The Impact of Pension Funding Mechanisms on the Stability and Payoff from DC Pension Schemes in Switzerland

Philipp Müller, Joël Wagner*

Abstract

Adequately funding occupational pension funds is a major concern for society in general and individual contributors in particular. The low returns accompanied with high volatility in capital markets have put many funds in distress. In such situations, the payoff for insureds at retirement strongly depends on the funding mechanisms in place. While the basic contributions are mostly defined by the state, the fund's situation may require additional contributions from the insureds or may allow the distribution of surpluses. In this paper, we focus on the accumulation phase of a defined contribution plan with minimum returns and annual solvency targets in terms of an assets-to-liabilities funding ratio. From the viewpoint of the pension fund, we evaluate the outcome of selected funding mechanisms on the solvency situation. Taking the perspective of the contributors, we analyze their payoff and utility. Combining both prospects, we discuss the optimal boundary values that trigger the various participation mechanisms. To do so, we calibrate our model and derive numerical results for pension funds in Switzerland. We find that remediation measures and surplus distribution play an important role in stabilizing the fund and increasing the utility of its insureds.

Key words DC pension schemes · funding mechanisms · stability · payoff

1 Introduction

Since the introduction of the Swiss pension system and occupational pension funds (2nd pillar) in particular, the demographic and capital market framework conditions have changed. Life expectancy is increasing, while birthrate is decreasing, causing the ratio of the number of active workers to the number of retirees to decline over the years (see also OECD (2015b)). In the financial markets, many asset classes have delivered historically low returns and at the same time exhibited increased volatility in the last two decades (see also OECD (2015a)).

The mentioned demographic issues and changing capital markets from the last financial crisis as well as ongoing turbulences are highlighted by most practitioners (see, Credit Suisse (2014)). However, there are also other factors that change the environment. At the society level, family and living structures along with work conditions have changed. Flexibility in work time management, statutory and effective retirement age and new disability and old-age dependency challenges need to be considered. Many technical parameters in Swiss pension funds (e.g., the minimum interest rate, the conversion rate) and their adaptations depend on political decisions.

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However, reforms have been strongly rejected in the last few years, and the definitions of technical and actuarial parameters have undergone lengthy political processes. The currently planned reform in Switzerland called "Altersvorsorge 2020"[1] only yields partial answers. In the wake of the last crisis, the solvency of many pension funds has been stressed. In fact, the funding ratio has dropped below 100% in many cases, which has put pressure on funds to move towards consolidation and sustainability considerations (see Swisscanto (2015)). This additional pressure comes along with operational risks with regard to compliance, higher transparency and governance requirements. Furthermore, one has to consider increasing wealth transfers between younger and older contributor groups (Eling (2012)) or potentially unfair mechanisms regarding employees who change their employer and pension fund.[2] The above changes pose challenges to pension fund systems in Switzerland and in many countries around the world. Many aspects of these problems have been discussed by practitioners and politicians, but they have been given much less attention in academic research. However, independent research and a solid academic foundation for such discussions are important in an area where the various stakeholders, contributors, pension funds, actors from the industry and politicians have diverging interests and opinions.

While our research holds true for pension funds in many countries (with certain adaptations), we apply our modeling more specifically to the Swiss 2nd pillar pension system, and we study different dimensions of risk that affect pension schemes and their members. We study the impact that remediation measures and surplus distribution have on the stability and the payoff of a fund and the utility of its members. This involves, among others, analyzing an adequate choice of parameters, the sensitivity of the model and the impact of capital market scenarios.
The academic literature analyzing different types of pension schemes is abundant. Sharpe (1976) is one of the first to rigorously analyze pension insurance provided by a sponsor. Black (1976) discusses both the optimal pattern of contributions and the optimal investment policy for the assets of a pension fund. Typically, stochastic pension fund modeling is used, as can be found in, e.g., O'Brien (1986), Bacinello (1988) and Dufresne (1989). The topic of asset allocation is studied from different perspectives in the literature as well. By using a simple stochastic model, Cairns et al. (2006) incorporate asset, salary and interest rate risk in the derivation of optimal investment strategies. While many actuarial papers analyze risk from demographic changes, financial risk in pension funds is less extensively considered in the existing literature: Most recently, by integrating assets and liabilities as well as solvency requirements, Berdin and Gründl (2015) consider a representative German life insurer and its asset allocation and outstanding liabilities. Generating a stochastic term structure of interest rates as well as stock market returns, the authors simulate investment returns for the business portfolio in a multi-period setting. Based on empirically calibrated parameters, the evolution of the balance sheet over time is observed with a special focus on the solvency situation. Looking at participating life insurance contracts, Schmeiser and Wagner (2014) try to find a suitable value for the guaranteed interest rate. Among others, their results show that as the risk-free interest rate approaches the guaranteed one, the equity capital falls to zero, as there is no longer any benefit from risky investments. This is relevant for pension funds too, as a minimum interest rate must be credited annually to the accounts of the contributors (see Broeders et al. (2011); Mirza and Wagner (2016)). A

[1] See `http://www.bsv.admin.ch/altersvorsorge_2020`.

[2] Contributors changing their employer must change to the pension linked to the new company. Thereby, the assets are transferred whereas, e.g. potential remediation measures to improve the overall state of the pension fund, remain with the previous institution.

study of the impact of product features and contributor types on lapse in life insurance contracts can be found in Eling and Kiesenbauer (2013). Using a data set from a German life insurer, they conclude that all considered characteristics have an impact on the lapse rate with the calendar year, with the contract age and the premium type being the most important. An analysis of the relationship between the liability structure and the asset allocation of defined benefit pension funds is performed by Alestalo and Puttonen (2006). Examining data from Finnish pension funds, the authors find that the liability structure does indeed influence the asset allocation, with the age structure of the members being one source of correlation. By combining a stochastic pension fund model with a traffic light approach, Braun et al. (2011) measure the shortfall probability of Swiss occupational pension funds in order to assist stakeholders in making decisions. Examining Dutch pension funds, Broeders et al. (2016) find empirical evidence for herding behavior in the asset allocation of institutions ranging from only weak links to great similarities. By analyzing the optimality of supervisory rules, Chen and Clever (2015) show that both the security mechanisms and risk measures used by regulators influence the optimality of the respective regulations.

Many recent statistical and industrial publications in Switzerland consider the current state of pension funds from a practical point of view. Often, they have a particular focus on the ongoing pension reforms, underline various challenges that the pension system is facing, and discuss relevant funding ratios or intergenerational wealth transfers. Some analyze the financial situation of funds and discuss possible reforms and ways to go forward (Bischofberger and Walser (2011)). Eling (2012) considers the current wealth distribution and transfer mechanisms among young and old generations in Switzerland. The aging population and the long-term (financial) perspectives are also in the focus of UBS (2014). In his recent book, Cosandey (2014) discusses reforms for fair intergenerational mechanisms and justice.

Our research aims at building on and extending the current state of knowledge by considering the framework of Swiss pension funds, accounting for the currently changing environmental conditions, and consistently including both the asset and liability perspectives. Using stochastic simulations and considering a contributor's account, we construct a model to assess the extra contribution and the surplus distribution mechanisms of defined contribution pension plans. From the institutional perspective, the funding ratio and the stability of the pension fund are taken into consideration. We use a simplified balance sheet approach that simulates the development of assets and liabilities of a Swiss pension fund. In particular, we look at the changes in the funding ratio over time for different funding mechanisms. This includes different limits for the distribution of bonuses as well as several methods for determining the required additional contributions. Taking these into account, we analyze what leads to greater stability of the overall fund as well as to higher contributor utility at the time of retirement. Additionally, this payoff is analyzed using internal rate of return and utility calculations. Going further into detail, we study several scenarios for the capital market returns in order to examine the ability of the fund to cope with extended periods of low and high rates of return. As a result, we obtain the distribution of the payoff at retirement and its sensitivity corresponding to different safety and distribution mechanisms.

Our main findings are as follows. First, it is deduced that charging remediation measures helps secure the stability of the fund in years following low market returns. We find that funds in good health can distribute bonuses to their clients in order to increase their utility while still maintaining their good state. For these methods to be fully effective, however, the right choice of parameters is crucial, as our sensitivity analysis shows. Moreover, long-lasting periods of low returns have a strong impact on the fund because remediation and bonus measures can strongly influence the contributor's payoff.

The remainder of the paper is structured as follows. The second section introduces the model framework and explains the processes that take place within the fund. An overview of the implementation and choice of parameters is given in the third part. Section four covers the numerical analysis. This includes examining several funding mechanisms as well as an analysis of the sensitivity of the results. Additionally, the valuation of the accounts of the insured at interim time points and capital market scenarios are studied. The final section discusses the results and concludes.

2 Model Framework

To properly control for actuarial gains and losses over time, a scenario-based stochastic approach seems natural. By performing numerical simulations, we examine how the accounts of members evolve over time. For these simulations, a representative portfolio of active insureds is looked at in a multi-period setting over a large number of years. We take the simplified balance sheet approach comparable to that in Eling and Holder (2013) or Broeders et al. (2011), which is depicted in Figure 1. We then model a pension fund by simulating the assets and liabilities of the fund limited to an individual contributor. On the asset side, this involves modeling the total contributions and a stochastic process for the rates of return. To model the asset process, a basic stock model is applied. Here, the market is simulated by a geometric Brownian motion with drift μ_{B} and volatility σ_{B}. The choice of the parameter values is based on historical data from a Swiss pension fund index (cf. Section 3.2). In this way, we are able simulate returns that correspond to those of portfolios common in the market. At the same time, the liabilities evolve according to the legally fixed minimum interest rate (cf. Section 3.1). We also look at predetermined market scenarios of high and low returns. In this way, we examine whether the pension fund models can persist over a longer period in an extreme market environment.

A	L
Assets A_t	Contributions C_t
Remediation measures K_t	Bonuses B_t

Figure 1: Simplified balance sheet in time t.

For every year that has passed, we compare how assets and liabilities relate to each other. This involves looking at the funding ratio $F_t = A_t/L_t$, which is the key regulatory measure for Swiss pension funds (BVV2, Art. 44). Depending on whether the funding ratio is higher or lower than some predefined threshold, actions are taken along predefined mechanisms (cf. Sections 2.2 and 3.3). Should the funding ratio fall below a certain limit, additional contributions by the insured may be required. Conversely, pension funds in very good health may be able to distribute a part of the existing surplus among their clients. Therefore, in our simulations, we compare different parameters and thresholds in order to identify which of them results in the highest stability with respect to the funding ratio of the pension fund and the utility of its members (cf. Section 4).
At the end of the considered time frame, the total value of the insured account, a stochastic outcome, is calculated. Applying internal rate of return calculations and suitable preference

functions, the associated utility of the contributor is derived (cf. Section 2.3). Different model mechanisms and sets of parameters lead to different levels of the utility of the insured.

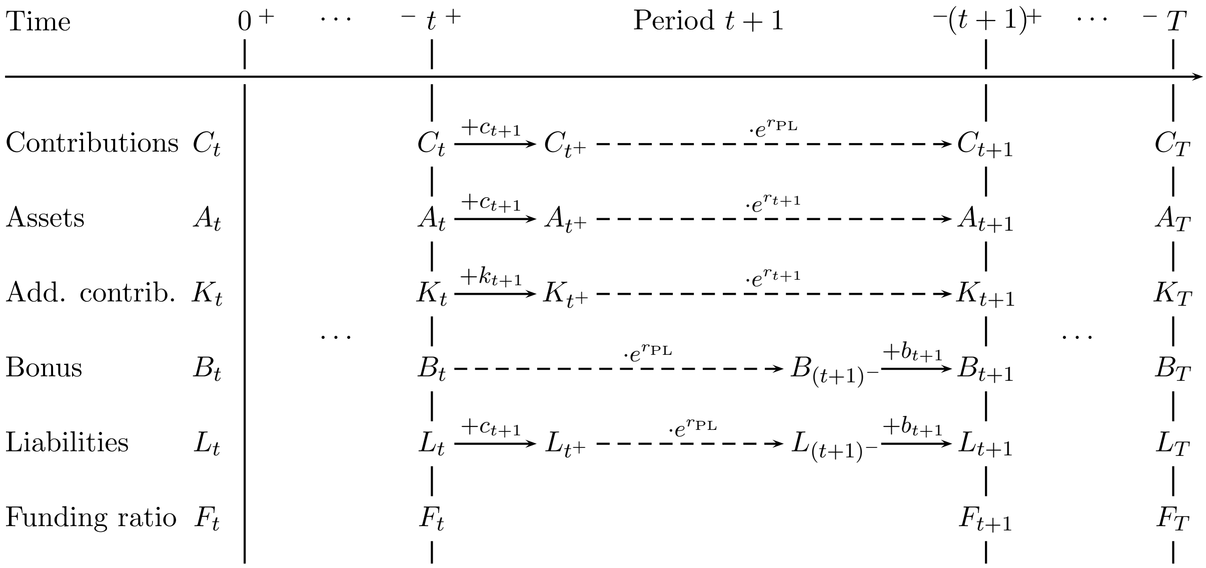


Figure 2: Illustration of the contract variables and the cash flows during the saving period along the contract timeline from time $t = 0$ until T. For the description of the variables see Sections 2.1 to 2.2.

2.1 Contributions, Asset Evolution and Funding Ratio

In the following, the processes that take place within the fund over time are outlined in detail. In our simulations, we look at an active insured that makes contributions to the fund during its working time. The age of the member is denoted by x. Furthermore, the time horizon ranges from periods one to T, i.e., $t = 0, \dots, T$. Adjustments of the key variables of the fund occur at the beginning and end of a period, i.e., in t^+ and $(t+1)^-$.

Basic Contributions The contributions c_t that the active insured makes depend on its salary as well as conversion factors linked to its age. Based on the salary S_{t+1} in period $t+1$, the coordinated period salary $\hat{S}_{t+1}$ is calculated by subtracting the coordination deduction S_{cd}. The calculated value, however, should be neither below a minimum value of $\hat{S}_{\min}$ nor above a maximum $\hat{S}_{\max}$ (BVG, Art. 8). For the coordinated salary in period $t+1$, $t = 0, \dots, T-1$, it thus holds that

$$\hat{S}_{t+1} = \min\left\{\max\left\{S_{t+1} - S_{\mathrm{cd}}; \hat{S}_{\min}\right\}; \hat{S}_{\max}\right\}. \tag{1}$$

Based on the coordinated salary $\hat{S}_{t+1}$, the contribution c_{t+1} to the fund is determined by multiplication with a contribution rate $f^{\mathrm{S}}_{x,t}$, which depends on the age x of the contributor for $t = 0, \dots, T-1$ (BVG, Art. 16), i.e.,

$$c_{t+1} = f^{\mathrm{S}}_{x,t} \cdot \hat{S}_{t+1}. \tag{2}$$

In our model, we assume that the members pay their contributions at the beginning of each period, i.e., for period $t+1$, the contribution c_{t+1} is paid at time t^+, which is the beginning of the period. During every period, the member receives a minimum interest rate r_{PL} on the sum of their contributions C_t. For the changes that take place in the sum of contributions C_t during period $t+1$, it therefore holds that the value of the contributions of the actives at time $t+1$ equals

$$C_{t+1} = (C_t + c_{t+1}) \cdot e^{r_{\mathrm{PL}}}, \tag{3}$$

with initial condition $C_0 = 0$.

Asset Evolution The assets A_t represent the funds that are available to the pension fund for paying the liabilities it has towards its members. They consist of the contributions paid as well as the returns from investing them on the capital market. As for the sum of contributions C_t, the contributions c_{t+1} in period $t+1$ are added to the assets at the beginning of each period, i.e.,

$$A_{t^+} = A_t + c_{t+1}, \tag{4}$$

with $A_0 = 0$. We assume that the fund starts with a capital of zero at time $t = 0$. The fund must be self-financing with the contributions and capital market earnings. In each period, the assets are invested on the capital market. To simulate the return on the assets, a basic stock model is applied. We use a geometric Brownian motion with drift μ_{B} and volatility σ_{B}. It thus holds that

$$dA_t = \mu_{\mathrm{B}} A_t dt + \sigma_{\mathrm{B}} A_t dW_t, \tag{5}$$

where $W = (W_t)_{t \geq 0}$ is a standard Brownian motion (cf. Björk (2004)). We assume that the pension fund only trades in a self-financing strategy. The return of the investment in period $t+1$, then, is

$$r_{t+1} = \ln \left[\frac{A_{t+1}}{A_t^+} \right] = \mu_{\mathrm{B}} - \frac{\sigma_{\mathrm{B}}^2}{2} + \sigma_{\mathrm{B}} \cdot N_{0,1}, \tag{6}$$

with $N_{0,1}$ being a standard normally distributed random variable.
For the changes that take place within the asset process in one period, it thus holds that

$$A_{t+1} = A_{t^+} \cdot e^{r_{t+1}} = (A_t + c_{t+1}) \cdot e^{r_{t+1}}. \tag{7}$$

Funding Ratio The funding ratio at the end of period $t+1$ represents an essential measure for the state of the pension fund. Its value is determined by dividing the pension fund's total assets, i.e., the equity and the sum of additional contributions K_{t+1} over the entire liabilities L_{t+1}. In fact, additional contributions from the insured may be due in case of earlier periods with underfunding (cf. Section 2.2). Hence, the funding ratio is calculated as

$$F_{t+1} = \frac{A_{t+1} + K_{t+1}}{L_{t+1}} = \frac{A_{t+1} + K_{t+1}}{C_{t+1} + B_{t+1}}, \tag{8}$$

where the liabilities L_{t+1} equal the member's total basic contributions C_{t+1} and surpluses B_{t+1} that have been distributed. Bonuses are distributed to the members when the fund is in good health (cf. Section 2.2.2). Because they represent an obligation that the pension fund has towards its insured, bonuses are attributed to the liabilities.
The funding ratio is calculated at the end of period $t+1$, i.e., at time $(t+1)^-$. Depending on its value, it is decided whether bonuses can be distributed or whether additional contributions need to be charged. In this context, a value below 100% corresponds to the fund being underfunded. Conversely, a ratio above 100% means that the pension fund is overfunded. In the following, the mechanisms of distributing bonuses and charging additional contributions are described.

2.2 Monitoring of the Funding Ratio and Funding Mechanisms

When the funding ratio exceeds a certain threshold, the existing surpluses can be distributed to the members of the fund. Conversely, in the case of underfunding, remediation measures in addition to the regular contributions may be necessary. In the following, the mechanisms that are used in our model in order to calculate distributed surpluses and additional contributions are explained in detail.

2.2.1 Situation of Underfunding and Additional Contributions

In our model, we consider a procedure that determines recovery contributions automatically if required. In practice, however, this is often not an automated process. First, the board of the pension fund needs to evaluate the underfunding with special consideration of the overall situation of the fund (e.g., capital market environment, investment portfolio structure and characteristics of the pool of members). If recovery measures have been decided upon, the employers of the insured may also be involved in covering existing deficits (BVG, Art. 65d).
Once the funding ratio drops below 100%, the assets do no longer suffice to meet all the obligations the pension fund has towards its members. Therefore, the insured may be requested to pay additional contributions in order to sustain adequate funding. In our model, we assume that whether the funding ratio is too low is determined at the end of each year. The additional contributions that are determined as a result are then paid by the clients at the beginning of the following year along with their regular contributions. In our simulations, we make use of two methods for calculating the additional contributions k_{t+1} in period $t+1$.
Our first method (UF1) is based on requesting a certain share z of the funding gap $L_t - (A_t + K_t)$ at time t from the members. This process comes into action once the funding ratio has dropped below a lower limit $F_{\min}$. The necessary amount for the additional contributions at time t^+ then, is

$$k_{t+1} = z \cdot (L_t - (A_t + K_t)), \tag{9}$$

where z represents the discretizing share of the funding gap that should be paid by the actives. The second method (UF2) for calculating the necessary remediation measures is based on a Value-at-Risk approach. Here, the additional contribution is set such that at the end of the next period, the funding ratio falls below 100% only with probability q. In this case, the expected funding ratio is evaluated through

$$\hat{F}_{t+1}(k_{t+1}) = \frac{(A_t + c_{t+1}) \cdot e^{r_{t+1}} + (K_t + k_{t+1}) \cdot e^{r_{t+1}}}{(L_t + c_{t+1}) \cdot e^{r_{\mathrm{PL}}}}, \tag{10}$$

where k_{t+1} denotes the required remediation measures. The additional contributions k_{t+1} must fulfill the equation

$$VaR_{1-q}\left(1 - \hat{F}_{t+1}\right) = \inf\left\{x \,\middle|\, \mathbb{P}\left(\hat{F}_{t+1}(k_{t+1}) < 1 - x\right) \leq q\right\} \stackrel{!}{=} 0. \tag{11}$$

Thus, in order to compute the necessary amount, the equation

$$\inf\left\{x \,\middle|\, \mathbb{P}\left(\frac{(A_t + c_{t+1}) \cdot e^{r_{t+1}} + (K_t + k_{t+1}) \cdot e^{r_{t+1}}}{(L_t + c_{t+1}) \cdot e^{r_{\mathrm{PL}}}} < 1 - x\right) \leq q\right\} \stackrel{!}{=} 0 \tag{12}$$

needs to be solved numerically for k_{t+1}. These contributions are not credited to the accounts of the insured, but add up to the assets. Because K_t is invested together with the regular assets on the capital market, the resulting market return during period $t+1$ is r_{t+1}. It therefore holds for the sum of additional contributions that

$$K_{t+1} = K_{t^+} \cdot e^{r_{t+1}} = (K_t + k_{t+1}) \cdot e^{r_{t+1}}. \tag{13}$$

2.2.2 Situation of Overfunding and Surplus Distribution

In years where market returns exceed r_{PL}, the assets of the pension fund can grow considerably. Consequently, the funding ratio will rise above 100%. In this situation, the fund can decide to distribute some of this surplus to its members (BVG, Art. 68a). In our framework, we assume that a bonus b_{t+1} is paid out at the end of a period if $F_{(t+1)^-}$ exceeds an upper limit F^{L}_{t+1}. Because it represents an obligation that the fund has towards its clients, the sum of all distributed surpluses B_{t+1} becomes part of the liabilities. The payments, therefore, lead to a growth of the liabilities and, consequently, to a drop in the funding ratio. In our model, it is assumed that the distributed amount is chosen such that from the threshold F^{L}_{t+1}, the decrease of F_{t+1} equals a value of ΔF_{t+1}. Subsequently, the distributed surplus b_{t+1} needs to be calculated such that

$$F_{t+1} = \frac{A_{t+1} + K_{t+1}}{C_{t+1} + B_{(t+1)^-} + b_{t+1}} \stackrel{!}{=} F^{\text{L}}_{t+1} - \Delta F_{t+1}. \tag{14}$$

Because the sum of bonuses B_t represents a liability, it is assumed that during a period it grows according to the interest rate r_{PL}. Thus, the value of the surplus account at time $t+1$ is

$$B_{(t+1)} = B_{(t+1)^-} + b_{t+1} = B_t \cdot e^{r_{\text{PL}}} + b_{t+1}. \tag{15}$$

In our work, we assume that surpluses are paid out to the members as a lump sum. In the market, however, it is more common to assign bonuses as increased interest rates on the contributions of the insured's. Because the second method can be interpreted through the first one, i.e.,

$$L_{t+1} = (L_t + c_{t+1}) \cdot e^{r_{\text{PL}}} + b_{t+1} \equiv (L_t + c_{t+1}) \cdot e^{r_{\text{PL}} + r_{\text{b}}} = (L_t + c_{t+1}) \cdot e^{r_{\text{eff}}}, \tag{16}$$

where it holds that

$$r_{\text{eff}} = r_{\text{PL}} + r_{\text{b}} \geq r_{\text{PL}}, \tag{17}$$

we focus only on the latter.

2.3 Contributor Valuation

To evaluate the payoff and utility of the members of the fund, we use several variables. To assess the return contributors receive on everything they have paid, we use the internal rate of return r_{c+b+k}. If the insured were only to receive the return r_{c+b+k} on their regular contributions, the value of their account at time T would be the same as if the bonuses and remediation measures were paid, i.e.,

$$\sum_{t=1}^{T} c_t \cdot e^{r_{c+b+k} \cdot (T-t+1)} \stackrel{!}{=} C_T + B_T - K_T. \tag{18}$$

To measure the utility of the clients, we use the certainty equivalent $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$. In our calculations, we use the constant relative risk aversion (CRRA) utility function, which is defined as (see Broeders et al. (2011))

$$u(x) = \frac{x^{1-\rho}}{1-\rho}, \text{ with } \rho > 0,\ \rho \neq 1. \tag{19}$$

3 Implementation and Parameterization

To simulate the evolution of both the assets and the liabilities over the course of every period, a Monte Carlo simulation is utilized. Numerical results are obtained using $N = 100\,000$ realizations in every simulation. In the following, we first introduce a reference case where we set the values for the various model parameters; this will serve as a starting point.

3.1 Legislation

We first focus on a reference case with one type of insured. For this individual, we assume that it begins to work at the age of 25 until retirement at age 65, corresponding to $T = 40$ working years. The salary is assumed to start at CHF 55 000 in the first period and to grow linearly up to CHF 82 300 in the final period. This corresponds to the average level and increase of employee salaries in Switzerland. The parameters for the reference case are described in the following and are summarized in Table 1.

Parameter	Variable	Value
Number of periods	T	40
Legislation		
Coordination deduction	S_{cd}	CHF 24 675
Minimum coordinated salary	$\hat{S}_{\min}$	CHF 3 525
Maximum coordinated salary	$\hat{S}_{\max}$	CHF 59 925
Contribution rate of age class 25 – 34	$f^{\text{S}}_{x,t}$	7%
Contribution rate of age class 35 – 44	$f^{\text{S}}_{x,t}$	10%
Contribution rate of age class 45 – 54	$f^{\text{S}}_{x,t}$	15%
Contribution rate of age class 55 – 65	$f^{\text{S}}_{x,t}$	18%
Minimum interest rate	r_{PL}	1.25%
Capital market		
Drift of the geometric Brownian motion process	μ_B	3.0%
Volatility of the geometric Brownian motion process	σ_B	5.0%
Pension fund governance		
Minimum funding ratio	$F_{\min}$	100%
Proportion of missing assets to be paid	z	90%
Quantile for additional contributions	q	1%
Upper bound for distributing surpluses	$F^{\text{L}}_{t+1} \equiv F^{\text{L}}$	110%
Difference of bonus bounds	$\Delta F_{t+1} \equiv \Delta F$	2%
Policyholder utility		
Risk aversion	ρ	30

Table 1: Input parameters for the reference case.

For the contributions, the values for all the parameters are set by the legislator. The coordination deduction S_{cd} for 2016 is fixed to CHF 24 675. The minimum and maximum coordinated salaries $\hat{S}_{\min}$ and $\hat{S}_{\max}$ amount to CHF 3 525 and CHF 59 925, respectively (BVV2, Art. 5). For the Swiss pension fund system, the contribution factor $f^{\mathrm{S}}_{x,t}$ is scaled according to the age of the contributor. Because both the salary S_t and the contribution rates $f^{\mathrm{S}}_{x,t}$, grow with the age of the client, the corresponding contributions c_t to the fund turn out to be higher in later years. For 2016, the minimum interest rate r_{PL} has been fixed at 1.25% (BVV2, Art. 12). We use this as a constant value throughout all our simulations except for when analyzing the impact of capital market scenarios in Section 4.4. There, we assume that the minimum interest rate is adjusted in order to match the prevalent capital market returns. Applying these values, it follows that the ratio of the sum of the contributions over the sum of coordinated salaries $\sum_{t=1}^{40} c_t / \sum_{t=1}^{40} \hat{S}_t$ is constant and amounts to 13.26%. Furthermore, in a setting where no bonuses are being paid, applying a fixed value for r_{PL} leads to a constant value for the liabilities at retirement. In our setting, it therefore holds that $L_{40} \equiv 283\,412$ for $b_t = 0$, $t = 1, \ldots, T$.

3.2 Capital Market

To calibrate the risk and return parameters of the asset process, the LPP-40 sub index of the Pictet LPP 2000 index, which began on 1 January 2000, is utilized. With an equity portion of 40%, this index is close to the current average of larger pension funds in Switzerland. It also contains approximately 40% of foreign currency investments.[3] Based on the monthly performance up to 31 December 2015, we choose to use the drift $\mu_{\mathrm{B}} = 3\%$ and volatility $\sigma_{\mathrm{B}} = 5\%$ for the geometric Brownian motion in our simulation.

3.3 Pension Fund Governance and Policyholder Utility

Should the returns from the capital market be low, the fund can become underfunded. Once the funding ratio falls below a lower limit $F_{\min}$, it can ask for remediation measures from its members. In the reference case, a lower limit of $F_{\min} = 100\%$ is used. In the first method for calculating the additional payments by the members, a proportion z of the $L_t - A_t$ funding gap is used. In the simulations, a value of $z = 90\%$ is employed.

The second method that is used for calculating remediation measures is based on a Value-at-Risk approach. Therein, the additional payment k_t is set such that in the following period, the pension fund becomes underfunded with a probability of q. In the base scenario, the value used for q is 1%.

Once the reserve has reached its target value and the funding ratio exceeds the limit F^{L}_{t+1}, the fund can distribute a bonus to its members. In our simulations, we use a constant upper limit of $F^{\mathrm{L}}_{t+1} \equiv F^{\mathrm{L}} = 110\%$. Furthermore, we assume that surpluses are distributed until the funding ratio has decreased to a value of $F^{\mathrm{L}} - \Delta F = 108\%$, which corresponds to $\Delta F_{t+1} \equiv \Delta F = 2\%$. In our numerical example, when calculating the policyholder utility (cf. Equation 19), we use a risk aversion factor of $\rho = 30$.

4 Numerical Analysis and Discussion

4.1 Funding Mechanisms: Impact over Time

In the following, we assess the impact of the introduction of remediation measures as well as surplus distributions. To this end, the funding ratio F_t, the additional contributions k_t and the

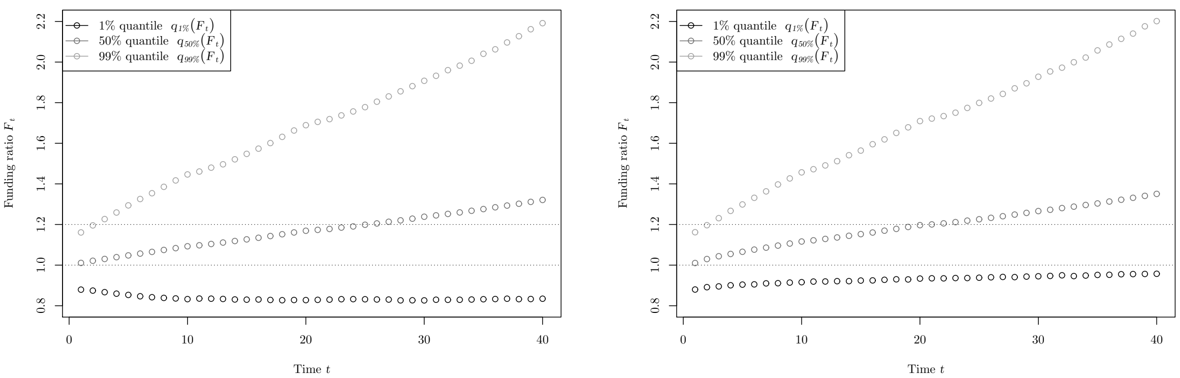
[3] For the composition of the index, see `https://www.pictet.com/corporate/en/home/institutional_investors/lpp_indices/lpp2000/`, February 2016.

bonuses b_t are analyzed. It is studied how both the average values as well as selected quantiles evolve over time. This is done for the following cases

- case (A), with neither additional contributions nor surplus distribution,
- case (B), with only additional contributions, and
- case (C), with both remediation measures and surplus distribution.

The additional contributions are calculated according to the method (UF1), and the parameters are as presented in Table 1.

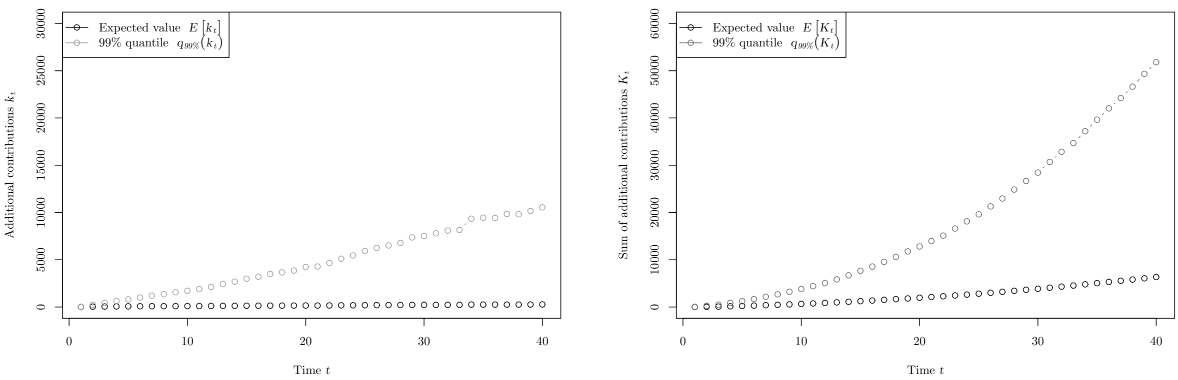
Funding Ratios and Remediation Measures In Figure 3, the 1%, 50% and 99% quantiles of the funding ratio F_t are given for the first two cases (A) and (B). Looking at the left graph, which depicts case (A) without additional contributions, it can be seen that the 1% quantile of F_t is underfunded during the whole timeframe. While it lies at approximately 90% at inception, its values quickly drop even lower and stay at approximately 80% for the following periods. The higher quantiles, however, are overfunded the whole time. The 50% quantile $q_{50\%}(F_t)$ starts at almost exactly 100% in the first period and subsequently grows regularly until it reaches approximately 130% in period 40. The introduction of remediation measures in case (B) is shown on the right. Comparing the two graphs, it can be see that the additional contributions only affect the 1% quantile. Instead of converging towards 80%, the 1% quantile now reaches almost 100% at the end of the time frame. Remediation measures lead to an improvement in the case of underfunding. For the other two quantiles, we can see that their values remain unchanged. The reason for this is that they stay well above the lower threshold $F_{\min}$ over time. The 99% quantile experiences very strong growth. Starting at a value of nearly 120%, it exceeds 220% at the end of the time frame. It can thus clearly be seen that an excess of funds exists. In these cases, the pension fund is in a position to distribute surpluses to its contributors.



(a) Quantiles of the funding ratio F_t in case (A) (b) Quantiles of the funding ratio F_t in case (B)

Figure 3: Illustration of the funding ratio F_t in the cases A (no additional contributions, no bonus payments) and B (additional contributions, no bonus payments). The graphs depict the 1%, 50% and 99% quantiles of F_t. The parameters are as in the reference case given in Table 1.

Expected Remediation Measures Staying with the same case (B), Figure 4 shows the development of the expected additional contributions k_t as well as their sum K_t from periods 1 to 40. The left graph shows the expected value of additional contributions k_t as well as the 99% quantile. It can be seen what the 99% quantile has to pay in addition in order to increase its funding ratio. Because both assets A_t and contributions C_t grow over time, the remediation measures will also grow. In the graph of Figure 4(a), we can see that the additional contributions of the 99% quantile grow and reach a maximum of more than CHF 10 000 at time $T = 40$. The expected value, however, always stays close to zero. As seen in Figure 3, the fund is always overfunded in the upper 50% of all cases, i.e., it holds that $F_t > 100\%$. Therefore, the threshold for remediation measures is never crossed, and thus, no additional payments need to be made. For the sum of contributions K_t (see Figure 4(b)), we can observe that the 99% quantile grows exponentially up to a value of more than CHF 50 000. The shape of the curve follows from the fact that the additional contributions are invested in the capital market and are credited at an interest rate of r_{t+1}. It can be seen that the expected value now exceeds zero. This case is also reported in line seven of Table 3 (see the sensitivity analysis in Section 4.2). The average annual additional contribution (if required) amounts to CHF 1 620. This amount is levied 3.9 times on average. Thus, the overall expected payments amount to approximately CHF 6 318, which corresponds to what we can see in Figure 4.



(a) Expected value and 99% quantile of the additional contributions k_t in case (B)

(b) Expected value and 99% quantile of the sum of additional contributions K_t in case (B)

Figure 4: Illustration of the remediation measures k_t and their sum K_t in case B (additional contributions, no bonus payments). The graphs depict the expected value as well as the 99% quantile of k_t and K_t. The parameters are as in the reference case given in Table 1.

Surplus Distribution In the previous case (B), the contributors pay remediation measures in years of low market returns but do not profit from better returns. Turning to case (C) of the members paying additional contributions according to method (UF1) and the fund distributing surpluses, we look at the course of the funding ratio. Figure 5 depicts the 1%, 50% and 99% quantiles of F_t from periods 1 to 40. The distribution of surpluses leads to a lower level for the 1% quantile $q_{1\%}(F_t)$. Compared to Figure 3(b), it is now located at approximately 90%. The 99% quantile $q_{99\%}(F_t)$ lies at exactly 110%, which is the value of the upper limit F^{L}. This is because surpluses are distributed once F_t exceeds this upper limit. Consequently, the funding ratio at time point t cannot surpass this value. The 50% quantile $q_{50\%}(F_t)$ starts at approximately 100% at time zero. Consequently, it converges to a value of approximately 107% and stays at this level until the end. We can thus see that the 50% quantile stays marginally below the border $F^{\mathrm{L}} - \Delta F = 110\% - 2\% = 108\%$.

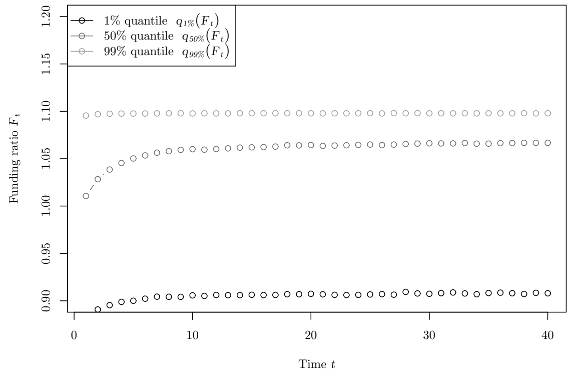


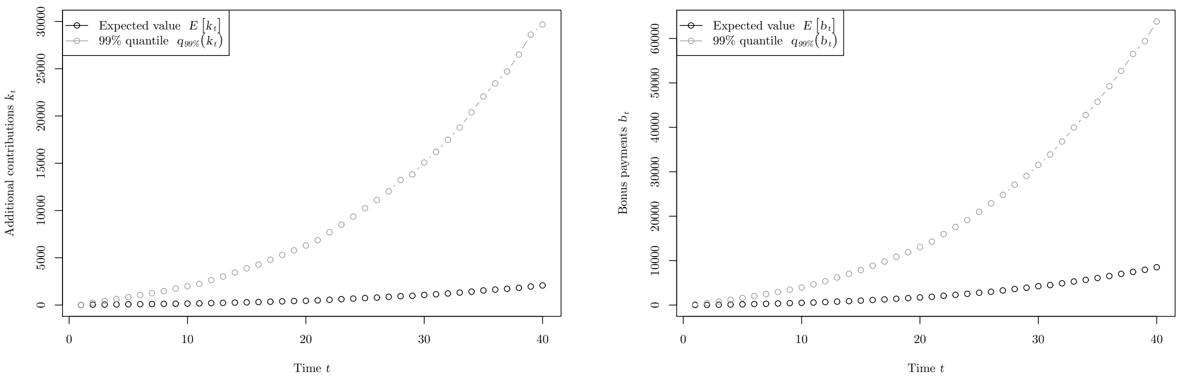
Figure 5: Illustration of the funding ratio F_t in case (C) (additional contributions, bonus payments). The graph depicts the 1%, 50% and 99% quantiles of F_t. The parameters are as in the reference case given in Table 1.

Additional Contributions and Bonus Payments Staying with case (C), Figure 6 depicts the expected value and the 99% quantile of the additional contributions k_t and the bonus payments b_t. It can be seen that both 99% quantiles, $q_{99\%}(k_t)$ and $q_{99\%}(b_t)$, show an exponential development. The reason for this is that the remediation measures are invested on the capital market and thus earn the return r_t. In contrast, the distributed surpluses are credited with the minimum interest rate r_{PL}. Subsequently, the 99% quantile of k_t reaches a maximum of approximately CHF 30 000 at time T, whereas $q_{99\%}(b_t)$ even exceeds CHF 60 000 at retirement. Thus, we can see that the surpluses that are distributed in the best cases are approximately double what the insured have to pay in the worst events. A similar conclusion can be made for the average values $\mathbb{E}[k_t]$ and $\mathbb{E}[b_t]$. While they only start to grow after time $t = 20$, their increase is exponential thereafter. At the end of the time frame in $t = 40$, the expected additional contributions have reached a value of approximately CHF 250. The expected bonus payments however, nearly reach CHF 10 000, thus exceeding $\mathbb{E}[k_t]$ by far. This is in line with the observations made regarding the funding ratio. There, the 50% quantile $q_{50\%}(F_t)$ was located at approximately 107%. Hence, the bonuses the members receive on average should be considerably higher than the additional contributions they have to pay on average. This, however, typically comes along with higher volatility in the payout stream, i.e., contributors receiving bonuses in some years will have to pay additional contributions in others (cf. discussion in Section 4.2).

4.2 Sensitivity Analysis

By performing analyses to compare the effects of various mechanisms and thresholds, optimal values for the model parameters and the funding ratio can be derived. Sensitivity analysis allows one to examine the robustness of the results. We study key indicators at the end of the savings phase. In particular, we look at the results from the perspectives of the contributors as well as the overall fund. Based on the parameters of the reference case, we analyze how sensitive the results are to changes in the input parameters. In Table 3, the columns labeled one to seven contain the input values, columns 8 to 13 the perspective of the insured, 14 to 19 the funding levels and 20 to 25 the information on the surplus distribution as well as the remediation measures. A detailed explanation of all the columns can be found in Table 2.

In the first line of Table 3, case (A) with neither a remediation measure nor a surplus distribution is analyzed. In rows two to eleven, we examine case (B) with additional contributions being paid. This includes an alteration of the minimum funding ratio $F_{\min}$ for the method for calculating



(a) Expected value and 99% quantile of the additional contributions k_t in case (C)

(b) Expected value and 99% quantile of the bonus payments b_t in case (C)

Figure 6: Illustration of the extra contributions k_t and the bonus payments b_t in case C (additional contributions, bonus payments). The graphs depict the expected value as well as the 99% quantile of k_t and b_t. The parameters are as in the reference case given in Table 1.

the additional contributions (UF1) as well as a change of $(1-q)$ for (UF2). Lines 12 to 32 cover case (C) with remediation measures and bonus payments. Therein, the upper bound for surplus distributions F^{L} is altered first. Next, we vary the difference of bonus bounds ΔF. Subsequently, the parameters $F_{\min}$ and $(1-q)$ are changed in the same way as previously in case (B). When changing a variable, we always keep the remaining ones constant, as in the reference case (cf. Table 1).

	Item	Description
1	Case	Method used for calculating the remediation measures and the distributed surpluses, (A): Neither additional contributions nor surplus distribution, (B): Only additional contributions, (C): Remediation measures and surplus distribution (cf. Section 4.1)
2	UF (indicator)	Method used for calculating the remediation measures, 1: Based on percentage of funding ratio, 2: Based on Value-at-Risk approach (cf. Section 2.2.1)
3	$F_{\min}$	Minimum funding ratio targeted (cf. Equation 8)
4	$(1-q)$	Probability for remediation measures according to the Value-at-Risk approach (cf. Equation 11)
5	Bonus (indicator)	Indication of the use of surplus distribution, 0: No bonus payments, 1: Bonus is being distributed (cf. Section 2.2.2)
6	F^{L}	Upper bound for distributing surpluses (cf. Equation 14)
7	ΔF	Difference of bonus bounds (cf. Equation 14)
8	$\mathbb{E}[L_{40}]$	Expected liabilities in $t=40$ (termination of the contribution period)

9	$\frac{\sigma[L_{40}]}{\mathbb{E}[L_{40}]}$	Relative volatility of L_{40}
10	$\gamma\left[L_{40}\right]$	Skewness of L_{40}
11	$u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$	Certainty equivalent of the insured in $t=40$ (for utility function u, cf. Equation 19)
12	$\frac{u^{-1}(\mathbb{E}[u(L_{40})])}{\mathbb{E}[C_{40}]+\mathbb{E}[K_{40}]}$	Relative certainty equivalent of the insured in $t=40$
13	$\mathbb{E}\left[r_{c+b+k}\right]$	Internal rate of return (cf. Equation 18)
14	$\mathbb{E}\left[F_t\right]$	Expected funding ratio
15	$\mathbb{E}\left[F_{F_t}^{-1}[1\%]\right]$	Expected 1% quantile of the funding ratio
16	$\mathbb{E}\left[F_{F_t}^{-1}[50\%]\right]$	Expected 50% quantile of the funding ratio
17	$\mathbb{E}\left[F_{F_t}^{-1}[99\%]\right]$	Expected 99% quantile of the funding ratio
18	$F^{-1}_{\sum_{t=1}^{40}\mathbb{1}_{\{F_t<1\}}}[1\%]$	1% quantile of the number of years in underfunding
19	$F^{-1}_{\sum_{t=1}^{40}\mathbb{1}_{\{F_t>F^{\mathrm{L}}\}}}[50\%]$	50% quantile of the number of years with F_t exceeding F^{L}
20	$\mathbb{E}\left[\sum_{t=1}^{40}k_t\right]/\sum_{t=1}^{40}c_t$	Ratio of the expected sum of additional contributions over the sum of regular contributions
21	$\mathbb{E}\left[\sum_{t=1}^{40}\mathbb{1}_{\{k_t>0\}}\right]$	Expected number of years with remediation measures being paid
22	$\mathbb{E}\left[k_t \vert k_t>0\right]$	Expected value of remediation measures when being paid
23	$\mathbb{E}\left[\sum_{t=1}^{40}b_t\right]/\sum_{t=1}^{40}c_t$	Ratio of the expected sum of distributed surpluses over the sum of regular contributions
24	$\mathbb{E}\left[\sum_{t=1}^{40}\mathbb{1}_{\{b_t>0\}}\right]$	Expected number of years with surpluses being distributed
25	$\mathbb{E}\left[b_t \vert b_t>0\right]$	Expected value of distributed surpluses when being paid

Table 2: Description of the items reported in Table 3.

Impact of Minimum Funding Ratio $F_{\mathbf{min}}$ Looking at the outcomes, the first line depicts case (A), where no additional contributions and bonuses are paid. Therefore, the mean effective return $\mathbb{E}\left[r_{c+b+k}\right]$ on the regular contributions in column 13 equals the minimum interest rate r_{PL} of 1.25%. The introduction of remediation measures according to the existing funding gap (UF1) in case (B) leads to higher overall payments by the members. Because the additional contributions k_t are not credited to the liabilities, the effective return decreases from 1.16% for a lower boundary of $F_{\min}$ equal to 90% down to 1.09% for $F_{\min}=100\%$.

At the same time, it holds for the additional contributions that a lower value for $F_{\min}$ leads to less-frequent payments of remediation measures, as can be seen in column 21. Conversely, the average amount k_t that needs to be paid is the highest for a low value of $F_{\min}$ and decreases as the boundary is raised (column 22). Taking both results together, column 20 contains the ratio of the mean sum of remediation measures over the sum of regular contributions $\mathbb{E}[\sum_{t=1}^{40} k_t]/\sum_{t=1}^{40} c_t$. Here, it can be stated that lower values of F_{min} lead to lower additional contributions on average, ranging from 2.7% for $F_{\text{min}} = 100\%$ to 1.6% for $F_{\text{min}} = 90\%$. While there are, as observed, considerable changes in the amount and frequency of additional contributions, the expected funding ratio stays nearly constant. As can be seen in column 14, the value of $\mathbb{E}[F_t]$ stays at approximately 1.21 for most values of F_{min}.

Value-At-Risk Approach Looking at the impact of a Value-at-Risk based approach (UF2) in the second case (B), we notice that the resulting changes are of greater scope. The remediation measures that result from $1-q$ increasing from 90% up to 99% are significantly higher than those for the first method. As can be seen in columns 21 and 22, additional contributions are significantly more frequent than in the previous case. The necessary payments, meanwhile, remain at a similar level as for high values of F_{min}. Consequently, the ratio of expected remediation measures over regular contributions reaches 8% in the highest case, nearly three times as much as the highest value for the funding gap approach (UF1). Due to the higher amount of additional contributions, there is an increase in the funding ratio as well. With $1-q$ rising from 90% to 99%, the mean of F_t grows from 1.25 to 1.30.

Impact of the Upper Distribution Limit $\boldsymbol{F^{\text{L}}}$ The payment of both additional contributions and surpluses is regarded in the last case (C). Overall, it can be seen that the effective returns are significantly higher than before due to the paid bonuses. A variation of the upper distribution limit F^{L} from 102% up to 118% on average leads to a less-frequent surplus distribution, as can be seen in column 24. Because the bonus that is paid on average decreases by more than CHF 1 000, the ratio of the expected sum of bonuses over the sum of contributions in column 23 decreases significantly as well. At the same time, however, the amount and the frequency of additional contributions both decrease. Looking at $\mathbb{E}[r_{c+b+k}]$, we see that, overall, a higher threshold for surplus distributions leads to a decrease in the expected effective return for the contributors. For the overall fund, however, the expected funding ratio rises together with the upper limit F^{L}, as surpluses are distributed at higher F_t values.

Bonus Bounds $\boldsymbol{\Delta F}$ A shift in the difference of bonus bounds is regarded as well. As seen in columns 23 to 25, an increase of ΔF from 0.01 to 0.06 leads to a strong increase in the bonuses that are paid on average. While the expected number of payments deceases, the ratio of the sum of the expected bonuses over the sum of contributions grows by more than 20%. Due to the larger amounts that are distributed to the clients, remediation measures become both higher and more frequent. Overall, it can be seen, however, that the expected relative return $\mathbb{E}[r_{c+b+k}]$ increases. An increase can be observed for the certainty equivalent $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$ as well. While a value of $\Delta F = 0.01$ leads to a certainty equivalent of approximately CHF 332 000, a change to $\Delta F = 0.06$ corresponds to an increase in $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$ of approximately CHF 12 000.

Insured Perspective When examining the perspective of the insured, we look at the absolute certainty equivalent $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$ and the relative certainty equivalent, which is denoted by $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)/\left(\mathbb{E}\left[C_{40}\right]+\mathbb{E}\left[K_{40}\right]\right)$. We see that in the case of raising the upper bound F^{L}, the certainty equivalent decreases together with the expected effective return $\mathbb{E}[r_{c+b+k}]$ by more than 20% from one extreme to the other. It therefore seems that clients should prefer lower boundaries for distributing surpluses. This is also supported by the relative certainty equivalent, which is the highest for a value of $F^{\mathrm{L}} = 102\%$. Contrary movements, however, can be observed when varying the difference of bonus bounds ΔF. While the absolute certainty equivalent grows together with the effective return as more surpluses are distributed, the relative certainty equivalent decreases by approximately 6%. Again, the reason for this is that, on average, higher bonus payments to the insured lead to an increased need for remediation measures in years with low capital market returns.

Impact of Remediation Measures on Surpluses Staying in the same case (C) and varying the lower border $F_{\min}$ for remediation measures, we observe that the changes for the additional contributions k_t correspond to the previous case (B). With an increasing value for $F_{\min}$, additional payments become more frequent while being lower in absolute terms. Consequently, the ratio of the expected sum of remediation measures over the sum of regular contributions is again increasing with a higher threshold $F_{\min}$. This higher threshold also leads to a slight increase in the expected funding ratio. Consequently, bonuses are, on average, distributed at approximately 1.3 more points in time. Together with an increase of the expected distributed surpluses by approximately 2%, the ratio of the sum of expected bonus payments over the sum of contributions grows by nearly 15% from one extreme to the other. Looking at the Value-at-Risk approach as well, we can see that for high values of $1-q$, both k_t and b_t grow considerably. For $1 - q = 95\%$, the ratio of the expected sum of bonuses over the sum of contributions already exceeds 100%. The remediation measures grow at a similar pace and reach a ratio of approximately 296.2% for $1 - q = 99\%$. The reason for this extreme development is that the high additional contributions cause a significant rise of the funding ratio. This can be seen from the fact that the expected funding ratio equals 107%. Subsequently, F_t often exceeds the upper limit F^{L} for distributing surpluses. Hence, there is an effect of the additional contributions being redistributed to the members as bonuses. While this leads to a strong increase in the certainty equivalent of approximately 80%, the relative certainty equivalent drops to less than 70%. This setting can therefore be seen as very unfavorable from the perspective of the members.

4.3 Interim Valuation

In today's working environment, employees change their jobs more frequently than in former times (see Cosandey (2014)). Often linked to this is a change of the pension fund associated with the employee. Therefore, the question of how the accounts of the insured are affected by these changes needs to be taken into account, especially in the case of underfunded and overfunded funds. From the fund's perspective, remediation is most important. If contributors leave, they are entitled by law to receive their regular contributions as well as the minimum interest rate that has been paid. Of particular interest are, therefore, the paid additional contributions as well as the distributed surpluses. Remediation measures are only credited to the assets A_t and not to their accounts. Therefore, they remain with the fund once the members leave. Bonuses that have been paid are credited to the accounts of the clients. They therefore take them with themselves once they change funds. It is thus of particular interest for us to examine the accounts of the contributors in early years. We also try to asses how the utility of the clients evolves with time. To do this, we look at the expected simulation results of the reference case after 10 and 20 years. Comparing them with the values at retirement, we try to assess to what degree members benefit or suffer from early changes. The results of our simulations are given in Table 4. We examine the contributions C_t paid until time t, the expected additional

	Cases and parameters							Valuation of final payoff and effective returns					
	1	2	3	4	5	6	7	8	9	10	11	12	13
	Case	UF (indicator)	$F_{\min}$	$(1-q)$ (in %)	Bonus (indicator)	F^{L}	ΔF	$\mathbb{E}[L_{40}]$ (in thousands)	$\frac{\sigma[L_{40}]}{\mathbb{E}[L_{40}]}$ (in %)	$\gamma[L_{40}]$	$u^{-1}(\mathbb{E}[u(L_{40})])$ (in thousands)	$\frac{u^{-1}(\mathbb{E}[u(L_{40})])}{\mathbb{E}[C_{40}]+\mathbb{E}[K_{40}]}$	$\mathbb{E}[r_{c+b+k}]$ (in %)
1	A							283.4	0		283.4	1.000	1.25
2	B	1	**0.90**		0			283.4	0		283.4	0.986	1.16
3	B	1	**0.92**		0			283.4	0		283.4	0.984	1.14
4	B	1	**0.94**		0			283.4	0		283.4	0.981	1.13
5	B	1	**0.96**		0			283.4	0		283.4	0.979	1.11
6	B	1	**0.98**		0			283.4	0		283.4	0.977	1.10
7	B	1	**1.00**		0			283.4	0		283.4	0.977	1.09
8	B	2		**90.0**	0			283.4	0		283.4	0.957	0.95
9	B	2		**95.0**	0			283.4	0		283.4	0.950	0.89
10	B	2		**97.5**	0			283.4	0		283.4	0.943	0.83
11	B	2		**99.0**	0			283.4	0		283.4	0.933	0.75
12	C	1	1.00		1	**1.02**	0.02	508.8	15.7	0.81	403.7	1.087	3.32
13	C	1	1.00		1	**1.06**	0.02	440.3	15.9	0.94	358.2	1.083	2.97
14	C	1	1.00		1	**1.10**	0.02	404.2	16.0	1.04	335.2	1.074	2.73
15	C	1	1.00		1	**1.14**	0.02	381.8	16.0	1.13	321.7	1.062	2.54
16	C	1	1.00		1	**1.18**	0.02	365.7	15.9	1.21	313.0	1.051	2.38
17	C	1	1.00		1	1.10	**0.01**	398.7	16.0	1.07	332.4	1.073	2.69
18	C	1	1.00		1	1.10	**0.02**	404.2	16.0	1.04	335.2	1.074	2.73
19	C	1	1.00		1	1.10	**0.03**	409.7	16.1	1.02	337.7	1.073	2.77
20	C	1	1.00		1	1.10	**0.04**	415.3	16.2	1.00	340.0	1.071	2.81
21	C	1	1.00		1	1.10	**0.05**	421.1	16.3	0.97	342.1	1.067	2.84
22	C	1	1.00		1	1.10	**0.06**	427.1	16.5	0.94	344.1	1.062	2.88
23	C	1	**0.90**		1	1.10	0.02	388.7	16.5	1.11	323.8	1.089	2.74
24	C	1	**0.92**		1	1.10	0.02	391.8	16.4	1.10	326.4	1.086	2.73
25	C	1	**0.94**		1	1.10	0.02	395.4	16.2	1.08	329.0	1.082	2.73
26	C	1	**0.96**		1	1.10	0.02	399.2	16.1	1.07	331.8	1.079	2.73
27	C	1	**0.98**		1	1.10	0.02	402.6	16.0	1.05	334.1	1.075	2.73
28	C	1	**1.00**		1	1.10	0.02	404.2	16.0	1.04	335.2	1.074	2.73
29	C	2		**90.0**	1	1.10	0.02	473.3	15.4	0.89	382.6	1.032	2.83
30	C	2		**95.0**	1	1.10	0.02	562.1	16.9	0.85	429.3	0.969	3.01
31	C	2		**97.5**	1	1.10	0.02	753.6	20.2	0.87	515.8	0.854	3.28
32	C	2		**99.0**	1	1.10	0.02	1 223.8	24.1	0.92	702.7	0.698	3.69

Table 3: Valuation of final payoff and effective returns in cases (A), (B) and (C) (see Section 4.1). The parameter values are as in Table 1.

	Funding levels, bonuses and additional contributions											
	14	15	16	17	18	19	20	21	22	23	24	25
	$\mathbb{E}[F_t]$	$\mathbb{E}[F_{F_t}^{-1}[1\%]]$	$\mathbb{E}[F_{F_t}^{-1}[50\%]]$	$\mathbb{E}[F_{F_t}^{-1}[99\%]]$	$F^{-1}_{\sum_{t=1}^{40} \mathbb{1}_{\{F_t<1\}}}[1\%]$	$F^{-1}_{\sum_{t=1}^{40} \mathbb{1}_{\{F_t>F^L\}}}[50\%]$	$\mathbb{E}\left[\frac{\sum_{t=1}^{40} k_t}{\sum_{t=1}^{40} c_t}\right]$ (in %)	$\mathbb{E}\left[\sum_{t=1}^{40} \mathbb{1}_{\{k_t>0\}}\right]$	$\mathbb{E}[k_t \mid k_t>0]$ (in thousands)	$\mathbb{E}\left[\frac{\sum_{t=1}^{40} b_t}{\sum_{t=1}^{40} c_t}\right]$ (in %)	$\mathbb{E}\left[\sum_{t=1}^{40} \mathbb{1}_{\{b_t>0\}}\right]$	$\mathbb{E}[b_t \mid b_t>0]$ (in thousands)
1	1.18	0.84	1.15	1.60	0	25						
2	1.20	0.90	1.16	1.61	0	26	1.6	0.5	6.97			
3	1.20	0.91	1.16	1.61	0	26	1.9	0.8	5.43			
4	1.21	0.91	1.16	1.61	0	27	2.1	1.2	4.17			
5	1.21	0.92	1.17	1.61	0	27	2.4	1.8	3.14			
6	1.21	0.93	1.17	1.61	0	27	2.6	2.6	2.32			
7	1.21	0.93	1.17	1.62	0	27	2.7	3.9	1.62			
8	1.25	0.98	1.20	1.64	0	32	5.0	6.3	1.83			
9	1.27	1.00	1.22	1.66	0	34	5.9	7.1	1.94			
10	1.28	1.01	1.23	1.67	0	35	6.8	7.8	2.04			
11	1.30	1.04	1.25	1.69	0	37	8.0	8.5	2.20			
12	0.98	0.88	1.00	1.02	10	19	35.8	15.6	5.36	85.6	18.8	10.61
13	1.01	0.90	1.03	1.05	4	15	19.2	10.9	4.13	59.6	14.7	9.42
14	1.04	0.91	1.05	1.09	1	12	11.7	7.8	3.47	45.9	11.7	9.14
15	1.07	0.91	1.07	1.12	0	9	7.9	6.1	3.00	37.5	9.5	9.16
16	1.09	0.92	1.09	1.16	0	8	5.8	5.2	2.61	31.5	7.9	9.30
17	1.05	0.91	1.06	1.09	1	13	10.6	7.4	3.35	43.8	12.7	8.05
18	1.04	0.91	1.05	1.09	1	12	11.7	7.8	3.47	45.9	11.7	9.14
19	1.04	0.90	1.05	1.09	2	11	12.7	8.3	3.58	48.0	10.8	10.32
20	1.03	0.90	1.05	1.09	2	10	13.9	8.8	3.69	50.1	10.1	11.59
21	1.03	0.90	1.04	1.09	2	9	15.1	9.2	3.82	52.3	9.4	12.94
22	1.03	0.90	1.03	1.09	3	9	16.5	9.7	3.95	54.6	8.9	14.36
23	1.03	0.87	1.04	1.09	1	10	5.7	1.0	12.80	40.0	10.4	8.98
24	1.03	0.88	1.05	1.09	1	10	6.9	1.5	10.47	41.2	10.7	9.01
25	1.04	0.89	1.05	1.09	1	11	8.3	2.3	8.41	42.6	11.0	9.05
26	1.04	0.90	1.05	1.09	1	11	9.8	3.5	6.58	44.0	11.3	9.08
27	1.04	0.90	1.05	1.09	1	11	11.1	5.2	4.95	45.3	11.6	9.12
28	1.04	0.91	1.05	1.09	1	12	11.7	7.8	3.47	45.9	11.7	9.14
29	1.06	0.94	1.07	1.09	0	17	35.4	16.6	4.96	72.2	16.7	10.07
30	1.06	0.96	1.09	1.09	0	21	64.8	37.5	4.04	106.1	20.7	11.94
31	1.07	0.98	1.11	1.11	0	26	130.8	39.0	7.82	180.1	25.4	16.50
32	1.07	1.00	1.13	1.13	0	31	296.2	39.0	17.71	363.4	30.7	27.64

Table 3: Valuation of final payoff and effective returns in cases (A), (B) and (C) (see Section 4.1). The parameter values are as in Table 1 (continued).

	Case	C_t (in thousands)	$\mathbb{E}[K_t]$ (in thousands)	$\mathbb{E}[L_t]$ (in thousands)	$\frac{\mathbb{E}[L_t]}{C_t+\mathbb{E}[K_t]}$	$\frac{u^{-1}(\mathbb{E}[u(L_t)])}{C_t+\mathbb{E}[K_t]}$
$t = 10$	A	25.1	0	25.1	1	1
	B	25.1	0.67	25.1	0.974	0.974
	C	25.1	0.85	27.3	1.055	1.008
$t = 20$	A	71.7	0	71.7	1	1
	B	71.7	2.03	71.7	0.972	0.972
	C	71.7	4.02	85.8	1.133	1.031
$t = 40$	A	283.4	0	283.4	1	1
	B	283.4	6.78	283.4	0.977	0.977
	C	283.4	28.72	404.2	1.295	1.074

Table 4: Simulation results for cases (A), (B) and (C) after 10, 20 and 40 years, respectively. The parameter values are as in Table 1. The case $t = 40$ yields the same results as in Table 3 (see marked rows).

contributions $\mathbb{E}[K_t]$ paid, the expected liabilities $\mathbb{E}[L_t]$, the ratio of expected liabilities over the sum of the contributions, the expected remediation measures $\mathbb{E}[L_t]/(C_t+\mathbb{E}[K_t])$ and the relative certainty equivalent $u^{-1}(\mathbb{E}[u(L_t)])/(C_t+\mathbb{E}[K_t])$. This is done for the three scenarios (A), (B) and (C) at time points $t = 10$, 20 and 40.

Valuation at Time $t = 10$ Additional contributions represent funds that the insured pay but cannot take with them once they leave the organization. It is therefore of particular interest for us to analyze what impact they have in those cases. Looking at the values after 10 years in cases (B) and (C), we see that the expected sum of remediation measures $\mathbb{E}[K_t]$ stays well below CHF 1 000. Because the sum of the contributions C_t already amount to more than CHF 25 000 at that time, this can be considered a low amount. Furthermore, we observe that in comparison to the values at retirement, less than 10% of $\mathbb{E}[K_t]$ is paid in the first ten years in case (B). The introduction of surplus distribution in (C) leads to a small increase of the remediation measures. At the same time, however, the ratio of the expected sum of additional contributions in years $t = 10$ and 40 decreases to less than 3%. Additionally, the bonuses the members receive lead to a growth in the expected liabilities of more than CHF 2 000, more than twice the amount of the remediation measures. Thus, we can see that the clients benefit significantly from the distribution of surpluses. Consequently, the ratio of expected liabilities over the sum of contributions and expected remediation measures $\mathbb{E}[L_t]/(C_t+\mathbb{E}[K_t])$ grows and is lager than one. The same holds for the relative certainty equivalent $u^{-1}(\mathbb{E}[u(L_t)])/(C_t+\mathbb{E}[K_t])$. However, the distribution of surpluses also leads to higher remediation measures and, thus, to a higher volatility. Due to the greater uncertainty connected to this, the relative certainty equivalent turns out to be almost 5% lower than the ratio of the expected liabilities and the sum of regular and expected additional contributions.

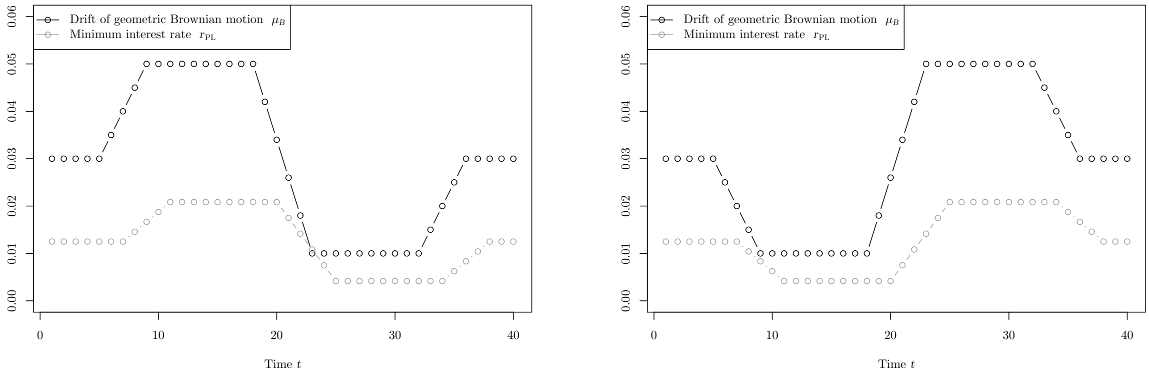
Valuation at Time $t = 20$ Having analyzed the simulation results in $t = 10$, we now look further ahead and examine the outcomes after 20 years. Here we assess how the previous insights develop over time. For the expected sum of additional contributions, it now holds that they exceed CHF 2 000 in case (B), corresponding to an increase by a factor of three. Furthermore, they reach approximately 30% of the expected value at the end of the time frame. Once again, distributing surpluses leads to an increase of $\mathbb{E}[K_t]$. While the absolute amount doubles, the ratio of the expected sum of additional contributions in year $t = 20$ and 40 decreases to less than 15%. Moreover, the distribution of bonuses leads to the expected liabilities gaining approximately CHF 14 000. This increase is more than five times larger than in time $t = 10$. We thus observe that while an increase of the remediation measures takes place, the growth of the distributed bonuses is even larger. This also extends to the ratio of expected liabilities over the sum of regular and expected additional contributions. Together with the strong increase in distributed surpluses, the ratio grows by approximately 7%. In the case without bonus distributions, it stays roughly the same at 0.972. The development of the relative certainty equivalent reflects this as well. While remaining nearly unchanged in case (B), it experiences a gain of more than 2% in case (C). As before, the relative certainty equivalent is smaller than the ratio of the expected liabilities and the sum of regular and expected additional contributions. The difference, which is again caused by a higher volatility, amounts to approximately 10%.

Discussion Overall, we see that in early years, the amounts that are being paid remain rather low. Due to the salary of the insured as well as the conversion factors growing over time, most of the contributions are paid towards the end of the time frame. Consequently, the absolute amounts that are necessary in the case of underfunding remain fairly low in early years. This holds especially for cases with payment of both additional contributions and bonuses. The consequence is that for changes of pension funds in early years, the extent to which contributors cannot take remediation measures with themselves remains fairly low. It can thus be said that clients' losses, which are connected to a change of pension funds, remain fairly moderate in early years. Additionally, we were able to see that a distribution of surpluses leads to a progressive growth of the relative certainty equivalent over time.

4.4 Impact of Capital Market Scenarios

In the following, we analyze specified capital market scenarios. Letting the returns on the assets follow a predefined course, we are able to examine how the model responds to periods of very low as well as very high returns. We consider the reference case and let the drift μ_B of the geometric Brownian motion follow a predefined path using two scenarios. In the first one, the value for μ_B is identical to the reference case for the first five periods. This is then followed by ten periods with very high returns of 5%. Subsequently, the drift drops to a low value of only 1% for another ten periods. After that, it recovers and returns to 3% again for the last five time points. For the second scenario, the course of the drift μ_B is mirrored. After the beginning, the drift drops to 1%, after which it rises to 5%. Similarly to the first case, a value of 3% is followed at the beginning and at the end. For the minimum interest rate r_{PL}, we assume that it follows μ_B at a ratio of $\mu_B/r_{\mathrm{PL}} = 1.25/3 = 41.67\%$ with a delay of two years. In this way, a deferred adaptation of r_{PL} to the market conditions is to be simulated.[4]
The paths of μ_B and r_{PL} for both scenarios are depicted in Figure 7. With our choices for the scenarios, we want to analyze what impact high and low capital market returns have. In particular, we try to assess their effects at early as well as late time points in our time frame.

[4]This reflects practice in many countries where the minimum interest rate is adapted through a political process (BVG, Art. 15).



(a) μ_B and r_{PL} for Scenario 1.

(b) μ_B and r_{PL} for Scenario 2.

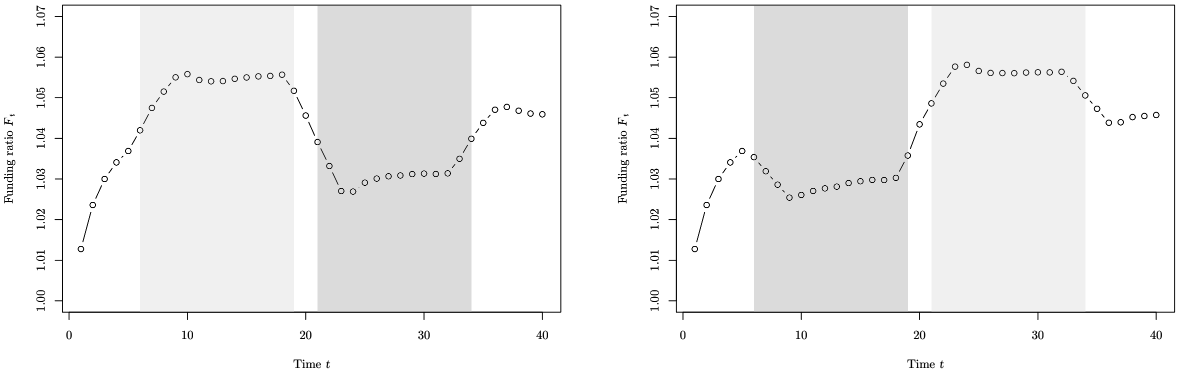
Figure 7: Illustration of the drift μ_B of the geometric Brownian motion and the minimum interest rate r_{PL} for scenarios 1 and 2.

Simulation Results The simulation results of the particular scenarios are shown in Table 5. The values for the reference case are also given. Figures 8 to 10 show the development of the means of the funding ratio F_t, the remediation measures k_t and the distributed surpluses b_t from periods 1 to 40. The periods of increased and decreased drift μ_B are shown as light and dark gray areas, respectively.

	$\mathbb{E}[F_t]$	$\mathbb{E}\left[\sum_{t=1}^{40} \mathbb{1}_{\{k_t>0\}}\right]$	$\mathbb{E}[k_t \| k_t > 0]$ (in thousands)	$\mathbb{E}\left[\sum_{t=1}^{40} \mathbb{1}_{\{b_t>0\}}\right]$	$\mathbb{E}[b_t \| b_t > 0]$ (in thousands)	$\mathbb{E}[L_{40}]$ (in thousands)	$\mathbb{E}[r_{c+b+k}]$ (in %)	$\frac{\sigma[L_{40}]}{\mathbb{E}[L_{40}]}$ (in %)	$u^{-1}(\mathbb{E}[u(L_{40})])$ (in thousands)	$\frac{u^{-1}(\mathbb{E}[u(L_{40})])}{\mathbb{E}[C_{40}]+\mathbb{E}[K_{40}]}$
Reference case	1.04	7.8	3.47	11.7	9.14	404.2	2.73	16.0	335.2	1.074
Scenario 1	1.04	8.1	4.25	11.8	8.02	377.1	2.18	15.0	319.6	1.037
Scenario 2	1.04	8.1	3.04	11.9	10.69	443.4	3.30	16.9	357.1	1.110

Table 5: Simulation results for the reference case (C) and scenarios 1 and 2 (cf. Table 1).

Funding Ratio F_t Looking at the first scenario in Figure 8(a), it can be seen that during the times of increased capital market returns, the funding ratio increases sharply. From its starting value of approximately 101%, it rises to more than 105% within less than ten periods. Once having arrived at this value, it stays nearly constant. The decrease of μ_B, which begins at period 20, also has an immediate impact on F_t. In response to the lowered capital market returns, the mean funding ratio falls to less than 103%. As in the case of high returns, F_t converges quickly to this value and subsequently changes only little. The recovery of μ_B to 3% at the end of the time frame also leads to the funding ratio returning to 104%. Despite the strong variation in the course of F_t over the 40 periods, it can be seen in Table 5 that, on average, $\mathbb{E}[F_t]$ equals 104%, which is exactly the same as in the reference case.
For the second scenario, which is given in Figure 8(b), the development of F_t is similar. After an increase during the first five periods, the low drift causes the mean funding ratio to drop to a value of approximately 102.5%. The subsequent periods of high capital market returns then



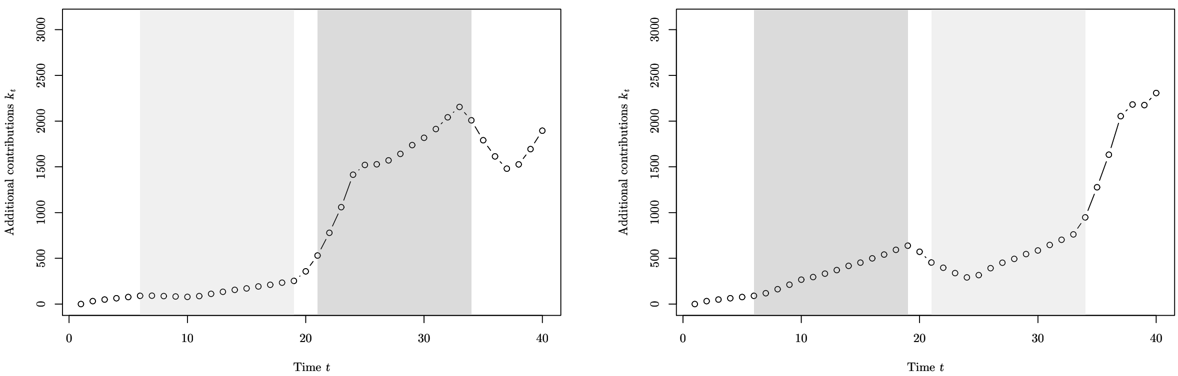
(a) Mean of the funding ratio F_t in scenario 1.

(b) Mean of the funding ratio F_t in scenario 2.

Figure 8: Illustration of the means of the funding ratio F_t in scenarios 1 and 2.

bring about an increase of F_t up to nearly 106%. With μ_B returning to 3% during the last five periods, the funding ratio decreases to approximately 104%. As in the first scenario, the mean of the funding ratio reacts quickly to changes in μ_B and stays nearly constant once the drift remains unchanged over a longer time. The expected value of F_t over all 40 periods is again equal to 104%. It can thus be seen that two periods of high and low capital market returns of similar severity and length lead to the expected funding remaining unchanged, regardless of how the two are ordered.

Remediation Measures k_t Turning to Figure 9, we examine the means of the remediation measures contributors are required to pay in each scenario. Looking at the graph for the first case, which is given on the left, it can be seen that k_t initially stays very low. The additional contributions clients have to pay, on average, hardly exceed CHF 200. This is because the high capital market returns lead to the fund being overfunded, as seen in Figure 8(a). Consequently, remediation measures are only required in rare cases. With the subsequent drop of the drift μ_B, the additional payments escalate very quickly. Following a strong increase, k_t reaches its maximum of more than CHF 2 000 right before the capital market returns start to recover. Towards the end of the time frame, it then settles at approximately CHF 1 800. The



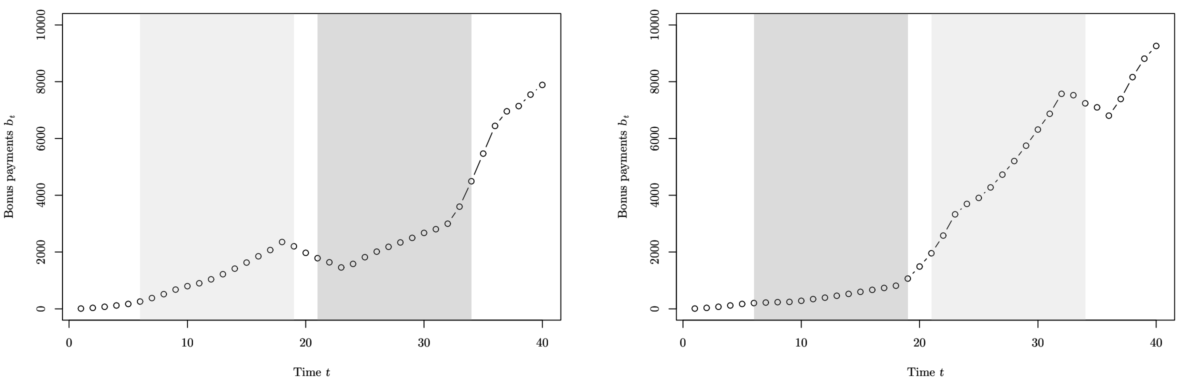
(a) Mean of the remediation measures k_t in scenario 1.

(b) Mean of the remediation measures k_t in scenario 2.

Figure 9: Illustration of the means of the remediation measures k_t in scenarios 1 and 2.

graph of the remediation measures for the second scenario shows a different progress. Being faced with a low drift μ_B shortly after the beginning, it grows regularly until the end of period 20, exceeding a value of CHF 600. The successive time points of higher capital market returns then cause the mean of the additional contributions to halve. While the drift μ_B equals 5%, the remediation measures again grow only slowly. As a response to the subsequent return of the drift to 3%, a strong increase in the mean of k_t takes place. In time $t = 40$, the mean of the remediation measures reaches more than CHF 2 000, which is close to the value of scenario 1. Comparing the two cases, it can be seen in Table 5 that in both cases the expected number of additional payments is exactly the same. However, they exceed the number of payments in the reference case. Furthermore, the expected amount that has to be paid is significantly higher for the first scenario, reaching more than CHF 4 250. The second scenario is approximately 30% lower than that value, reaching approximately CHF 3 000. The reference case is located almost in the middle between the two cases, with a value of CHF 3 470. The reason for the differences in $\mathbb{E}\left[k_t | k_t > 0\right]$ can be found in the development of the contributor accounts over time. Should the capital market returns be very low at early time points, the absolute amounts that are needed to compensate for this are still relatively low. This can be seen in Figure 9(b), where the rise of the mean of the remediation measures is rather small. When, in contrast, there is underfunding at later points in time, the required monetary amounts are much larger. The reason for this is that the accounts of the assets A_t and the contributions C_t are much higher. Therefore, the same change in percentages requires higher monetary amounts.

Distributed Surpluses b_t For the bonuses b_t in Figure 10, we can observe a similar pattern as for the additional contributions k_t. For the first scenario, the mean of the bonus payments increases very little at the beginning, reaching a maximum of approximately CHF 2 000. The low capital market returns in the second half of the time frame then cause a slight decline, which is followed by an increase similar to that at the beginning. The recovery of μ_B to 3% at the end then leads to a strong increase to nearly CHF 8 000. For the second case, the payments



(a) Mean of the distributed surpluses b_t in scenario 1.

(b) Mean of the distributed surpluses b_t in scenario 2.

Figure 10: Illustration of the means of the distributed surpluses b_t in scenarios 1 and 2.

during the first 20 periods only reach approximately CHF 1 000 due to the low capital market returns. The high returns in later times, however, lead to very high surpluses being distributed. Due to the higher accounts of the members at those later time points, a maximum of CHF 7 500 is reached. The subsequent return of μ_B to 3%, then, only leads to a small decrease in the paid bonuses.

The simulation results in Table 5 again show a similar pattern as for the remediation measures k_t.

While the expected number of bonus payments amounts to nearly 12 for both scenarios, the expected distributed surpluses are substantially higher in the second scenario, reaching CHF 10 690 in comparison to the CHF 8 020 of the first one. Again, the reference case has both fewer time points where bonuses are being paid and an expected value for b_t that falls between the two cases.

Liabilities and Certainty Equivalent The results we see for the bonuses are the same for the liabilities. Because the second scenario has higher distributed surpluses than the first, the expected values of its liabilities $\mathbb{E}[L_{40}]$ are higher as well. Overall, the difference in the expected liabilities amounts to nearly CHF 65 000. For the effective return $\mathbb{E}\left[r_{c+b+k}\right]$, the differences in the remediation measures as well as the distributed surpluses have the same effects and thus reinforce each other. In the second scenario, members receive an effective return of 3.30% on their overall payments. The first case, however, only reaches a return of 2.18%, with the reference case once again falling in between, with a value of 2.73%. With regard to the certainty equivalent $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)$, the results are similar to the expected liabilities. While the second scenario reaches a value of nearly CHF 360 000, the first one only manages to exceed CHF 319 000. The reference case is located between the two cases with a certainty equivalent of approximately CHF 335 000. This trend continues when looking at the relative certainty equivalent $u^{-1}\left(\mathbb{E}\left[u\left(L_{40}\right)\right]\right)/\left(\mathbb{E}\left[C_{40}\right]+\mathbb{E}\left[K_{40}\right]\right)$. Here, the reference case has a value of 0.774. The first scenario is comparatively close to this, reaching 1.037. The second scenario however, has a considerably higher value and reaches 1.11. Another important aspect can be seen when analyzing the relative volatility $\sigma\left[L_{40}\right]/\mathbb{E}\left[L_{40}\right]$. While the first scenario has the lowest fluctuation, with a value of 15%, the reference case already has more variation, reaching 16%. While obtaining the highest effective returns, the second scenario is also coupled with strong fluctuations, leading to a relative volatility of 16.9%. Consequently, contributors have to face a more volatile payoff in this case.

5 Discussion and Conclusion

In the following, we discuss and summarize the main results of our simulations. This involves trying to assess the reasons for certain outcomes as well as the implications that follow from them.

Remediation Measures In the first part of our work, we analyze the effects of remediation measures as well as surplus distribution. Looking at scenarios without (A) and with additional contributions (B), we observe a distinct improvement of funding levels connected with their charging. In a graphical analysis, the 1%, 50% and 99% quantiles of the funding ratio F_t are examined over the whole savings phase. While the two higher quantiles were in good funding the whole time, the 1% quantile remained well below 100% when no remediation measures were charged. With the introduction of additional contributions in case (B), we are able to see a distinct improvement of this subgroup, moving from approximately 80% up to nearly 100% at time T. Furthermore, we can see in Table 3 that the additional expenditures the members have, remain fairly low compared with their regular contributions. At the same time, even in scenarios with very low returns on assets, the payment of additional contributions enables the funding ratio to stay at a sufficiently high level. Thus, utilizing remediation measures leads to a stabilization of the whole fund.

Surplus Distribution While additional contributions are charged with regard to the health of the fund, the distribution of surpluses addresses the utility of the insured. In our simulations, we see that without bonuses being paid, the funding ratio rises well above 100% on average. It is thus possible to distribute excessive funds to the clients while keeping the fund in good health. Looking at case (C) with remediation measures and surplus distribution, our results show that a distinct increase in the certainty equivalent of the insured takes place. Additionally, the bonuses the clients receive exceed the remediation measures they have to pay to the fund. The average funding ratio, meanwhile, remains stable and well above 100%.

Calibration Charging remediation measures leads to an improved stability of pension funds. While distributing excessive funds (to a reasonable extent), the utility of the contributors increases. For these mechanisms to be fully effective, however, an adequate assignment of all model parameters is essential. We see this when analyzing the sensitivity of our model. To do this, we vary the selected parameters and examine the changes in the output variables. The conclusion is that small changes in variables can already lead to strong alterations of the outcomes. For example, among other things, a decrease in the minimum funding ratio $F_{\min}$ of only 2% in the reference case (C) leads to an increase in the expected remediation measures k_t of more than 40%. It is thus of great importance to calibrate the respective model.

Interim Valuation With remediation measures possibly reaching large sums, the processes of entering and leaving pension funds are of great importance. The reason for this is that additional contributions remain with the fund when members leave. Because employees tend to change their jobs more often in early years, we analyze the state of the fund in the reference case after 10 and 20 years, respectively. Among others, the results show that the accounts of the insured are still fairly low at these points in time. Therefore, the additional contributions that had to be paid thus far remain relatively small as well. The same conclusion can be drawn for the distributed bonuses. For early years, it can therefore be concluded that a change of pension funds can be made without a large impact on the savings of the members. However, because the majority of contributions is paid in later years, this circumstance changes over time.

Capital Market Scenarios Prompted by recent crashes on the capital market, we analyze the impact of such events on pension funds. We let the capital market returns of our model follow a predefined path, imitating periods of both very low and very high returns. The results show that additional contributions can rise substantially if a funding gap occurs. This holds especially for later years, when the insured have already accumulated a large amount of capital on their accounts. The same is true for the distributed surpluses. When there is an excess of assets due to high market returns, this will turn out to be much higher in late years (e.g., shortly before retirement) than in early ones. Furthermore, the amounts that are charged or distributed can make up a great percentage of the overall cash flows. In one of the market scenarios we look at, the remediation measures reach nearly 10% of the overall liabilities, while in another scenario, the expected distributed surpluses reach nearly 30% of the expected liabilities. Capital market scenarios therefore have a great impact and need to be taken into account with close attention.

Concluding Remarks This work analyzes the value of the accounts of the insured at retirement. In our simulations, we are able to see that the funding mechanisms and their calibration have a strong impact on both the stability of the overall fund as well as the utility of its contributors. However, in this work, we do not take the mortality of members into account. Moreover, the charging of remediation measures and the distribution of surpluses are assumed to be automated. In practice, however, decisions concerning these actions would ultimately be made by the board of the fund. Also, higher administrative costs should be accounted for. Furthermore, a simplified capital market model without a detailed investment strategy is implemented. While our model is designed to fit the Swiss pension fund system, we assume that an adaptation to other countries and extension of the results is straightforward.

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