a because $-x'(a)$ is decreasing with p, so that a rises with p.

3 A model of optimal coverage under ambiguity

Our model of ambiguity is based on Klibanoff et al. [10]. We consider a risk averse and ambiguity averse decision maker who faces a loss x. The individual can implement self-insurance technology. The hypothesis for self-insurance under ambiguity are similar to those presented under risk: $x = x(a), x'(a) < 0, x''(a) > 0$. The probability q of the loss is ambiguous and is represented by a stochastic variable $\tilde{q} = q + \tilde{\varphi}$ with $E(\tilde{\varphi}) = 0$, where E is the term of expectation for $\tilde{\varphi}$. The individual is characterized by a vNM utility function $V(.)$, strictly increasing and concave ($V'(.) > 0$ and $V''(.) < 0$), which captures the individual's risk preferences. Klibanoff et al. [10] define ambiguity aversion by an increasing and concave function $\phi(.)$ defined over the expectation of $V(.)$. In addition, the decision maker may purchase an insurance contract to a risk neutral and ambiguity neutral insurer that specifies three factors: the insurance premium P, the compensation I and the deductible F. The unit price of insurance is denoted p.

Thus, the decision maker maximizes the following expected utility:

$$Max_{(F,a)} EV(W) = E_{\tilde{\varphi}}\phi[V(F, \tilde{\varphi})] = E_{\tilde{\varphi}}\phi[(1-\tilde{q})V(W_1) + \tilde{q}V(W_2)] \tag{16}$$

where W_1 is the final wealth with no loss, and W_2 the final wealth when a loss occurs. In this context, we analyze the link between insurance and self-insurance under two conditions, first when the insurer observes the decision maker's self-insurance and second, when the insurer does not observe the decision maker's self-insurance.

3.1 Observability of self-insurance

When the insurer can observe the decision maker's self-insurance, then the compensation is $I = x(\hat{a}) - \hat{F}$. The wealths in the two states of the world are identical to conditions (2) and (3) respectively. The decision maker's maximisation programme is the same as in condition (16).

The first-order conditions are:

$$\frac{\partial V(\hat{F})}{\partial \hat{F}} = E_{\tilde{\varphi}} \left\{ \phi'[V(\hat{F}^{*}, \tilde{\varphi})][(1-\tilde{q})V'(\hat{W}_1^{*})p - \tilde{q}V'(\hat{W}_2^{*})(1-p)] \right\} = 0 \tag{17}$$

$$\frac{\partial V(\hat{F})}{\partial \hat{a}} = E_{\tilde{\varphi}} \left\{ \phi'[V(\hat{F}, \tilde{\varphi})][\tilde{q}V'(\hat{W}_2^{*})(-px'(\hat{a}^{*}) - 1) + (1-\tilde{q})V'(\hat{W}_1^{*})(-px'(\hat{a}^{*}) - 1)] \right\} = 0 \tag{18}$$

Condition (18) allows to display that:

$$E_{\tilde{\varphi}} \left\{ \phi'[V(\hat{F}, \tilde{\varphi})][\tilde{q}V'(\hat{W}_2^{*}) + (1-\tilde{q})V'(\hat{W}_1^{*})] \right\} = -px'(\hat{a}^{*})E_{\tilde{\varphi}} \left\{ \phi'[V(\hat{F}, \tilde{\varphi})][\tilde{q}V'(\hat{W}_2^{*}) + (1-\tilde{q})V'(\hat{W}_1^{*})] \right\} \tag{19}$$

Consequently, we obtain the following equilibrium:

$$-x'(\hat{a}^{*}) = \frac{1}{p} \tag{20}$$

At the equilibrium, the marginal return of self-insurance equalizes the reverse of the marginal return of insurance.

3.2 Unobservability of self-insurance

When the insurer does not observe the decision maker's self-insurance, then the compensation is $I = x(0) - F$. The wealths in the two states of the world are identical to conditions (9) and (10).

In this context, the first-order conditions are:

$$\frac{\partial V(W)}{\partial F} = E_{\tilde{\varphi}} \left\{ \phi'[V(F^{*}, \tilde{\varphi})][\tilde{q}V'(W_2^{*})(p-1) + (1-\tilde{q})V'(W_1^{*})p] \right\}$$

$$= E_{\tilde{\varphi}} \left\{ \phi'[V(F^{*}, \tilde{\varphi})] \right\} E_{\tilde{\varphi}} \left\{ [\tilde{q}V'(W_2^{*})(p-1) + (1-\tilde{q})V'(W_1^{*})p] \right\} + cov(\phi'[V(F^{*}, \tilde{\varphi})], EV'_F) = 0 \tag{21}$$

where $EV'_F = [\tilde{q}V'(W_2^{*})(p-1) + (1-\tilde{q})V'(W_1^{*})p]$.

$$\frac{\partial V(W)}{\partial a} = E_{\tilde{\varphi}}\left\{\phi'[V(F^*,\tilde{\varphi})][\tilde{q}V'(W_2^*)(-1-x'(a^*))-(1-\tilde{q})V'(W_1^*)]\right\}$$

$$= E_{\tilde{\varphi}}\left\{\phi'[V(F^*,\tilde{\varphi})]\right\} E_{\tilde{\varphi}}\left\{[\tilde{q}V'(W_2^*)(-1-x'(a^*))-(1-\tilde{q})V'(W_1^*)]\right\} + cov(\phi'[V(F^*,\tilde{\varphi})], EV_a') = 0 \quad (22)$$

where $EV_a' = [\tilde{q}V'(W_2^*)(-1-x'(a^*))-(1-\tilde{q})V'(W_1^*)]$.

Conditions (21) and (22) may be rewritten as follows:

$$\tilde{q}V'(W_2^*)+(1-\tilde{q})V'(W_1^*) = \frac{\tilde{q}V'(W_2^*)}{p} - \frac{cov(\phi'[V(F^*,\tilde{\varphi})], EV_F')}{E_{\tilde{\varphi}}\phi'[V(F^*,\tilde{\varphi})]} \times \frac{1}{p} \quad (23)$$

$$\tilde{q}V'(W_2^*)+(1-\tilde{q})V'(W_1^*) = -\tilde{q}x'(a^*)V'(W_2^*) + \frac{cov(\phi'[V(F^*,\tilde{\varphi})], EV_a')}{E_{\tilde{\varphi}}\phi'[V(F^*,\tilde{\varphi})]} \quad (24)$$

We can easily observe that the LHS of these two last conditions are identical, then equalizing the RHS leads to:

$$\frac{1}{p} + x'(a^*) = \left[\frac{cov(\phi'[V(F^*,\tilde{\varphi})], EV_a')}{E_{\tilde{\varphi}}\phi'[V(F^*,\tilde{\varphi})]} + \frac{1}{p} \times \frac{cov(\phi'[V(F^*,\tilde{\varphi})], EV_F')}{E_{\tilde{\varphi}}\phi'[V(F^*,\tilde{\varphi})]}\right] \times \frac{1}{\tilde{q}V'(W_2^*)} \quad (25)$$

The optimal level of self-insurance depends on the comparison between the marginal benefit and the marginal cost of self-insurance but also on a term which sign is function of the ambiguity preferences. If the individual is ambiguity neutral ($\phi'' = 0$), then the optimal self-insurance condition is such that the RHS of condition (25) is equal to zero. Under ambiguity aversion ($\phi'' < 0$), then the two covariance terms, $cov(\phi'[V(F^*,\tilde{\varphi})], EV_a')$ and $cov(\phi'[V(F^*,\tilde{\varphi})], EV_F')$, are positive. Indeed, the two terms, $\phi'[V(F^*,\tilde{\varphi})]$ and EV', vary in the same direction with an increase in the loss x. When x raises, the wealth W reduces, the utility reduces and then, $\phi'[V(F^*,\tilde{\varphi})]$ increases. In the same manner, when x raises, the wealth W reduces, and as $V' > 0$, then EV' increases. Consequently, the RHS of condition (25) is positive and the optimal level of self-insurance is higher than under ambiguity neutrality. If the individual is ambiguity prone ($\phi'' > 0$), then the RHS of condition (25) is negative and the optimal level of self-insurance is lower than under ambiguity neutrality.

3.3 Observability, ambiguity aversion and substitution

Under ambiguity, the impact of the observability or not of self-insurance on insurance decision may be summarized by the following proposition:

Proposition 2 *Under ambiguity, for an ambiguity averse (prone or neutral respectively) decision* *maker, the optimal level of self-insurance when self-insurance is unobservable is higher (lower or equal respectively) than the optimal level when self-insurance is observable.*

This result depends on the comparison on conditions (20) and (25), depending on the RHS of condition (25) which is positive under ambiguity aversion, negative under ambiguity prone and equal to zero under ambiguity neutrality, as previously proved.

Another interesting result emerges from the comparison of optimal conditions obtained depending on the observability or not of self-insurance. Indeed both conditions (23), defining the optimal level of insurance, and (25), defining the optimal level of self-insurance depend on risk aversion and ambiguity aversion. On the contrary, conditions (17) and (20) show that the optimal level of insurance depends on risk and ambiguity aversion, while the optimal level of self-insurance is independent both on risk aversion and ambiguity aversion. This result highlights that the observability of self-insurance activities by the insurer removes any heterogeneity in terms of preferences towards risk and ambiguity.

In addition, we find that, under ambiguity, the result of Ehrlich and Becker [8] is valid under observable self-insurance but may fail under unobservable self-insurance. Under unobservability, the substitution depends on the ambiguity preferences. These results are summarized in the following proposition (proof in Appendix A):

Proposition 3 *Under ambiguity, when self-insurance is observable, self-insurance and insurance are always substitutes, while, when self-insurance is unobservable, the link between self-insurance and insurance depends on the decision maker's ambiguity preferences :*

- *If the decision maker is ambiguity neutral, then self-insurance and insurance are substitutes*
- *If the decision maker is ambiguity prone or averse, then self-insurance and insurance may be or not substitutes.*

Under ambiguity, when self-insurance activities are unobservable, the hypothesis about the decision maker's ambiguity preferences do not allow us to always conclude about the link between insurance and self-insurance. This result highlights the importance of both the decision maker's ambiguity preferences and the observability or not of the self-insurance activity.

4 Risk *versus* ambiguity

The comparison between the optimal self-insurance decisions under risk and under ambiguity leads to the following proposition:

Proposition 4 *The optimal levels of self-insurance activities chosen by the decision maker under risk and under ambiguity when self-insurance is observable are identical. When self-insurance is unobservable, the optimal level of self-insurance under ambiguity is higher (lower or identical respectively) than the optimal level of self-insurance under risk if the decision maker is ambiguity averse (prone or neutral respectively).*

The proof of this proposition is immediate due to the previous results obtained. Consequently, when self-insurance is observable by the insurer, ambiguity do not allow to justify the self-insurance clauses included in some insurance contracts. Indeed, under such assumptions, the optimal level of self-insurance is identical to the ones obtained under risk. Both optimal self-insurance levels are independent of the decision maker's risk and ambiguity preferences. In addition, when self-insurance is unobservable, under ambiguity, the optimal level of self-insurance activities strongly depends on the ambiguity preferences of the decision maker. If the agent is risk averse and ambiguity averse, then it is optimal to increase the optimal prevention level under ambiguity compared to a situation under risk. This result is in line with Snow [15] who show that the level of self-insurance that is optimal for an ambiguity averse decision maker is higher in the presence of ambiguity than in its absence.

5 Conclusion

This paper deals with the self-insurance clauses associated to some insurance contracts. More precisely, we question the relevancy of these clauses and we wonder if the ambiguity and/or the observability or not of the self-insurance effort may explain their existence. To do that, we develop a theoretical model of insurance economics under risk and under ambiguity, and we analyze the effect of the observability of the self-insurance on the optimal level of prevention and insurance.

Our results suggest that, under risk, the self-insurance clauses are never justified. Under ambiguity, such clauses are also not relevant when the self-insurance is observable. At the opposite, under ambiguity, when the self-insurance is unobservable by the insurer, we show that optimal self-insurance depends on ambiguity preferences. We also show that insurance and self-insurance are substitutes under risk and under ambiguity, only when the self-insurance is observable.

The extension of this article that we want to focus on is the experimental test of the theoretical predictions.

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References

[1] D. Alary, C. Gollier, and N. Treich. The effect of ambiguity aversion on insurance and self-protection. The Economic Journal, 123(573):1188–1202, 2013.

[2] K.J. Arrow. Essays in the theory of risk bearing. Markham Publishing Co., Chicago, 1971.

[3] M. Brunette, L. Cabantous, S. Couture, and A. Stenger. The impact of governmental assistance on insurance demand under ambiguity: A theoretical model and an experimental test. Theory and Decision, 75:153–174, 2013.

[4] E. Bryis, H. Schlesinger, and J-M.G. Schulenburg. Reliability of risk management: market insurance, self-insurance and self-protection reconsidered. Geneva Papers on Risk and Insurance Theory, 16:45–58, 1991.

[5] J. Carson, K. McCullough, and D. Pooser. Deciding whether to invest in mitigation measures: Evidence from florida. The Journal of Risk and Insurance, 80(2):309–327, 2013.

[6] C. Courbage. Market-insurance, self-insurance and self-protection within the dual theory of choice. Geneva Papers on Risk and Insurance Theory, 26(1):43–56, 2001.

[7] G. Dionne and L. Eeckhoudt. Self-insurance, self-protection and increased risk aversion. Economics Letters, 17(1):39–42, 1985.

[8] I. Ehrlich and G.S. Becker. Market insurance, self-insurance, and self-protection. Journal of Political Economy, 80(4):623–648, 1972.

[9] J. Etner and S. Spaeter. Self-protection, private insurance, and public prevention for non pecuniary risks under ambiguity. Working Paper EconomiX, 2011.

[10] P. Klibanoff, M. Marinacci, and S. Mukerjji. A smooth model of decision making under ambiguity. Econometrica, 73(6):1849–1892, 2005.

[11] F. Pannequin and A. Corcos. How self-insurance reduces the market power of an insurer? Miméo, 2010.

[12] F. Pannequin, A. Corcos, and C. Montmarquette. The trade-off between insurance and self-insurance: An experimental evidence of the substitutability property. Miméo, 2014.

[13] J. Pratt. Risk aversion in the small and in the large. Econometrica, 32:122–136, 1964.

[14] S. Shavell. On moral hazard and insurance. Quarterly Journal of Economics, 93(4):541–562, 1979.

[15] A. Snow. Ambiguity aversion and the propensities for self-insurance and self-protection. Journal of Risk and Uncertainty, 42:27–43, 2011.

[16] J. Stiglitz. Monopoly, nonlinear pricing, and imperfect information: the insurance market. Review of Economic Studies, 4:407–430, 1977.

[17] N. Treich. The value of a statistical life under ambiguity aversion. Journal of Environmental Economics and Management, 59:15–26, 2001.

Appendix A. Proof of Proposition 4 : substitutability of insurance and self-insurance

Observability of self-insurance

Recall that condition (20) is as follows:

$$-x'(\hat{a}^*) = \frac{1}{p}$$

It is immediate to prove that an increase in the unit price of insurance p raises the optimal self-insurance $\hat{a}^*$. Under observability, insurance and self-insurance are always substitutes.

Unbservability of self-insurance

We study the impact of a variation of the cost of self-insurance a on the optimal insurance, F^*, defined by condition (21). By differentiating this equation, we obtain: $\frac{dF^*}{da} = -\frac{\partial(\partial V(F)/\partial F)/\partial a}{\partial^2 V(F)/\partial F^2}$.

As $\partial^2 V(F)/\partial F^2$ is the second order condition and is negative, then the sign of $\frac{dF^*}{da}$ only depends on the sign of $\partial(\partial V(F)/\partial F)/\partial a$.

$$\partial(\partial V(F)/\partial F)/\partial a = E_{\tilde{\varphi}}\{\phi''[V(F^*,\tilde{\varphi})][[\tilde{q}V'(W_2^*)(p-1) + (1-\tilde{q})V'(W_1^*)p]]$$

$$[[\tilde{q}V'(W_2^*)(-x'(a)-1)-(1-\tilde{q})V'(W_1^*)]]+\phi'[V(F^*,\tilde{\varphi})][\tilde{q}V''(W_2^*)(-x'(a)-1)(p-1)-(1-\tilde{q})pV''(W_1^*)]\}$$

The sign of $\partial(\partial V(F)/\partial F)/\partial a$ is ambiguous. Assuming that $(-1-x'(a)) > 0$, *i.e.*, the marginal benefit of self-insurance is greater than its marginal cost. If the decision maker is ambiguity neutral ($\phi'' = 0, \phi' = constant$), then the sign of $\partial(\partial V(F)/\partial F)/\partial a$ is positive. In this case, insurance and self-insurance are substitutes under ambiguity. An increase of the self-insurance's cost raises the demand for insurance. Under ambiguity aversion or prone, the sign of $\partial(\partial V(F)/\partial F)/\partial a$ stays ambiguous, indicating that insurance and self-insurance may be or not substitutes under ambiguity.