Capital Asset Pricing Model

Based on a Generalized Economic Index of Riskiness*

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Abstract

This paper establishes a generalized economic index of riskiness which includes the risk measures proposed by Aumann and Serrano (2008), Foster and Hart (2009) and Bali et al. (2011) as special cases. Based on the index, we derive the capital pricing equilibrium in a reward-risk framework and propose the statistical estimation and tests. The empirical study shows that our model can better capture the cross section of stock returns than the traditional mean-variance capital asset pricing model. The findings suggest that the traditional pricing model could overestimate the magnitude of the abnormal portfolio returns by mis-evaluating the systematic risk.

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1 Introduction

Asset pricing models have been one of the cornerstones in the finance literature since the seminal contributions of Sharpe (1964), Lintner (1965) and Mossin (1966). Following their mean-variance Capital Asset Pricing Model (CAPM), the literature has been attempting to adopt different risk measurements such as general lower partial moments (Jean, 1975; Bawa, 1975; Unser, 2000), value at risk (Jorion, 1997), expected shortfall (Acerbi and Tasche, 2002; Rockafellar and Uryasev, 2002) and so on, to better capture the cross section of stock returns. Recently, De Giorgi and Post (2008) have developed a general mean-risk framework for portfolio selection from an axiomatic definition of reward and risk measures to derive the capital market equilibrium. However, all of the above risk measures share a common drawback as pointed out by Aumann and Serrano (2008) who claim that most well-known riskiness measures in the literature are not monotonic with respect to second-degree stochastic dominance (SSD).

SSD is the necessary and sufficient condition for all risk-averse investors preferring one risky asset to another. According to the definition of "risk" proposed by Machina and Rothschild (2008, 7:193), "risk is what risk-averters hate." Thus, if portfolio A dominates portfolio B in terms of SSD, then portfolio A should be considered less risky than portfolio B. In other words, a normatively appealing measure of riskiness should give portfolio A a better risk ranking than portfolio B. This property is referred to as "monotonicity with respect to SSD".[1] Since the above commonly-used riskiness measures are not monotonic with SSD,[2] it is possible to conclude that portfolio A is riskier than B while using these indices. This mis-ranking could further weaken the empirical performance of capturing the cross section of stock returns.

The main purpose of this paper is to establish a CAPM based on a risk index which is monotonic with respect to SSD. Specifically, there are four tasks in this paper. First, we propose a generalized economic index of riskiness which satisfies the monotonicity with respect to SSD and includes the recently developed indices[3] with economic foundations as special cases. The

[1] De Giorgi (2005) refers to this property as "isotonicity with respect to SSD".

[2] Some of the above risk indices, e.g., the index proposed by De Giorgi and Post (2008), are monotonic with respect to SSD under the condition that the means of the evaluated portfolios are the same.

[3] These recently developed riskiness indices with economic foundations have been applied to several problems in finance. For example, Homm and Pigorsch (2012) modify the Sharpe ratio by replacing the standard deviation by the index proposed by Aumann and Serrano (2008). They further use this new performance measure to evaluate

indices satisfying the monotonicity with respect to SSD in the literature have two types of economic meaning. In Aumann and Serrano (2008), the index of riskiness can be viewed as the reciprocal of the critical degree of absolute risk aversion of the decision maker with an exponential utility function who is indifferent between accepting and rejecting the gamble.[4] On the other hand, the indices constructed by Foster and Hart (2009) and Bali et al. (2011) can be viewed as the critical wealth level for an agent with respectively a log and power utility function who is indifferent between acceptance and rejection. Our paper combines both of these two economic interpretations by bridging our index as an affine function of the wealth, as well as an affine function of the reciprocal of the degree of absolute risk aversion. To do so, we adopt a hyperbolic absolute risk aversion (HARA) utility function. Intuitively, the integration can succeed due to the properties of HARA utility functions whereby (1) the reciprocal of the degree of absolute risk aversion is a linear function of wealth, and (2) the exponential, log and power utility functions belong to the HARA utility function family.[5] We then show that the proposed index satisfies several desirable properties, such as homogeneity, subadditivity and convexity. Moreover, it satisfies monotonicity with respect to SSD.

Second, we examine the equilibrium asset price in a mean-risk framework based on our newly-proposed index. By employing the riskiness index of the excess return as the risk measure in the asset allocation problem, we find that the well-known two-fund monetary separation theorem still holds. A new one-factor asset pricing model is then established. The model is denoted as a "generalized risk CAPM" (denoted as GR-CAPM) since it is obtained by using a generalized risk index. Recently, due to requiring risk measures that satisfy certain properties, Kadan et al. (2015) have generalized systematic risk by adoping both equilibrium and axiomatic approaches. Our paper, however, differs from Kadan et al. (2015) in three ways. First, we innovate a new risk

the performance of mutual funds and hedge funds. Chen et al. (2014) use the index of Aumann and Serrano (2008) as the risk measure and suggest the optimal hedge ratio. Kadan and Liu (2014) interpret the riskiness measures of Aumann and Serrano (2008) and Foster and Hart (2009) as performance indices. They provide the moment properties of these indices, and apply these indices to explain several anomalous investment strategies.

[4] Aumann and Serrano (2008) show that their index satisfies duality and positive homogeneity. Moreover, they also show that any index satisfying these two axioms is a positive multiple of their index.

[5] In the literature, there are several other popular functional forms for utility which can represent a wide variety of risk preferences, for example, the double-exponential (Bell, 1986), expo-power (Saha, 1993), and power risk aversion (Xie, 2000) functional forms. However, the relationship between the reciprocal of the degree of absolute risk aversion and wealth is more complex than HARA utility and is not necessarily monotonic. Thus, the HARA utility function is adopted in our paper.

measure with an economic foundation. Our risk measure does not satisfy all of the sufficient conditions for the risk measures required in Kadan et al. (2015). Instead, our risk measure satisfies other important properties such as monotonicity with respect to SSD. Second, when the risk measure does not satisfy the risk-free property, Kadan et al. (2015) adopt an axiomatic approach instead of an equilibrium approach to derive the systematic risk. We overcome this problem by proposing a method to derive the equilibrium pricing formula under a mean-risk framework.[6] Third, we also establish a test for our CAPM and provide empirical support for it.

The GR-CAPM shares a similar form to the traditional mean-variance CAPM (denoted as MV-CAPM), i.e., the expected excess return of a capital asset i is equal to the systematic risk of the capital asset i times the expected market excess return. However, unlike that for the MV-CAPM, in which the systematic risk depends on the covariance of the portfolio return and the market portfolio return, we find that in the GR-CAPM the systematic risk depends on the covariance of the portfolio return and a non-linear function of the market portfolio return. Moreover, the non-linear function is determined by the newly-proposed risk index of the market portfolio.

Third, we show that the traditional ordinary least squares (OLS) method and the associated zero-alpha test are inapplicable to estimating and evaluating the GR-CAPM. To circumvent this problem, we use the method-of-moments to develop a new set of estimation and testing methods for the GR-CAPM. The proposed test is not only applicable to assessing the zero-alpha restriction, but is also applicable to testing the GR-CAPM against the misspecification of ignoring pricing factors, such as the size and book-to-market factors. In addition, we also show that the proposed method includes the OLS method and the associated zero-alpha test as a special case.

Fourth, based on our estimation and testing method, we empirically analyze the equilibrium monthly return for 50 portfolios: the 25 Fama-French size and book-to-market and the 25 Fama-French size and momentum portfolios. Our data period extends from January 1981 to December 2014. We find that in general the GR-CAPM outperforms the MV-CAPM. Furthermore, we

[6] When the risk measure does not satisfy the risk-free property, Kadan et al. (2015) adopt an axiomatic approach instead of an equilibrium approach to derive the systematic risk. We overcome this problem and the detailed discussions are provided in Section 3.3.

find that when the parameters in the HARA utility function are close to those used in the log utility function, the model can better explain the portfolio returns. In other words, by using the riskiness index proposed by Foster and Hart (2009) for portfolios, the asset pricing model generally performs better among the GR-CAPM family. The findings also suggest that, compared to the GR-CAPM, the MV-CAPM could overestimate the magnitude of the abnormal portfolio returns by mis-evaluating the systematic risk. In addition, in spite of the performance of the GR-CAPM, the test results suggest that our one-factor model might be further improved by considering size and book-to-market factors. The above findings are robust for the data period from January 1961 to December 2014.

2 A Generalized Index of Riskiness

In this section, an approach to integrate the indices of riskiness constructed by Aumann and Serrano (2008), Foster and Hart (2009), and Bali et al. (2011) is proposed.

Let $g \in \mathcal{G}$ denote a gamble which has some positive and negative values where the expected value of the gamble is positive. Let L denote the maximum loss of the g. We assume that $\text{Prob}\,(g = -L) > 0$. We make no additional assumptions regarding the underlying probability distribution of the payoff of the gamble and impose no other restrictions on the random payoff. For any given parameters $a > 0$ and $\gamma \leq 0$, we define our index for gamble g as:

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\left(\frac{\frac{a}{1-\gamma}g}{R_g}+1\right)^{\gamma}\right]-1\right\}=0, \tag{1}$$

where $\mathbb{E}$ is the expectations operator.

In Equation (1), R_g could be interpreted as an affine function of the reciprocal of the critical degree of absolute risk aversion such that a HARA decision maker is indifferent between accepting and rejecting gamble g. To see this, let W denote the wealth level. A HARA utility function has the following form:

$$u\,(W)=\frac{1-\gamma}{\gamma}\left(\frac{aW}{1-\gamma}+b\right)^{\gamma}, \tag{2}$$

where a, b and γ are parameters. The restrictions $a > 0$ and $\frac{aW}{1-\gamma} + b > 0$ are imposed to ensure that the utility function is increasing and concave.[7] The reciprocal of the degree of absolute risk aversion is a linear function of wealth since

$$-\frac{u'(W)}{u''(W)} = \frac{W}{(1-\gamma)} + \frac{b}{a}. \tag{3}$$

That a HARA decision maker is indifferent between accepting and rejecting gamble g means that

$$\frac{1-\gamma}{\gamma}\mathbb{E}\left[\frac{1-\gamma}{\gamma}\left(\frac{a(W+g)}{1-\gamma}+b\right)^{\gamma}\right] = \frac{1-\gamma}{\gamma}\left[\left(\frac{W}{1-\gamma}+b\right)^{\gamma}\right], \tag{4}$$

Rearranging the above equation yields

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\left(\frac{\frac{a}{1-\gamma}g}{\frac{aW}{1-\gamma}+b}+1\right)^{\gamma}\right]-1\right\} = 0. \tag{5}$$

By defining

$$R_g = \frac{aW}{1-\gamma} + b, \tag{6}$$

Equation (5) can be further rewritten as Equation (1). From Equation (3), we have

$$R_g = a \times \left[-\frac{u'(W)}{u''(W)}\right]. \tag{7}$$

i.e., the index is a times the reciprocal of the critical degree of absolute risk aversion at W.

The following Definition formally defines the index of riskiness R_g of a gamble g:

Definition 1 *For any gamble $g \in \mathcal{G}$, its riskiness R_g is determined by Equation (1).*

In Definition 1, we require that the evaluated gamble must be in the set of $\mathcal{G}$. For risky gambles with a negative expectation, we set the risk index of these gambles as ∞. As argued by

[7] If $\gamma = 2$, then the utility is quadratic, i.e., $u(W) = -\frac{1}{2}(-aW+b)^2$. If γ goes to 0, then $u(W) = \ln(aW+b)$. If $b = 1$ and γ goes to negative infinity, then the exponential utility function is obtained. If $\gamma < 1$ and $b = 0$, then the power utility function is observed.

Aumann and Serrano (2008), no risk-averse agent would accept a risky gamble with a negative expectation. Thus, these types of gambles could be viewed as having very high risk. For gambles with no negative payoff and gambles with a zero payoff for sure, we define the risk index of these gambles as 0 since these gambles can be viewed as no risk.

The advantage of our index is that it includes the existing indices as special cases. Aumann and Serrano's (2008) index (hereafter AS index) for gamble g is defined as

$$\mathbb{E}\left[\exp\left(-\frac{g}{R_g^{AS}}\right)\right] = 1. \tag{8}$$

The index can be obtained by setting γ as approaching negative infinity and $a = 1$ in our model since

$$\lim_{\gamma\to-\infty} \mathbb{E}\left[\frac{\frac{ag}{1-\gamma}}{R_g} + 1\right]^{\gamma} = \mathbb{E}\left[\exp\left(-\frac{g}{R_g}\right)\right].$$

It is worth noting that although our index can be reduced to the AS index, our index is not ordinally equivalent to the AS index since our index does not satisfy the duality axiom proposed by Aumann and Serrano (2008).

Foster and Hart (2009) define their index (hereafter the FH index) as

$$\mathbb{E}\left[\ln\left(\frac{g}{R_g^{FH}} + 1\right)\right] = 0. \tag{9}$$

It can be obtained by setting $a = 1$ and γ as approaching 0 in our model since

$$\lim_{\gamma\to 0} \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\left(\frac{\frac{1}{1-\gamma}g}{R_g} + 1\right)^{\gamma}\right] - 1\right\} = \lim_{\gamma\to 0} \frac{1}{\gamma}\left\{\mathbb{E}\left[\left(\frac{g}{R_g} + 1\right)^{\gamma}\right] - 1\right\}.$$

By the L'Hopital rule, the above equation equals $\mathbb{E}\left[\ln\left(\frac{g}{R_g} + 1\right)\right]$.

The index proposed by Bali et al. (2011) (hereafter the BCCY index) is defined as

$$\frac{1}{\gamma}\left\{\mathbb{E}\left[\frac{g}{R_g^{BCCY}} + 1\right]^{\gamma} - 1\right\} = 0. \tag{10}$$

By setting $a = 1 - \gamma$ and dividing Equation (1) by $1 - \gamma$, our index reduces to R_g^{BCCY}.

The following Theorem further characterizes our index.

Theorem 1 *Assume $\gamma \leq 0$. For any given parameters a and γ, the index for g defined by Equation (1) is unique and continuous.*

Proof. Please see Appendix A. ■

In Theorem 1, the concept of continuity is the same as in Aumann and Serrano (2008), and Foster and Hart (2009): an index is continuous if $R_{g_n} \to R_g$ whenever the g_n are uniformly bounded and converge to g in probability.

In addition, our index shares similar properties to the AS, FH and BCCY indices. First, our index is measured in the same units as the outcomes. Second, it has a lower bound, i.e., $\frac{a}{1-\gamma}L$, where L denotes the maximum loss,[8] whereas both the FH and BCCY indices are greater than L. Moreover, our index has the following important properties:

Proposition 1 *Let $g, h \in \mathcal{G}$. For any given parameters $a > 0$ and $\gamma < 0$, we have*

1. *homogeneity: $R_{\theta g} = \theta R_g$ for every $\theta > 0$*
2. *dilution: let z be a compound gamble that yields g with probability p and zero with probability $1 - p$, then $R_z = R_g$*
3. *subadditivity: $R_{g+h} \leq R_g + R_h$*
4. *convexity: $R_{\theta g + (1-\theta)h} \leq \theta R_g + (1 - \theta) R_h$ for every $\theta \in (0, 1)$*
5. *monotonicity with respect to first- and second-degree stochastic dominance*

Proof. Please see Appendix B. ■

In addition, by taking the Taylor expansion of Equation (1) with respect to g around zero, Equation (1) can be written as

$$\frac{1-\gamma}{\gamma} \left\{ \sum_{n=1} \frac{\prod_{k=0}^{n-1} (\gamma - k)}{n!} \left(\frac{\frac{a}{1-\gamma}}{R_g} \right)^n \mathbb{E}\left[g^n\right] \right\} = 0, \tag{11}$$

[8] See Appendix B for the proof.

if the higher moments exist. This means that the index R_g is determined implicitly by the moments of the distribution of g. In other words, not only does the second moment matter, but the higher moments also matter.

3 A General Capital Asset Pricing Model

3.1 The Riskiness of a Portfolio

Assume that there are n assets in the market. Let the nth asset be safe with a rate of return X_f. Furthermore, let X_i and X_p denote the rate of return of the ith risky asset, $i = 1, ..., n-1$, and a portfolio p, where x_i and x_p respectively denote the excess return of asset i and portfolio p, i.e., $x_i = X_i - X_f$ and $x_p = X_p - X_f$. Thus,

$$x_p = \sum_{i=1}^{n-1} w_i x_i,$$

where w_i denotes the investment in asset i and $\sum_{i=1}^{n} w_i = 1$. Short-sale constraints are not imposed, i.e., w_i could be negative.

To properly evaluate the risk of a portfolio p, we use the excess return x_p as the payoff of the portfolio while calculating the riskiness index of this portfolio, R_p, i.e.,

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right]^{\gamma}-1\right\}=0. \tag{12}$$

It is worth noting that the above formula is for portfolios in the set of $\mathcal{G}$, i.e., having some positive and negative excess returns and a positive expected excess return.

One reason for using excess returns to evaluate the risk of a portfolio is that X_f can be viewed as the opportunity cost of the investment. It is natural for an investor to consider x_p rather than X_p only. The second reason is that even by using the excess return to calculate the risk index as in Equation (12), Proposition 1 still holds. The demanded properties of R_p for the equilibrium pricing formula include: homogeneity of degree one, convexity, the zero-payoff property, and monotonicity with respect to first- and second-degree stochastic dominance. R_p

also satisfies continuity and uniqueness as in Theorem 1 for portfolios in the set of $\mathcal{G}$.[9] In the following section, we will show that using the excess return to calculate the risk index is crucial for the two-fund monetary separation (see, for example, Huang and Litzenberger, 1988), which means that investors' optimal portfolios are always linear combinations of a risk-free asset and a fixed portfolio of risky assets, i.e., the market portfolio.[10]

3.2 Portfolio Selection and Capital Market Equilibrium

We follow the standard assumptions and the setting employed in the analysis of capital market equilibrium, such as homogeneous expectations regarding future returns, a complete market and others.[11] In a departure from the literature, we assume that the investors use our index of riskiness to evaluate the risk of assets. We assume that all investors have a mean-riskiness type preference, i.e., investors evaluate portfolio p according to the mean-risk pair, $(\mathbb{E}[X_p], R_p)$. Investors prefer portfolios with a higher mean return and a lower index of riskiness. For portfolios with the same expected returns, investors prefer the one with the lowest index of riskiness. For portfolios with the same index of riskiness, investors prefer the one with the highest expected return. Each investor will choose the optimal portfolio according to his or her own preference regarding the trade-off between the expected return and risk.

Thus, the efficient portfolio allocation is the solution to the following optimization problem:

$$\begin{aligned} \min_{w_i, i=1,..n} \quad & R_p \\ \text{s.t.} \quad & \mathbb{E}[X_p] = \textstyle\sum_{i=1}^n w_i \mathbb{E}[X_i] \geq \mu \\ & \textstyle\sum_{i=1}^n w_i = 1 \\ & \tfrac{1-\gamma}{\gamma} \left\{ \mathbb{E}\left[\frac{\frac{a}{1-\gamma} x_p}{R_p} + 1 \right]^{\gamma} - 1 \right\} = 0, \end{aligned} \tag{13}$$

[9] We set R_p as infinity for portfolios with a negative expected excess return. This setting gives rise to an issue in that the continuity property does not hold in general. However, this issue will not affect the equilibrium pricing as shown in the next section.

[10] The literature (for example, see De Giorgi et al. (2011)) suggests that in a reward-risk framework, if there exist strictly increasing transformations of the reward and the risk function such that the transforming reward and risk measures satisfy positive homogeneity of degree 1, and are translation invariant or translation equivalent, then the two-fund separation holds. Since our risk measure does not satisfy the translation invariance (or, risk-free condition), i.e., $R_{g+c} = R_g$, where c is a constant, we cannot apply the findings in the literature to conclude that the two-fund separation holds.

[11] Please refer to Sharpe (1964), Lintner (1965) and Mossin (1966) for more details.

where μ is a positive constant. It is important to note that if there exists any asset with a positive random excess return for sure, then the demand for all other risky assets in the market will be zero. To see this, assume that an asset j has $\text{Prob}(X_j < X_f) = 0$ and $\mathbb{E}[X_j] > X_f$. Thus, $R_j = 0$ for this asset. To obtain the solution of the optimization problem (13), an investor can construct a portfolio with zero risk and a positive excess return by short-selling the risk-free asset and investing all his or her wealth in this asset j. Thus, all other portfolios with a positive index of riskiness are dominated by this particular portfolio. In other words, there will be zero demand for all risky assets except for asset j and the risk-free asset. Similar results will be drawn if investors can construct a portfolio with zero risk and a positive expected excess return by investing in some risky assets only. To avoid this extreme situation, we assume that, except for the risk-free asset, there does not exist any other asset or portfolio in the market such that the index of riskiness is zero and the expected excess return is positive.

In what follows, we use figures to analyze the optimization problem (13). In all figures in this section, the x-axis represents the index of riskiness, R, whereas the y-axis denotes the expected return $\mathbb{E}[X]$. First, we would like to show that, while considering risky assets only, the investment opportunity set is weakly convex. Let i and j denote risky assets with $(\mathbb{E}[X_i], R_i)$ and $(\mathbb{E}[X_j], R_j)$, respectively. Assume that $\mathbb{E}[X_j] \geq \mathbb{E}[X_i] > X_f$ and $R_j \geq R_i > 0$. Let I and J in Figure 1 denote assets i and j, respectively. Assume that portfolio q is a linear combination of i and j, i.e., $X_q = wX_i + (1-w)X_j$, where $w \in [0,1]$ is a constant. Thus, we have $\mathbb{E}[X_q] = w\mathbb{E}[X_i] + (1-w)\mathbb{E}[X_j]$. In this case, the index satisfies continuity. Thus, we have $R_i \leq R_q$. Furthermore, our risk index satisfies convexity, i.e., $R_q \leq wR_i + (1-w)R_j$. Thus, portfolio q is located to the left-hand side of line IJ and could be denoted as Q in Figure 1.

When short-selling is allowed, i.e., w is not restricted in the range of $[0,1]$, the combinations containing assets i and j are not limited to being between I and J. For example, S in Figure 1 can be reached by setting $w < 0$. Note that O in Figure 1 represents the portfolio with $w_o > 1$ such that $w_o\mathbb{E}[X_i] + (1-w_o)\mathbb{E}[X_j] = X_f$. For this portfolio and the portfolios with $w > w_o$, the excess return is non-positive and by definition, the index of riskiness is positive infinity. Thus, no investors with mean-riskiness preferences will invest in any portfolio with $w > w_o$. In other words, the possible investment opportunity containing assets i and j can be represented by curve $OIQJ$ in Figure 1, which does not include point O.

While investing in different individual assets, another curve similar to $OIQJ$ is obtained. Thus, by considering all risky assets in the market, we can generate the investment opportunity set. Since our risk measure satisfies homogeneity of degree one, convexity and continuity for portfolios with positive excess returns, the investment opportunity set is weakly convex as shown by the shaded part in Figure 2. Note that all investors exhibit mean-riskiness preferences. Thus, without considering the risk-free asset, all investors will invest in the portfolios which lie along the upper left-hand side of the investment opportunity set, i.e., the investment opportunity curve depicted as curve QSP in Figure 2.

Second, we would like to show that the efficient frontier is line $X_f P$ as shown in Figure 2, which is the line through the point $(0, X_f)$ that is tangential to the investment opportunity curve QSP in Figure 2. Since our index satisfies the zero-payoff condition, i.e., the index of riskiness of the risk-free asset is 0, the risk-free asset will be located on the vertical line as shown in Figure 2. Let portfolio q denote a portfolio which lies along the investment opportunity curve indicated by Q in Figure 2. Furthermore, let portfolio k be a linear combination of q and the risk-free asset, i.e., $X_k = \theta X_q + (1-\theta) X_f$, where θ is a constant. In other words, investors can invest some money in the risk-free asset and some in q. Thus, we have

$$\mathbb{E}[X_k] = \theta \mathbb{E}[X_q] + (1-\theta) X_f, \tag{14}$$

and $x_k = X_k - X_f = \theta (X_q - X_f) = \theta x_q$. Since the excess return is considered in calculating the index of riskiness and the index satisfies homogeneity of degree one, we have $R_k = \theta R_q$ or $\theta = R_k / R_q$. Thus, Equation (14) can be rewritten as

$$\mathbb{E}[X_k] = \frac{\mathbb{E}[X_q] - X_f}{R_q} R_k + X_f = \frac{\mathbb{E}[x_q]}{R_q} R_k + X_f. \tag{15}$$

In other words, all combinations involving the risk-free asset and q must be a straight line between the points representing the two components as indicated by line $X_f Q$ in Figure 2.

Furthermore, these portfolios are dominated by all combinations which contain the risk-free asset and a portfolio which is located on the right-hand side of q and lies along the investment opportunity curve, such as point S in Figure 2 since the investment opportunity set is convex. Thus, it is obvious to see that the corresponding portfolios on the line tangential to the curve have the lowest index of riskiness for any given μ in Problem 13. Let P in Figure 2 denote the tangent from X_f to the investment opportunity curve. This tangential line represents the efficient frontier.

Third, we would like to show that the two-fund monetary separation theorem holds. Since the efficient frontier is linear, any linear combination of the risk-free asset and an efficient portfolio is also an efficient portfolio. Let m denote the market portfolio and R_m denote the riskiness of the market portfolio which satisfies

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right]^{\gamma}-1\right\}=0. \tag{16}$$

Note that the market portfolio m is a convex combination of all individuals' portfolios when the market is clear. Since all investors will only consider efficient portfolios, the market portfolio is a convex and linear combination of the frontier portfolios and the risk-free asset. In other words, the market portfolio is also on the frontier. Therefore, regardless of which efficient portfolio investors prefer, all optimal portfolio choices are linear combinations of the risk-free asset and the market portfolio. In sum, the traditional two-fund monetary separation theorem holds as shown in the following theorem:

Theorem 2 *The optimal portfolio choice in the optimization problem (13) admits separation between the risk-free asset and the market portfolio.*

Under the traditional assumptions in the literature, the GR-CAPM is characterized by the following theorem:

Theorem 3 *Under the standard assumptions employed in the analysis of capital market equilibrium, if all investors evaluate portfolios in the generalized mean-risk framework, then the market* *equilibrium prices satisfy the following*

$$\mathbb{E}[x_i] = \beta_i(R_m)\,\mathbb{E}[x_m], \tag{17}$$

where

$$\beta_i(R_m) = \frac{\operatorname{cov}\left[x_i, \left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\operatorname{cov}\left[x_m, \left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}. \tag{18}$$

Proof. Please see Appendix C. ∎

The traditional systematic risk in the MV-CAPM is

$$\beta_i^{MV} = \frac{\operatorname{cov}[x_i, x_m]}{\operatorname{cov}[x_m, x_m]}.$$

By comparing β_i^{MV} and $\beta_i(R_m)$, we find that in both models the systematic risk has a similar form. However, the newly-proposed systematic risk depends on a non-linear function of x_m with parameter R_m.

The traditional CAPM can be obtained in a representative agent economy by assuming that the stochastic discount factor δ is a linear function of X_m. In that framework, our GR-CAPM can be obtained by assuming that δ is a non-linear function of X_m as

$$\delta = \left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}.$$

From Equation (11), we know that R_m is affected by all of the higher moments of x_m. In other words, the GR-CAPM coincides with the setting in a representative agent economy by assuming that the stochastic discount factor is a non-linear function of x_m and R_m, which are affected by all of the higher moments of x_m.[12]

Since our risk index includes the AS, FH and BCCY indices under different settings of a and γ, the GR-CAPM can include "AS-CAPM", "FH-CAPM" and "BCCY-CAPM". Specifically,

[12] Harvey and Siddique (2000) assumed that δ is quadratic in the market return. They found that the systematic risks are functions of the market variance, skewness, covariance, and coskewness.

the GR-CAPM includes the AS-CAPM as a limiting special case where $a = 1$ and γ approaches ∞, and the FH-CAPM as another limiting special case where $a = 1$ and γ approaches 0. It also contains the BCCY-CAPM as a subclass of models where $a = 1 - \gamma$. Also note that, if $\gamma = 2$, then Equations (17) and (18) reduce to the MV-CAPM. Therefore, we can unify the MV-CAPM and the GR-CAPM and its particular examples into a general form, which is the systematic risk β_i for asset i as follows:

$$\beta_i(R_m) = \frac{\operatorname{cov}[x_i, k(x_m, R_m)]}{\operatorname{cov}[x_m, k(x_m, R_m)]}, \tag{19}$$

where

$$k(x_m, R_m) = \begin{cases} x_m, & MV-CAPM, \\ \exp\left(-x_m/R_m^{AS}\right), & AS\text{-}CAPM, \\ \left(1 + x_m/R_m^{FH}\right)^{-1}, & FH\text{-}CAPM, \\ \left(1 + x_m/R_m^{BCCY}\right)^{\gamma-1}, & BCCY\text{-}CAPM, \\ \left(1 + \left(x_m/R_m^{GR}\right)(a/(1-\gamma))\right)^{\gamma-1}, & GR\text{-}CAPM. \end{cases} \tag{20}$$

R_m can be obtained by setting

$$\mathbb{E}[\phi(x_m, R_m)] = 0, \tag{21}$$

where

$$\phi(x_p, R_m) = \begin{cases} \exp\left(-x_p/R_m^{AS}\right) - 1, & AS, \\ \ln\left(1 + x_p/R_m^{FH}\right), & FH, \\ \frac{1}{\gamma}\left[\left(1 + x_p/R_m^{BCCY}\right)^{\gamma} - 1\right], & BCCY, \\ \frac{1-\gamma}{\gamma}\left[\left(1 + \left(x_p/R_m^{GR}\right)(a/(1-\gamma))\right)^{\gamma} - 1\right], & GR. \end{cases} \tag{22}$$

From the previous discussion, we know that R_m in Equation (21) is defined when $\mathbb{E}[x_m] > 0$ and $\operatorname{Prob}(x_m < 0) > 0$. In addition, since R_m^{FH} and R_m^{BCCY} are greater than the maximum loss, we have $\operatorname{Prob}\left(x_p > -R_m^{FH}\right) = 1$ and $\operatorname{Prob}\left(x_p > -R_m^{BCCY}\right) = 1$. For R_m^{GR}, the lower bound of the index is $\frac{a}{1-\gamma}L$. Thus, we have $\text{Prob}\left(x_p > -R_m^{GR}(1-\gamma)/a\right) = 1$ for GR-CAPM.

3.3 A Related Systematic Risk

Our finding in Theorem 3 is closely related to the findings in Kadan et al. (2015). Let us rewrite their main theorem (Theorem 4) by using the notations in this paper. Denote R_p^{KLL} as the risk index in Kadan et al. (2015) for portfolio p. In Kadan et al. (2015), the risk measure R_p^{KLL} does not have a specific formula. They require that R_p^{KLL} be homogeneous of some degree k, smooth and convex, and that it satisfy the risk-free property.

Using the mean-risk framework as in our paper, Theorem 4 in Kadan et al. (2015) shows that if the risk measure R_p^{KLL} satisfies the above properties, then the equilibrium pricing model can be expressed as

$$\mathbb{E}[x_i] = \beta_i\left(R_m^{KLL}\right)\mathbb{E}[x_m], \tag{23}$$

where

$$\beta_i\left(R_m^{KLL}\right) = \frac{\frac{\partial R_m^{KLL}}{\partial w_i}}{\sum_{j=1}^{n} w_j \frac{\partial R_m^{KLL}}{\partial w_j}}. \tag{24}$$

Our systematic risk in Theorem 3 actually shares a similar form to theirs. By Equation (A.7) in Appendix C, Equation (18) can be written as

$$\beta_i(R_m) = \frac{\mathbb{E}\left[x_i\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[x_m\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}. \tag{25}$$

Recall Equation (16) and let

$$\mathcal{L} = \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right]^{\gamma}-1\right\} = 0.$$

Thus,

$$\frac{\partial R_m}{\partial w_i} = -\frac{\frac{\partial \mathcal{L}}{\partial w_i}}{\frac{\partial \mathcal{L}}{\partial R_m}} = -\frac{a\mathbb{E}\left[x_i\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\frac{\partial \mathcal{L}}{\partial R_m}}.$$

On the other hand,

$$\sum_{j=1}^{n} w_j \frac{\partial R_m}{\partial w_j} = -\frac{a\mathbb{E}\left[\left(\sum_{j=1}^{n} w_j x_j\right)\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\frac{\partial \mathcal{L}}{\partial R_m}} = -\frac{a\mathbb{E}\left[x_m\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\frac{\partial \mathcal{L}}{\partial R_m}}.$$

Thus, Equation (25) can be rewritten as

$$\beta_i\left(R_m\right) = \frac{\frac{\partial R_m}{\partial w_i}}{\sum_{j=1}^{n} w_j \frac{\partial R_m}{\partial w_j}}, \tag{26}$$

which shares a similar form to Equation (24).

It is worth noting that the risk index R_m^{KLL} in Equation (24) is a function of X_m, whereas our index R_m in Equation (26) is a function of x_m. This difference between R_m^{KLL} and R_m is not trivial. In Kadan et al. (2015), the equilibrium pricing model requires that R_p^{KLL} satisfy the risk-free property for any portfolio p. Our risk index (including the AS, FH and BCCY indices) does not satisfy the risk-free property and does not belong to R_p^{KLL}. Thus, Theorem 4 in Kadan et al. (2015) cannot be directly applied to derive the CAPM when our index is employed.

On the other hand, since our risk index does not satisfy the risk-free property, the two-fund separation theorem does not hold in our framework. However, in our paper, we propose measuring the risk index on the basis of the excess return x_p for portfolio p. Because of this design, we show that the two-fund monetary separation theorem holds and the CAPM in Theorem 3 can be obtained.

Kadan et al. (2015) noted that there are some limitations to the equilibrium approach, since the risk measure to make Theorem 4 hold should satisfy homogeneity of some degree k, smoothness, convexity, and the risk-free property. To overcome the limitation where the risk measure does not satisfy the above properties, Kadan et al. (2015) further proposed an axiomatic approach to derive the systematic risk. They proposed four axioms for systematic risk measures and derived the systematic risk directly rather than examining the equilibrium price.

Differing from Kadan et al. (2015), our paper demonstrates that the equilibrium approach under the mean-risk framework could still be useful for deriving the CAPM, when the risk measure is a function of the excess returns rather than the gross returns. Note that one important task of our paper is to derive the equilibrium CAPM under the mean-risk framework by employing risk indices satisfying monotonicity with respect to SSD. However, the barrier to this task is that the indices do not satisfy the risk-free property. Our paper's contribution to the literature is that it proposes a method to overcome this problem.

4 Estimation and Testing

Corresponding to x_i and x_m, we let x_{it} and x_{mt} be the excess returns of a risky asset and the market portfolio at time t, respectively, with $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$. Let R be a general value in $\mathbb{R}_+$. The generalized CAPM is associated with the one-factor model:

$$x_{it} = \alpha_i + \beta_i(R)\, x_{mt} + u_{it}, \tag{27}$$

where u_{it} is a zero-mean error term. The intercept of (27) is the same as

$$\alpha_i = \mathbb{E}[x_{it}] - \beta_i(R)\ \mathbb{E}[x_{mt}]. \tag{28}$$

Correspondingly, condition (28) at $R = R_m$ can be rewritten as the zero-alpha hypothesis:

$$H_o : \alpha_{o,i} = 0 \tag{29}$$

for an arbitrary i, where α_{oi} is defined by evaluating α_i at $R = R_m$. In this context, we may assess various CAPMs by testing H_o based on the associated $k(\cdot)$'s and $\phi(\cdot)$'s.

Importantly, although it is common to estimate and test the MV-CAPM by the OLS method, this method is inapplicable to other CAPMs. Let $\widehat{\text{cov}}[\cdot]$ and $\widehat{\text{var}}[\cdot]$ be, respectively, the sample covariance and variance operators, and denote $\bar{x}_i := T^{-1}\sum_{t=1}^{T} x_{it}$ and $\bar{x}_m := T^{-1}\sum_{t=1}^{T} x_{mt}$. In Appendix D.1, we show that, for the MV-CAPM, we can evaluate H_o by directly examining the significance of $\hat{\alpha}_{i,OLS}$; however, for other CAPMs, $\hat{\alpha}_{i,OLS}$ will converge in probability to a "spurious alpha," which is caused by a "misspecified beta," even when H_o is true. Thus, we need a new estimation and testing method for the generalized model.

4.1 Estimation

In this study, we propose a three-stage estimator for $\lambda_{o,i} := (\alpha_{o,i}, \beta_i(R_m), R_m)^\top$, which is the parameter vector of the generalized model. This estimator is denoted by $\hat{\lambda}_i := \left(\hat{\alpha}_i, \hat{\beta}_i, \hat{R}\right)^\top$ and is composed of

1. the first-step method-of-moments (MM) estimator $\hat{R}$ for R_m which is the solution to the estimating equation:

$$\frac{1}{T}\sum_{t=1}^{T} \phi\left(x_{mt}, \hat{R}\right) = 0, \tag{30}$$

2. the second-step plug-in estimator $\hat{\beta}_i$ for $\beta_i(R_m)$:

$$\hat{\beta}_i = \frac{\widehat{\text{cov}}\left[x_{it}, \hat{k}_{mt}\right]}{\widehat{\text{cov}}\left[x_{mt}, \hat{k}_{mt}\right]} = \frac{T^{-1}\sum_{t=1}^{T}(x_{it}-\bar{x}_i)(\hat{k}_{mt}-\bar{k}_m)}{T^{-1}\sum_{t=1}^{T}(x_{mt}-\bar{x}_m)(\hat{k}_{mt}-\bar{k}_m)}, \tag{31}$$

where $\hat{k}_{mt} := k(x_{mt}, \hat{R})$ and $\bar{k}_m := T^{-1}\sum_{t=1}^{T}\hat{k}_{mt}$, and

3. the third-step plug-in estimator for $\alpha_{o,i}$:

$$\hat{\alpha}_i = \bar{x}_i - \hat{\beta}_i \bar{x}_m. \tag{32}$$

In Appendix D.2, we define the asymptotic covariance matrix V_{λ_i} and show the following result:

Proposition 2 *Given H_o and Assumption 1 listed in Appendix D.2,*

$$\hat{\lambda}_i \xrightarrow{\text{p}} \lambda_{o,i}$$

and

$$\sqrt{T}(\hat{\lambda}_i - \lambda_{o,i}) \xrightarrow{\text{d}} N(0, V_{\lambda_i}), \tag{33}$$

with the asymptotic covariance matrix V_{λ_i}, hold for all $i = 1, 2, \ldots, N$.

This proposition indicates that, under H_o and suitable conditions, $\hat{\lambda}_i$ is consistent for $\lambda_{o,i}$ and has an asymptotically normal distribution.

This result is applicable to assessing the joint significance of $\hat{\lambda}_i$ or the individual significance of $\hat{\alpha}_i$, $\hat{\beta}_i$ and $\hat{R}_i$. Let V_{α_i}, V_{β_i} and V_R be, respectively, the asymptotic covariance matrices of $T^{1/2}(\hat{\alpha}_i - \alpha_{o,i})$, $T^{1/2}(\hat{\beta}_i - \beta_i(R_m))$ and $T^{1/2}(\hat{R} - R_m)$. These moments are, respectively, the first diagonal element, the second diagonal element and the last diagonal element of V_{λ_i}. According to (A.8), (A.9) and (A.12), we can understand that V_{α_i} and V_{β_i} are, respectively, dependent on V_{β_i} and V_R. This reflects the fact that, in the three-stage estimation, $\hat{\alpha}_i$ and $\hat{\beta}_i$ are, respectively, dependent on $\hat{\beta}_i$ and $\hat{R}$.

4.2 Specification Tests

By construction, the generalized model relies on the market factor x_{mt} and the generalized beta in explaining the cross-sectional variation of risky asset returns. This model could be misspecified when the true market model is of a multi-factor form:

$$x_{it} = \beta_i(R_m)x_{mt} + z_t^\top \delta + \epsilon_{it}, \tag{34}$$

where z_t stands for an $r \times 1$ vector of non-zero mean asset pricing factors in addition to the market factor x_{mt}, δ is an $r \times 1$ vector of regression coefficients and ϵ_{it} represents a zero-mean error term which is orthogonal to x_{mt} and z_t. Clearly, (34) encompasses the multi-factor models considered by Fama and French (1996, 2015) and related studies when $\beta_i(R_m)$ is free of R_m. Given (34), we have

$$\mathbb{E}[x_{it}] = \beta_i(R_m)\ \mathbb{E}[x_{mt}] +\ \mathbb{E}[z_t^\top]\delta;$$

correspondingly, condition (28) holds at $R = R_m$ when $\delta = 0$ (fails to hold when $\delta \neq 0$). By denoting the error term of (27) under H_o:

$$u_{o,it} := x_{it} - \beta_i(R_m)x_{mt}, \tag{35}$$

(34) implies that $z_t u_{o,it} = z_t z_t^\top \delta + z_t \epsilon_{it}$ and hence $\mathbb{E}[z_t u_{o,it}] = \mathbb{E}[z_t z_t^\top]\delta$. Thus, we can test the adequacy of the generalized model against the misspecification of ignoring pricing factors by examining the hypothesis:

$$H_{z,o}\ :\ \mathbb{E}[z_t u_{o,it}] = 0 \tag{36}$$

for any arbitrary i. Clearly, $H_{z,o}$ includes H_o as a special case where $z_t = 1$. Thus, a test for $H_{z,o}$ is directly applicable to examining H_o by setting $z_t = 1$ for all t's. This also means that the test for H_o amounts to examining condition (28) at $R = R_m$ without explicitly specifying a set of potential pricing factors in addition to x_{mt}. In what follows, we refer to the test for H_o as the "zero-alpha test" and the test for $H_{z,o}$ with $z_t \neq 1$ as the "omitted variable test."

Corresponding to (35), we define the residual:

$$\hat{u}_{it} := x_{it} - \hat{\beta}_i x_{mt},$$

and estimate $\mathbb{E}[z_t u_{o,it}]$ by its sample analogue:

$$\hat{\alpha}_{z,i} := \frac{1}{T}\sum_{t=1}^{T} z_t \hat{u}_{it}.$$

Denote the $Nr \times 1$ vector $\hat{\boldsymbol{\alpha}}_z := \left(\hat{\alpha}_{z,1}^\top, \ldots, \hat{\alpha}_{z,N}^\top\right)^\top$. Since the formula of the covariance matrix

$V_{\boldsymbol{\alpha}_z}$ is complicated, we define the matrix and prove the following result in Appendix D.3:

Proposition 3 *Given $H_{z,o}$ and Assumptions 1-2 listed in Appendix D.3, the test statistic:*

$$\mathcal{J}_{z,T} := T\hat{\boldsymbol{\alpha}}_z^\top \hat{V}_{\boldsymbol{\alpha}_z}^{-1}\hat{\boldsymbol{\alpha}}_z, \tag{37}$$

with $\hat{V}_{\boldsymbol{\alpha}_z}$ denoting a positive definite matrix which is consistent for $V_{\boldsymbol{\alpha}_z}$, has the asymptotic null distribution: $\mathcal{J}_{z,T} \xrightarrow{\text{d}} \chi^2(Nr)$, as $T \to \infty$.

We denote this test by the $\mathcal{J}_z$ test. A formula for $\hat{V}_{\boldsymbol{\alpha}_z}$ for computing the $\mathcal{J}_z$ test statistic is provided in Appendix D.3. The $\mathcal{J}_z$ test is the zero-alpha test for H_o when $z_t = 1$ for all t's; otherwise, it is the omitted variable test. Note that, if $V_{\boldsymbol{\alpha}_z}$ is allowed to be singular, we need to redefine the $\mathcal{J}_z$ test statistic by replacing $\hat{V}_{\boldsymbol{\alpha}_z}^{-1}$ with a generalized inverse of $\hat{V}_{\boldsymbol{\alpha}_z}$ in $\mathcal{J}_{z,T}$ and replacing Nr with $N^* := \text{rank}(V_{\boldsymbol{\alpha}_z})$ as the degrees of freedom of the asymptotic null distribution of $\mathcal{J}_{z,T}$; see, e.g., Moore (1977, Theorem 1).

The proposed method is flexibly applicable to estimating and testing the GR-CAPM and its special cases. In Appendix D.4, we show that it encompasses the OLS method for estimating and testing the MV-CAPM; see, e.g., Cochrane (2000, Equation 168) for a popular test based on the OLS method.

5 Empirical Study

In this section, we analyze the equilibrium monthly returns for 50 portfolios: the 25 Fama-French size and book-to-market and the 25 Fama-French size and momentum portfolios of value-weighted NYSE, AMEX and NASD stocks. The market return is the value-weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. The risk-free rate is the one-month Treasury bill rate. The data are obtained from the data library of Kenneth French. Our data period is from January 1981 to December 2014.

The MM estimators for the riskiness index of the market portfolio under different models are shown in Table 1. Note that the BCCY index is a function of γ where $1-\gamma$ may be viewed as the degree of relative risk aversion. According to the estimation in Bali et al. (2011), $1-\gamma$

is about 3. Thus, we use $\gamma = -2$ as in Bali et al. (2011).[13] We further use different values of a and γ to estimate the riskiness index of the market portfolio: a is an integer between 1 and 10; γ is $-10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. For the sake of brevity, we tabulate only the indices of $(a = 1, \gamma = -10)$, $(a = 1, \gamma = -0.1)$, $(a = 10, \gamma = -10)$, and $(a = 10, \gamma = -0.1)$ for the GR-CAPM models in Table 1. From Table 1, we can find that different models give different values of R_m. Note that R_m^{AS} and R_m^{FH} can be obtained by setting $(a = 1, \gamma = -\infty)$ and $(a = 1, \gamma = 0)$, respectively. Table 1 shows that, in both periods, R_m^{AS} is close to R_m^{GR} at $(a = 1, \gamma = -10)$ and R_m^{FH} is close to R_m^{GR} at $(a = 1, \gamma = -0.1)$. In addition, we also find that R_m increases in γ for both periods.

Table 1 also indicates that the risk of the market portfolio is higher for the period from 1961 to 2014 than for the period from 1981 to 2014. Note that this result is different from the case where the standard deviation is used as a proxy for the risk. The standard deviation of the market portfolio for the period from 1961 to 2014 is 4.34%, which is lower than that for the period from 1981 to 2014 of 4.45%. The different conclusion to a large extent arises from the fact that our index accounts for all moments as shown in Equation (11). The market portfolio for the period from 1981 to 2014 has higher mean and standard deviation and lower central 3rd moment than for the period from 1961 to 2014.

5.1 Zero-Alpha Test

Table 2 presents the $\hat{\alpha}_i$'s estimated by Equation (27) and the zero-alpha test statistics for the 25 size and book-to-market portfolios. The 25 size and book-to-market portfolios are labeled as SjBk, j, k= $1, 2, 3, 4, 5$. For example, S1B1 comprises the small and low book-to-market stocks and S5B5 includes the large and high book-to-market stocks.

If the proposed model is able to explain the patterns of the average returns, then the zero-alpha test statistic will be smaller than the critical values of $\chi^2(1)$ as shown in Proposition 3. Thus, we report "GR-min" (the 6th column) and "GR-max" (the 7th column), which are, respectively, the model with the minimum zero-alpha test statistics and that with the maximum zero-alpha test statistics in the class of GR-CAPMs with $a = 1, 2, ..., 10$ and

[13] We also try $\gamma = -1$ and find that the results are similar.

$\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. In other words, GR-min is the model with the best performance and GR-max is the one with the worst performance among the GR-CAPM family under the parameter setting. The corresponding parameters for the GR-min and GR-max models are shown in the last two blocks of columns of Table 2.

Table 2 indicates that the GR-min model captures most of the pattern in average returns and outperforms the MV-CAPM. Among these 25 portfolios, the absolute values of the zero-alpha test statistics in the GR-min model are smaller than those in the MV-CAPM except for 6 portfolios, i.e., S1B1, S2B1, S3B1, S4B4, S5B1 and S5B4. We further find that if the zero-alpha test does not reject the MV-CAPM, then it does not reject the GR-min model either. Moreover, for some portfolios, the zero-alpha test rejects the MV-CAPM, but it does not reject the GR-min model. For example, for portfolios S1B4, S1B5 and S2B4, the zero-alpha test rejects the MV-CAPM at the 5% level, whereas it does not reject the GR-min model. For portfolios S2B3 and S3B5, the zero-alpha test rejects the MV-CAPM at the 1% level, whereas it does not reject the GR-min model at the 1% level. It seems that our model performs better than the MV-CAPM for small and value firms.

Table 2 also shows that the performance of the GR-max model is similar to that of the MV-CAPM. If the zero-alpha test does not reject the MV-CAPM, then it does not reject the GR-max model either. On the other hand, if the zero-alpha test rejects the MV-CAPM at the p level, then it also rejects the GR-max model at the same level except for portfolio S2B1. For portfolio S2B1, the zero-alpha test rejects the MV-CAPM at the 5% level, whereas it rejects the GR-max model at the 1% level.

Table 3 reports the $\hat{\alpha}_i$'s estimated by Equation (27) and the zero-alpha test statistics for the 25 Fama-French size and momentum portfolios. For these 25 portfolios, we also find that if the zero-alpha test does not reject the MV-CAPM, then it does not reject the GR-min model either. Furthermore, the zero-alpha test cannot reject the GR-min model for all of these portfolios except for S1M1. There are 12 portfolios for which the zero-alpha test rejects the MV-CAPM at either the 1% or 5% level, i.e., S1M1, S1M4, S1M5, S2M1, S2M3, S2M4, S2M5, S3M4, S3M5, S4M1, S4M3, and S4M4. However, it does not reject the GR-min model even at the 1% level for these portfolios except for S1M1. The results in Table 3 support the view that our models are useful for explaining the average returns of momentum portfolios.

Table 3 further shows that the performance of the GR-max model is slightly better than that of the MV-CAPM. Among these 25 portfolios, the absolute values of the zero-alpha test statistics in the GR-max model are smaller than those in the MV-CAPM except for 7 portfolios, i.e., S1M1, S1M2, S3M2, S4M2, S5M2, S5M3 and S5M4. In addition, the zero-alpha test rejects the MV-CAPM at the 5% level for portfolios S2M3 and S4M1, but it does not reject the GR-max model for these two portfolios. For portfolio S1M5, the zero-alpha test rejects the MV-CAPM at the 1% level, but it rejects the GR-max model only at the 5% level.

From Tables 2 and 3, we find that an equilibrium pricing model based on these indices which satisfy monotonicity with respect to SSD strengthens the empirical performance in terms of capturing the cross section of stock returns. Among all of the proposed models, we could further examine whether there exists a model which consistently outperforms other models. Strictly speaking, the GR-min models in Tables 2 and 3 for different portfolios do not share the same parameters a and γ. For the portfolios for which the zero-alpha test rejects the MV-CAPM, but does not reject the GR-min model, the corresponding a and γ are, respectively, 1 and -0.1. Note that the largest amount of γ in our setting is -0.1. Also note that the GR-CAPM will reduce to the FH-CAPM for $a = 1$ and as γ approaches 0. Thus, we observe that the performance of the FH-CAPM is close to that of the GR-min model in Tables 2 and 3 and performs generally better than the AS-CAPM and BCCY-CAPM.

5.2 Systematic Risk

Now, let us turn our focus to the systematic risk $\hat{\beta}_i$ as reported in Table 4. Since the FH-CAPM generally performs better, let us compare the systematic risk estimated by the FH-CAPM, $\hat{\beta}_i^{FH}$, with that estimated by the MV-CAPM, $\hat{\beta}_i^{MV}$. At first glance, it seems that no specific pattern exists in Table 4. Thus, we turn to Table 5. Table 5 summarizes the comparison of the $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s for different models in the cases where the zero-alpha test rejects the MV-CAPM but accepts the FH-CAPM.

From Table 5, we find that for most portfolios, if $\hat{\alpha}_i^{MV} > 0$, then $\hat{\alpha}_i^{MV} > \hat{\alpha}_i^{FH}$ and $\hat{\beta}_i^{MV} < \hat{\beta}_i^{FH}$. If $\hat{\alpha}_i^{MV} < 0$, then $\hat{\alpha}_i^{MV} < \hat{\alpha}_i^{FH}$ and $\hat{\beta}_i^{MV} > \hat{\beta}_i^{FH}$. In other words, for the portfolios where the MV-CAPM predicts that there are positive abnormal returns, on the basis of the FH-CAPM, in most cases, the MV-CAPM overestimates the positive abnormal returns by underestimating the systematic risk.

For example, for the portfolio S1B4, we have $\hat{\alpha}^{MV}_{S1B4} = 0.444$ and $\hat{\beta}^{MV}_{S1B4} = 0.927$. For the same portfolio, we have $\hat{\alpha}^{FH}_{S1B4} = 0.272$ and $\hat{\beta}^{FH}_{S1B4} = 1.204$. Note that for the portfolio S1B4, $\hat{\alpha}^{MV}_{S1B4} > 0$ which is significant at the 5% level. If we use the MV-CAPM, we will conclude that the portfolio has a significant positive abnormal return. However, we find that $\hat{\alpha}^{MV}_{S1B4} = 0.444 > \hat{\alpha}^{FH}_{S1B4} = 0.272$ and $\hat{\beta}^{MV}_{S1B4} = 0.927 < \hat{\beta}^{FH}_{S1B4} = 1.204$. In other words, if FH-CAPM is the "correct" model, then the true systematic risk of S1B4 is 1.204 and the abnormal return is 0.272 which is insignificantly different from zero.

On the other hand, for the portfolios where the MV-CAPM predicts that there are negative abnormal returns, on the basis of the FH-CAPM, in most cases the MV-CAPM overestimates the magnitude of the negative excess returns by overestimating the systematic risk. For example, for portfolio S4M1, $\hat{\alpha}^{MV}_{S4M1} = -0.499$ and $\hat{\beta}^{MV}_{S4M1} = 1.388$, whereas $\hat{\alpha}^{FH}_{S4M1} = -0.120$ and $\hat{\beta}^{FH}_{S4M1} = 0.774$. Note that in the portfolio $S4M1$, $\hat{\alpha}^{MV}_{S4M1} < 0$. We find that $\left|\hat{\alpha}^{MV}_{S4M1}\right| = 0.499 > \left|\hat{\alpha}^{FH}_{S4M1}\right| = 0.120$ and $\hat{\beta}^{MV}_{S4M1} = 1.388 > \hat{\beta}^{FH}_{S4M1} = 0.774$. Thus, if the FH-CAPM is the "correct" model, then the true systematic risk of S4M1 is much smaller than the systematic risk estimated by the MV-CAPM. Adopting the MV-CAPM rather than the FH-CAPM will lead to a biased conclusion that S4M1 has a significant negative abnormal return and a high systematic risk. However, according to the FH-CAPM, the systematic risk of S4M1 is low and the negative abnormal return is insignificant.

The above findings might rest on the following reason. These two CAPMs are, respectively, based on the fitted regressions: $\bar{x}_i = \hat{\alpha}^{MV}_i + \hat{\beta}^{MV}_i \bar{x}_m$ and $\bar{x}_i = \hat{\alpha}^{FH}_i + \hat{\beta}^{FH}_i \bar{x}_m$. Thus, $\text{sign}\left(\hat{\alpha}^{MV}_i - \hat{\alpha}^{FH}_i\right) = -\text{sign}\left(\hat{\beta}^{MV}_i - \hat{\beta}^{FH}_i\right)$ when $\bar{x}_m > 0$.

5.3 Omitted Variable Test

In this section, we further examine whether we still need a multi-factor model. By adopting the statistics proposed by Proposition 3, we can test the adequacy of the generalized model against the misspecification of ignoring pricing factors. To conduct the test, we assume that the

Fama-French size and book-to-market factors are the missing common factors.[14] The omitted variable test statistics are shown in Table 6.

Table 6 indicates that the performance of the GR-min model is similar to but slightly better than that of the MV-CAPM. Consistent with the above findings, for most portfolios, the statistics in the GR-min model are smaller than those in the MV-CAPM. We also find that if the omitted variable test rejects the MV-CAPM, then it does not reject the GR-min model. Furthermore, among these portfolios for which the omitted variable test rejects the MV-CAPM at the 5% level, we find that it does not reject the GR-min model for portfolios S4M1, S4M5 and S5M5.

In addition, by comparing the results in Table 6 and Tables 2 and 3, we find that most of the omitted variable test statistics in the GR-CAPM family of models shown in Table 6 are significantly different from zero at the 5% level but several of the one-factor test statistics in the GR-CAPM family of models shown in Tables 2 and 3 are not significant.

5.4 The Effect of the Sample Period

We further examine whether the above findings are robust by employing a longer data period from January 1961 to December 2014. The results are shown in Tables 7 to 11. These tables show that the results in this longer period are similar to the previous findings. In other words, we also confirm that our general model captures most of the pattern in average returns and outperforms the MV-CAPM.

Specifically, Tables 7 and 8 indicate that the GR-min model outperforms the MV-CAPM for these 50 portfolios. If the zero-alpha test does not reject the MV-CAPM, then it does not reject the GR-min model either. Moreover, for some portfolios, the zero-alpha test rejects the MV-CAPM, but it does not reject the GR-min model. These two tables also demonstrate that the performance of the FH-CAPM is close to that of the GR-min model and is generally better than that of the AS-CAPM and BCCY-CAPM.

Tables 9 and 10 confirm that in this longer period, for the portfolios where the MV-CAPM predicts that there are positive abnormal returns, on the basis of the FH-CAPM, in most cases,

[14] We also test the models by assuming the missing factors including the momentum factor. The results are similar and are thus not unreported.

the MV-CAPM overestimates the positive abnormal returns by underestimating the systematic risk. In other words, compared to the FH-CAPM, the MV-CAPM could overestimate the magnitude of the abnormal portfolio returns by mis-evaluating the systematic risk. In addition, by comparing Table 11 and Tables 7 and 8, the results also suggest that although the GR-CAPM outperforms the MV-CAPM, the omitted variable tests still reject the GR-CAPM in most cases.

6 Conclusion

A generalized index of riskiness has been proposed. This index not only includes the risk measures proposed by Aumann and Serrano (2008), Foster and Hart (2009) and Bali et al. (2011) as special cases, but also satisfies several desirable properties such as homogeneity, subadditivity, convexity, and monotonicity with respect to second-degree stochastic dominance.

Based on the index, a capital pricing equilibrium in a mean-risk framework has been analyzed. Sharing a similar form to the traditional CAPM, the newly-proposed CAPM, the GR-CAPM, indicates that the expected excess return of a capital asset i is equal to the systematic risk of the capital asset i times the expected market excess return. On the other hand, in a way that differs from the traditional CAPM, the systematic risk depends on the covariance of the portfolio return and a non-linear function of the market portfolio return, where the non-linear function is determined by the newly-proposed risk index of the market portfolio.

In addition, we have proposed statistical estimation and tests for the GR-CAPM. Using the tests, the empirical studies show that our model can better capture the cross section of stock returns than the traditional mean-variance capital asset pricing model. The findings also suggest that, compared to our model, using the traditional pricing model could overestimate the magnitude of the excess portfolio returns by mis-evaluating the systematic risk. Since our CAPM includes the traditional CAPM as a special case and performs better in general, many empirical hypotheses under the one-factor model in the literature could be re-examined under the generalized CAPM.

Appendix A Proof of Theorem 1

When $\gamma = 0$, we have $R_g = a \times R_g^{FH}$. Since R_g^{FH} is unique according to Foster and Hart (2009), R_g is also unique for each a. On the other hand, when γ approaches negative infinity, we have $R_g = a \times R_g^{AS}$. Since R_g^{AS} is unique based on Aumann and Serrano (2008), R_g is also unique for each a.

The following shows the uniqueness when any finite $\gamma < 0$. Let us denote

$$F_g(\lambda) = \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\lambda}{1-\gamma}g + 1\right]^{\gamma} - 1\right\}, \tag{A.1}$$

where $\lambda = \frac{a}{R_g} \geq 0$. Thus, $F_g'(\lambda) = \mathbb{E}\left[g\left(\frac{\lambda}{1-\gamma}g+1\right)^{\gamma-1}\right]$, and $F_g''(\lambda) = -\mathbb{E}\left[g^2\left(\frac{\lambda}{1-\gamma}g+1\right)^{\gamma-2}\right]$. Since the HARA utility function requires that $\frac{\lambda}{1-\gamma}g + 1 > 0$ for all payoffs, $F_g(\lambda)$ is a concave function. In addition, let L denote the maximum loss from the gamble. Therefore, we have $\frac{\lambda}{1-\gamma}(-L) + 1 > 0$, or, $\lambda < \frac{1-\gamma}{L}$.

Furthermore, we have $F_g(0) = 0$ and $F_g'(0) = \mathbb{E}[g] > 0$. Note that $\Pr(g = -L) > 0$. We have $\lim_{\lambda \to \left(\frac{1-\gamma}{L}\right)^-} F_g(\lambda) = -\infty$. Since $F_g(\lambda)$ is a strictly concave function, $F_g(0) = 0$, $F_g'(0) > 0$ and $F_g(\lambda)$ approaches negative infinity when λ approaches $\frac{1-\gamma}{L}$, there exists a unique solution λ^* such that $F_g(\lambda^*) = 0$. Since $R_g = \frac{a}{\lambda^*}$, R_g is also unique for given parameters.

For the proof of continuity, since this is very similar to the proof of continuity in Foster and Hart (2009), for the sake of brevity, this technical proof is omitted from the paper.

Appendix B Proof of Proposition 1

1. Homogeneity: $R_{\theta g} = \theta R_g$ for every $\theta > 0$

   Proof: From the definition of R_g, we have

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}g}{R_g} + 1\right]^{\gamma} - 1\right\} = \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}\theta g}{\theta R_g} + 1\right]^{\gamma} - 1\right\} = 0. \tag{A.2}$$

   Let gamble $h = \theta g$. According to Theorem 1, R_h is uniquely defined. Thus, we have $R_h = R_{\theta g} = \theta R_g$.

2. Dilution: let z be a compound gamble that yields g with probability p and zero with probability $1-p$, then $R_z = R_g$.

   Proof: Note that for gamble z, we have

$$\frac{1-\gamma}{\gamma}\left\{p\mathbb{E}\left[\frac{\frac{a}{1-\gamma}g}{R_z}+1\right]^{\gamma}+(1-p)\,\mathbb{E}\,[1]^{\gamma}-1\right\}=0.$$

   Or, equivalently, we have $\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}g}{R_z}+1\right]^{\gamma}-1\right\}=0$. Since the index is uniquely defined, and R_g satisfies $\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}g}{R_g}+1\right]^{\gamma}-1\right\}=0$, we have $R_z = R_g$.

3. Subadditivity: $R_{g+h} \leq R_g + R_h$.

   Proof: For $\gamma = 0$, we have $R_g = a \times R_g^{FH}$. Since the FH index satisfies the property of subadditivity, our index also has this property. Let us show the property under the case where $\gamma < 0$. Denote $\theta = \frac{R_g}{R_g+R_h}$, and $\Phi\left(\frac{g}{R}\right) = \frac{1-\gamma}{\gamma}\left\{\left(\frac{a}{1-\gamma}\frac{g}{R}+1\right)^{\gamma}-1\right\}$. Note that Φ is a concave function of $\frac{g}{R}$. Let h denote the payoff of a gamble h. Thus, the concavity ensures that

$$\Phi\left(\theta\frac{g}{R_g}+(1-\theta)\frac{h}{R_h}\right)\geq\theta\Phi\left(\frac{g}{R_g}\right)+(1-\theta)\,\Phi\left(\frac{h}{R_h}\right).$$

   Taking expectations yields

$$\mathbb{E}\left[\Phi\left(\theta\frac{g}{R_g}+(1-\theta)\frac{h}{R_h}\right)\right]\geq\theta\mathbb{E}\left[\Phi\left(\frac{g}{R_g}\right)\right]+(1-\theta)\,\mathbb{E}\left[\Phi\left(\frac{h}{R_h}\right)\right].$$

   Based on the definitions of R_g and R_h, the right-hand side of the above inequality equals 0. Furthermore, R_{g+h} satisfies $\mathbb{E}\left[\Phi\left(\frac{g+h}{R_{g+h}}\right)\right]=0$. Thus,

$$\mathbb{E}\left[\Phi\left(\theta\frac{g}{R_g}+(1-\theta)\frac{h}{R_h}\right)\right]=\mathbb{E}\left[\Phi\left(\frac{g+h}{R_g+R_h}\right)\right]\geq 0=\mathbb{E}\left[\Phi\left(\frac{g+h}{R_{g+h}}\right)\right].$$

   By Equation (A.1), we have

$$F_{g+h}\left(\frac{a}{R_g+R_h}\right)=\mathbb{E}\left[\Phi\left(\frac{g+h}{R_g+R_h}\right)\right]\geq\mathbb{E}\left[\Phi\left(\frac{g+h}{R_{g+h}}\right)\right]=F_{g+h}\left(\frac{a}{R_{g+h}}\right)=0.$$

From Appendix A, we know that F_{g+h} is a strictly concave function with

$$\begin{aligned} F_{g+h}(\lambda) &\geq F_{g+h}(\lambda^*) \text{ for all } \lambda \leq \lambda^* \\ F_{g+h}(\lambda) &< F_{g+h}(\lambda^*) \text{ for all } \lambda > \lambda^* \end{aligned}$$

where $F_{g+h}(\lambda^*) = 0$. Thus, we have $\frac{a}{R_g+R_h} \leq \frac{a}{R_{g+h}}$. Since $a > 0$, we have $R_{g+h} \leq R_g + R_h$.

4. Convexity: $R_{\theta g+(1-\theta)h} \leq \theta R_g + (1-\theta)R_h$ for every $\theta \in (0,1)$.

   Proof: By subadditivity, we have $R_{\theta g+(1-\theta)h} \leq R_{\theta g} + R_{(1-\theta)h}$. Since the index is homogeneous of degree one, $R_{\theta g} = \theta R_g$ and $R_{(1-\theta)h} = (1-\theta)R_h$, we have $R_{\theta g+(1-\theta)h} \leq \theta R_g + (1-\theta)R_h$.

5. Monotonicity with respect to first- and second-degree stochastic dominance.

   Proof: Define a utility function as $v(x) = \frac{1-\gamma}{\gamma}\left\{\left(\frac{a}{1-\gamma}\frac{x}{R_g}+1\right)^\gamma - 1\right\}$. Thus, v is an increasing and concave function in x. If a gamble g first- or second-degree stochastically dominates another gamble h, then $\mathbb{E}[v_g] \geq \mathbb{E}[v_h]$ because v is an increasing and concave function. Since $\mathbb{E}[v_g] = 0$, we have $\mathbb{E}[v_h] \leq 0$. According to Equation (A.1), we have $F_h\left(\frac{a}{R_g}\right) = \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\left(\frac{a}{1-\gamma}\frac{h}{R_g}+1\right)^\gamma\right] - 1\right\} \leq 0 = \frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\left(\frac{a}{1-\gamma}\frac{h}{R_h}+1\right)^\gamma\right] - 1\right\} = F_h\left(\frac{a}{R_h}\right)$. Since

$$\begin{cases} F_h(\lambda) \geq F_h(\frac{a}{R_h}) \text{ for all } \lambda \leq \frac{a}{R_h} \\ F_h(\lambda) < F_h(\frac{a}{R_h}) \text{ for all } \lambda > \frac{a}{R_h}, \end{cases}$$

   we have $\frac{a}{R_g} \geq \frac{a}{R_h}$, i.e., $R_g \leq R_h$.

Appendix C Proof of Theorem 3

As argued by Sharpe (1964), all curves represent the linear combination of a risky asset and an efficient portfolio must be tangential to the efficient frontier in equilibrium. This requirement can lead us to the capital pricing formula. We follow Sharpe's argument to demonstrate the new equilibrium pricing model.

In Figure 3, I represents a risky asset i and P denotes portfolio p, which is an efficient portfolio. The curve IP refers to all the means and riskiness of all linear combinations of i and p. Let portfolio τ be such a portfolio, i.e.,

$$X_\tau = \theta X_p + (1-\theta) X_i, \tag{A.3}$$

where θ is a constant. Portfolio τ can be represented as $x_\tau = X_\tau - X_f = \theta X_p + (1-\theta) X_i - X_f = \theta x_p + (1-\theta) x_i$. Furthermore, since $\mathbb{E}[X_\tau] = \theta \mathbb{E}[X_p] + (1-\theta)\mathbb{E}[X_i]$, we have $\theta = \frac{\mathbf{E}[X_\tau]-\mathbf{E}[X_i]}{\mathbf{E}[X_p]-\mathbf{E}[X_i]}$. The index of riskiness of τ satisfies

$$\frac{1-\gamma}{\gamma}\left\{\mathbb{E}\left[\frac{\frac{a}{1-\gamma}(\theta x_p + (1-\theta) x_i)}{R_\tau}+1\right]^\gamma - 1\right\} = 0.$$

By the implicit function theorem, we have

$$\begin{aligned}
\frac{d\mathbb{E}[X_\tau]}{dR_\tau} &= -\frac{\gamma \mathbb{E}\left[-\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau^2}\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]}{\gamma \mathbb{E}\left[\frac{x_p-x_i}{\mathbf{E}[X_p]-\mathbf{E}[X_i]}\frac{\frac{a}{1-\gamma}}{R_\tau}\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]} \\
&= \frac{\mathbb{E}[X_p]-\mathbb{E}[X_i]}{R_\tau}\frac{\mathbb{E}\left[(\theta x_p + (1-\theta) x_i)\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[(x_p - x_i)\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]} \\
&= \frac{\mathbb{E}[x_p]-\mathbb{E}[x_i]}{R_\tau}\frac{\mathbb{E}\left[(\theta x_p + (1-\theta) x_i)\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[(x_p - x_i)\left(\frac{\frac{a}{1-\gamma}(\theta x_p+(1-\theta)x_i)}{R_\tau}+1\right)^{\gamma-1}\right]}.
\end{aligned} \tag{A.4}$$

At $\theta = 1$ and $R_\tau = R_p$, the above slope becomes

$$\left.\frac{d\mathbb{E}[X_\tau]}{dR_\tau}\right|_{\theta=1, R_\tau=R_p} = \frac{\mathbb{E}[x_p]-\mathbb{E}[x_i]}{R_p}\frac{\mathbb{E}\left[x_p\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[(x_p - x_i)\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}. \tag{A.5}$$

According to Equation (15), the slope of the frontier at P is $\mathbb{E}[x_p]/R_p$. Since P is the tangent, we have

$$\frac{\mathbb{E}[x_p]-\mathbb{E}[x_i]}{R_p}\frac{\mathbb{E}\left[x_p\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[(x_p-x_i)\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}=\frac{\mathbb{E}[x_p]}{R_p}.$$

Rearranging the above equation yields

$$\mathbb{E}[x_i]=\frac{\mathbb{E}\left[x_i\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[x_p\left(\frac{\frac{a}{1-\gamma}x_p}{R_p}+1\right)^{\gamma-1}\right]}\mathbb{E}[x_p]. \tag{A.6}$$

Furthermore, since two-fund monetary separation holds, p is a linear combination of the market portfolio and the risk-free asset. Let $X_p=\eta X_m+(1-\eta)X_f$, where η is a constant. Thus, we have $x_p=\eta x_m$. Note that our index satisfies homogeneity of degree one. Thus, $R_p=\eta R_m$. Therefore, Equation (A.6) can be rewritten as

$$\mathbb{E}[x_i]=\frac{\mathbb{E}\left[x_i\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[x_m\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}\mathbb{E}[x_m]. \tag{A.7}$$

Note that

$$\frac{\mathbb{E}\left[x_i\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\mathbb{E}\left[x_m\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}=\frac{\mathbb{E}[x_i]\,\mathbb{E}\left[\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]+\operatorname{cov}\left[x_i,\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\mathbb{E}[x_m]\,\mathbb{E}\left[\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]+\operatorname{cov}\left[x_m,\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}.$$

Equation (A.7) can be rewritten as

$$\mathbb{E}[x_i]=\frac{\operatorname{cov}\left[x_i,\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}{\operatorname{cov}\left[x_m,\left(\frac{\frac{a}{1-\gamma}x_m}{R_m}+1\right)^{\gamma-1}\right]}\mathbb{E}[x_m],$$

which completes the proof.

Appendix D Proofs of the Econometrics Part

D.1 The OLS method is inapplicable to the generalized CAPM

Let $\hat{\alpha}_{i,OLS}$ and $\hat{\beta}_{i,OLS}$ be, respectively, the OLS estimators for the coefficients of the linear regression: x_{it} on 1 and x_{mt}:

$$\hat{\alpha}_{i,OLS} := \bar{x}_i - \hat{\beta}_{i,OLS}\bar{x}_m$$

and

$$\hat{\beta}_{i,OLS} := \frac{\widehat{\text{cov}}[x_{it}, x_{mt}]}{\widehat{\text{var}}[x_{mt}]}.$$

By a suitable law of large numbers which implies that $\bar{x}_i \xrightarrow{\text{p}} \mathbb{E}[x_{it}]$, $\bar{x}_m \xrightarrow{\text{p}} \mathbb{E}[x_{mt}]$, $\widehat{\text{cov}}[x_{it}, x_{mt}] \xrightarrow{\text{p}} \text{cov}[x_{it}, x_{mt}]$ and $\widehat{\text{var}}[x_{mt}] \xrightarrow{\text{p}} \text{var}[x_{mt}]$, we have the result:

$$\hat{\alpha}_{i,OLS} \xrightarrow{\text{p}} \alpha_i^{MV} := \mathbb{E}[x_{it}] - \beta_i^{MV}\ \mathbb{E}[x_{mt}]$$

and

$$\hat{\beta}_{i,OLS} \xrightarrow{\text{p}} \beta_i^{MV} := \frac{\text{cov}[x_{it}, x_{mt}]}{\text{var}[x_{mt}]}.$$

By a decomposition, we can further see that, under H_o,

$$\hat{\alpha}_{i,OLS} \xrightarrow{\text{p}} -\left(\beta_i^{MV} - \beta_i(R_m)\right)\ \mathbb{E}[x_{mt}]$$

and

$$\hat{\beta}_{i,OLS} \xrightarrow{\text{p}} \beta_i(R_m) + \left(\beta_i^{MV} - \beta_i(R_m)\right).$$

This shows that $\hat{\alpha}_{i,OLS}$ would converge in probability to a non-zero value provided that $\beta_i^{MV} \neq \beta_i(R_m)$, even if H_o is true.

D.2 Proof of Proposition 2

Denote the following transformations of x_{it} and x_{mt}:

$$\phi_t := \phi(x_{mt}, R_m),\ \frac{\partial}{\partial R}\phi_t := \frac{\partial}{\partial R}\phi(x_{mt}, R_m),\ k_{mt} := k(x_{mt}, R_m),\ \frac{\partial}{\partial R}k_{mt} := \frac{\partial}{\partial R}k(x_{mt}, R_m),$$

$$\psi_{R,t} := -\mathbb{E}[\frac{\partial}{\partial R}\phi_t]^{-1}\phi_t, \tag{A.8}$$

$$\psi_{\beta_i,t} := \frac{1}{\mathrm{cov}[x_{mt}, k_{mt}]}\psi^A_{\beta_i,t} - \frac{\mathrm{cov}[x_{it}, k_{mt}]}{\mathrm{cov}[x_{mt}, k_{mt}]^2}\psi^B_{\beta_i,t}, \tag{A.9}$$

$$\begin{aligned}\psi^A_{\beta_i,t} &:= (x_{it}k_{mt} - \mathbb{E}[x_{it}k_{mt}]) - \mathbb{E}[x_{it}](k_{mt} - \mathbb{E}[k_{mt}]) - \mathbb{E}[k_{mt}](x_{it} - \mathbb{E}[x_{it}]) \\ &\quad + \mathbb{E}[(x_{it} - \mathbb{E}[x_{it}])\frac{\partial}{\partial R}k_{mt}]\psi_{R,t}\end{aligned} \tag{A.10}$$

and

$$\begin{aligned}\psi^B_{\beta_i,t} &:= (x_{mt}k_{mt} - \mathbb{E}[x_{mt}k_{mt}]) - \mathbb{E}[x_{mt}](k_{mt} - \mathbb{E}[k_{mt}]) - \mathbb{E}[k_{mt}](x_{mt} - \mathbb{E}[x_{mt}]) \\ &\quad + \mathbb{E}[(x_{mt} - \mathbb{E}[x_{mt}])\frac{\partial}{\partial R}k_{mt}]\psi_{R,t},\end{aligned} \tag{A.11}$$

$$\psi_{\alpha_i,t} := (x_{it} - \beta_i(R_m)x_{mt}) - \mathbb{E}[x_{mt}]\psi_{\beta_i,t}, \tag{A.12}$$

and

$$\psi_{\lambda_i,t} := \left(\psi_{\alpha_i,t}, \psi_{\beta_i,t}, \psi_{R,t}\right)^\top. \tag{A.13}$$

Let us make the following assumption:

Assumption 1 Assume that

- **(i)** $\hat{R} \xrightarrow{\mathrm{p}} R_m$ as $T \to \infty$.
- **(ii)** $\sup_{R\in\Theta} |\hat{\beta}_i(R) - \beta_i(R)| \xrightarrow{\mathrm{p}} 0$ as $T \to \infty$ for all i's.
- **(iii)** $k(\cdot)$ and $\phi(\cdot)$ are continuously differentiable with respect to R.
- **(iv)** $\mathbb{E}[\phi(x_t, R_m)]$ is defined.
- **(v)** $T^{-1}\sum_{t=1}^T \frac{\partial}{\partial R}\phi(x_t, R)$, $T^{-1}\sum_{t=1}^T \frac{\partial}{\partial R}k(x_{mt}, R)$ and $T^{-1}\sum_{t=1}^T x_{it}\frac{\partial}{\partial R}k(x_{mt}, R)$ are uniformly positive definite.

(vi) A law of large numbers ensures that $\bar{x}_i$, $\bar{x}_m$, $\widehat{\text{cov}}[x_{it}, x_{mt}]$ and $\widehat{\text{var}}[x_{mt}]$ have the probability limits: $\mathbb{E}[x_{it}]$, $\mathbb{E}[x_{mt}]$, $\text{cov}[x_{it}, x_{mt}]$, $\text{var}[x_{mt}]$, respectively, as $T \to \infty$, for all i's. In addition, there is a uniform law of large numbers ensuring that $T^{-1}\sum_{t=1}^{T}\frac{\partial}{\partial R}\phi(x_{mt}, R)$, $T^{-1}\sum_{t=1}^{T}\frac{\partial}{\partial R}k(x_{mt}, R)$ and $T^{-1}\sum_{t=1}^{T}x_{mt}\frac{\partial}{\partial R}k(x_{mt}, R)$ uniformly converge in probability to $\mathbb{E}[\frac{\partial}{\partial R}\phi(x_{mt}, R)]$, $\mathbb{E}[\frac{\partial}{\partial R}k(x_{mt}, R)]$ and $\mathbb{E}[x_{it}\frac{\partial}{\partial R}k(x_{mt}, R)]$, respectively, as $T \to \infty$, for all i's.

(vii) Under H_o, $\{\psi_{\lambda_i,t}\}$, with the $\psi_{\lambda_i,t}$ that will be defined in (A.13), obeys a central limit theorem which ensures that $T^{-1/2}\sum_{t=1}^{T}\psi_{\lambda_i,t} \xrightarrow{\text{d}} N(0, V_{\lambda_i})$ as $T \to \infty$, for all i's, where $V_{\lambda_i} := \lim_{T\to\infty} \text{var}\left[T^{-1/2}\sum_{t=1}^{T}\psi_{\lambda_i,t}\right]$.

To justify this assumption, note that because $\hat{R}$ is an MM estimator for R_m, it is standard to argue the consistency of $\hat{R}$ for R_m in A.1(i) by the generalized MM (GMM) theory; see, e.g., Newey and McFadden (1994, Theorem 2.6). The uniform convergence in A.1(ii) is similar to a result in Hall (2005, Lemma 3.1). A.1(iii), (iv) and (v) are ensured to be satisfied when $k(\cdot)$ and $\phi(\cdot)$ are suitably defined. A.1(vi) needs a suitable (uniform) law of large numbers. Because $\psi_{\lambda_i,t}$ is a zero-mean transformation of x_{it} and x_{mt} under A.1(v) and (21), it is not restrictive to make A.1(vii).

Given A.1(i) and (ii), we have $\hat{\beta}_i := \hat{\beta}_i(\hat{R}) \xrightarrow{\text{p}} \beta_i(R_m)$. The result: $\hat{\alpha}_i \xrightarrow{\text{p}} \alpha_{o,i}$ also holds because α_i is a linear transformation of $\beta_i(R)$. Thus, $\hat{\lambda}_i \xrightarrow{\text{p}} \lambda_{o,i}$. It is standard to further show the asymptotic normality by using the mean-value expansion; see, e.g., Newey and McFadden (1994). Given the continuous differentiability of $\phi(x_{mt}, R)$ with respect to R in A.1(iii), we can use this expansion to see that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\phi(x_{mt}, \hat{R}) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\phi(x_{mt}, R_m) + \left[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial R}\phi(x_{mt}, R^{\dagger})\right]\sqrt{T}(\hat{R} - R_m), \quad \text{(A.14)}$$

for some $R^{\dagger} \in \mathbb{R}_+$ such that $\|R^{\dagger} - R_m\| \leq \|\hat{R} - R_m\|$. Given (30) and A.1(v), we can rewrite (A.14) as:

$$\sqrt{T}(\hat{R} - R_m) = -\left[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial R}\phi(x_{mt}, R^{\dagger})\right]^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\phi(x_{mt}, R_m).$$

By A.1(i), (iii), (vi): $\sup_{R\in\Theta} \|T^{-1}\sum_{t=1}^{T}\frac{\partial}{\partial R}\phi(x_{mt},R) - \mathbb{E}[\frac{\partial}{\partial R}\phi(x_{mt},R)]\| \overset{\text{p}}{\to} 0$ as $T \to \infty$, we can reexpress this result as:

$$\begin{aligned}\sqrt{T}(\hat{R}-R_m) &= -\mathbb{E}\left[\frac{\partial}{\partial R}\phi(x_{mt},R_m)\right]^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\phi(x_{mt},R_m)+o_p(1), \\ &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{R,t}+o_p(1) \qquad \text{(A.15)}\end{aligned}$$

By definition, we also have

$$\sqrt{T}(\hat{\beta}_i-\beta_i(R_m)) = \sqrt{T}\left(\frac{\hat{A}}{\hat{B}}-\frac{A}{B}\right) = \frac{1}{\hat{B}B}\left(B\sqrt{T}(\hat{A}-A)-A\sqrt{T}(\hat{B}-B)\right), \qquad \text{(A.16)}$$

where

$$\hat{A} := \widehat{\text{cov}}[x_{it},\hat{k}_{mt}] = \frac{1}{T}\sum_{t=1}^{T}(x_{it}-\bar{x}_i)(\hat{k}_{mt}-\bar{k}_m),$$

$$A := \text{cov}[x_{it},k_{mt}] = \mathbb{E}\left[(x_{it}-\mathbb{E}[x_{it}])(k_{mt}-\mathbb{E}[k_{mt}])\right],$$

$$\hat{B} := \widehat{\text{cov}}[x_{mt},\hat{k}_{mt}] = \frac{1}{T}\sum_{t=1}^{T}(x_{mt}-\bar{x}_m)(\hat{k}_{mt}-\bar{k}_m)$$

and

$$B := \text{cov}[x_{mt},k_{mt}] = \mathbb{E}\left[(x_{mt}-\mathbb{E}[x_{mt}])(k_{mt}-\mathbb{E}[k_{mt}])\right].$$

By construction, we have

$$\begin{aligned}\sqrt{T}(\hat{B}-B) &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(x_{mt}\hat{k}_{mt}-x_{mt}\bar{k}_m-\bar{x}_m\hat{k}_{mt}+\bar{x}_m\bar{k}_m-\mathbb{E}[x_{mt}k_{mt}]+\mathbb{E}[x_{mt}]\mathbb{E}[k_{mt}]\right) \\ &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(x_{mt}\hat{k}_{mt}-\mathbb{E}[x_{mt}k_{mt}]\right)-\bar{x}_m\sqrt{T}(\bar{k}_m-\mathbb{E}[k_{mt}])-\mathbb{E}[k_{mt}]\sqrt{T}(\bar{x}_m-\mathbb{E}[x_{mt}]) \\ &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\left(x_{mt}\hat{k}_{mt}-\mathbb{E}[x_{mt}k_{mt}]\right)-\mathbb{E}[x_{mt}](\hat{k}_{mt}-\mathbb{E}[k_{mt}])-\mathbb{E}[k_{mt}](x_{mt}-\mathbb{E}[x_{mt}])\right) \\ &\quad +o_p(1), \qquad \text{(A.17)}\end{aligned}$$

where the last equality utilizes A.1(vi): $\bar{x}_m \xrightarrow{\text{p}} \mathbb{E}[x_{mt}]$. Similar to the derivation of (A.15), we can use the mean-value expansion, A.1(i), (iii), (v) and (vi) to show that

$$\begin{aligned}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(x_{mt}\hat{k}_{mt}-\mathbb{E}[x_{mt}k_{mt}]\right) &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}(x_{mt}k_{mt}-\mathbb{E}[x_{mt}k_{mt}]) \\ &\quad +\mathbb{E}\left[x_{mt}\frac{\partial}{\partial R}k_{mt}\right]\sqrt{T}(\hat{R}-R_m)+o_p(1) \end{aligned} \tag{A.18}$$

and

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\hat{k}_{mt}-\mathbb{E}[k_{mt}]\right)=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}(k_{mt}-\mathbb{E}[k_{mt}])+\mathbb{E}\left[\frac{\partial}{\partial R}k_{mt}\right]\sqrt{T}(\hat{R}-R_m)+o_p(1). \tag{A.19}$$

By first introducing (A.15) in (A.18) and (A.19) and then plugging the results in (A.17), we can obtain that

$$\sqrt{T}(\hat{B}-B)=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi^{B}_{\beta_i,t}+o_p(1). \tag{A.20}$$

By the same method, we can also show that, under A.1(i), (iii), (v) and (vi),

$$\sqrt{T}(\hat{A}-A)=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi^{A}_{\beta_i,t}+o_p(1), \tag{A.21}$$

By introducing (A.17) and (A.21) in (A.16) and using $\hat{B} \xrightarrow{\text{p}} B$, which is implied by (A.20) and a requirement of A.1(vii): $T^{-1/2}\sum_{t=1}^{T}\psi^{B}_{\beta_i,t}=O_p(1)$, we have the result:

$$\sqrt{T}(\hat{\beta}_i-\beta_i(R_m))=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{\beta_i,t}+o_p(1). \tag{A.22}$$

In addition, by using H_o and (32), we can write that

$$\begin{aligned}\sqrt{T}\,(\hat{\alpha}_i - \alpha_{o,i}) &= \sqrt{T}\hat{\alpha}_i = \sqrt{T}(\bar{x}_i - \hat{\beta}_i \bar{x}_m) = \sqrt{T}(\bar{x}_i - \beta_i(R_m)\bar{x}_m - \hat{\beta}_i \bar{x}_m + \beta_i(R_m)\bar{x}_m) \\ &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} (x_i - \beta_i(R_m)x_{mt}) - \bar{x}_m \sqrt{T}(\hat{\beta}_i - \beta_i(R_m)). \end{aligned} \tag{A.23}$$

By introducing A.1(vi): $\bar{x}_m \xrightarrow{\text{p}} \mathbb{E}[x_{mt}]$ and (A.22) in (A.23), we obtain that

$$\sqrt{T}\,(\hat{\alpha}_i - \alpha_{o,i}) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \psi_{\alpha_i,t} + o_p(1). \tag{A.24}$$

From (A.15), (A.22) and (A.24), we obtain that

$$\sqrt{T}\left(\hat{\lambda}_i - \lambda_{o,i}\right) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \psi_{\lambda_i,t} + o_p(1). \tag{A.25}$$

The result in (33) follows (A.25) and A.1(vii).

D.3 Proof of Proposition 3

Denote the $Nr \times 1$ vector: $\psi_{\boldsymbol{\alpha_z},t} := \left(\psi^\top_{\alpha_{z1,t}}, \ldots, \psi^\top_{\alpha_{zN,t}}\right)^\top$, where

$$\psi_{\alpha_{zi},t} := z_t\,(x_{it} - \beta_i(R_m)x_{mt}) - \mathbb{E}[z_t x_{mt}]\psi_{\beta_i,t}, \tag{A.26}$$

and define the $Nr \times Nr$ asymptotic covariance matrix $V_{\boldsymbol{\alpha}_z} := \lim_{T\to\infty} \text{var}\left[T^{-1/2} \sum_{t=1}^{T} \psi_{\boldsymbol{\alpha}_z,t}\right]$. By comparing (A.12) with (A.26), we can observe that $\psi_{\alpha_{zi},t}$ degenerates to $\psi_{\alpha_i,t}$ when $z_t = 1$.

In addition to Assumption 1, we make the following assumption:

Assumption 2 Assume that

- **(i)** $T^{-1} \sum_{t=1}^{T} z_t x_{mt} \xrightarrow{\text{p}} \mathbb{E}[z_t x_{mt}]$, as $T \to \infty$.
- **(ii)** Under $H_{z,o}$, the sequence $\{\psi_{\boldsymbol{\alpha}_z,t}\}$ obeys a central limit theorem which ensures that $T^{-1/2} \sum_{t=1}^{T} \psi_{\boldsymbol{\alpha}_z,t} \xrightarrow{\text{d}} N(0, V_{\boldsymbol{\alpha}_z})$, as $T \to \infty$.
- **(iii)** There exists an $Nr \times Nr$ matrix $\hat{V}_{\boldsymbol{\alpha}_z}$ which is positive definite and consistent for $V_{\boldsymbol{\alpha}_z}$.

Note that

$$
\begin{aligned}
\sqrt{T}\hat{\boldsymbol{\alpha}}_z &= \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t\hat{u}_{1t} \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t\hat{u}_{2t} \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t\hat{u}_{Nt} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{1t}-\hat{\beta}_1 x_{mt}) \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{2t}-\hat{\beta}_2 x_{mt}) \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{Nt}-\hat{\beta}_N x_{mt}) \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{1t}-\beta_1(R_m)x_{mt}) - \left[\frac{1}{T}\sum_{t=1}^{T} z_t x_{mt}\right]\sqrt{T}(\hat{\beta}_1-\beta_1(R_m)) \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{2t}-\beta_2(R_m)x_{mt}) - \left[\frac{1}{T}\sum_{t=1}^{T} z_t x_{mt}\right]\sqrt{T}(\hat{\beta}_2-\beta_2(R_m)) \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t(x_{Nt}-\beta_N(R_m)x_{mt}) - \left[\frac{1}{T}\sum_{t=1}^{T} z_t x_{mt}\right]\sqrt{T}(\hat{\beta}_N-\beta_N(R_m)) \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\Big(z_t(x_{1t}-\beta_1(R_m)x_{mt}) - \mathbb{E}[z_t x_{mt}]\psi_{\beta_1,t}\Big) \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\Big(z_t(x_{2t}-\beta_2(R_m)x_{mt}) - \mathbb{E}[z_t x_{mt}]\psi_{\beta_2,t}\Big) \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\Big(z_t(x_{Nt}-\beta_N(R_m)x_{mt}) - \mathbb{E}[z_t x_{mt}]\psi_{\beta_N,t}\Big) \end{bmatrix} + o_p(1) \\
&= \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{\alpha_{z1},t} \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{\alpha_{z2},t} \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{\alpha_{zN},t} \end{bmatrix} + o_p(1) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_{\boldsymbol{\alpha}_{\boldsymbol{z}},t} + o_p(1), \qquad \text{(A.27)}
\end{aligned}
$$

where the third equality is due to A.2(i). Given $H_{z,o}$, we can use (A.27) and A.2(ii) to obtain the asymptotic null distribution of $\hat{\boldsymbol{\alpha}}_z$:

$$\sqrt{T}\hat{\boldsymbol{\alpha}}_z \xrightarrow{\text{d}} N(0, V_{\boldsymbol{\alpha}_z}) \qquad \text{(A.28)}$$

as $T \to \infty$. Given A.2(iii), the statistic $\mathcal{J}_z$ in (37) is defined and has the asymptotic null distribution $\chi^2(Nr)$ because of (A.28).

To facilitate the $\mathcal{J}_z$ test, we conduct the heteroskedasticity-autocorrelation-consistent (HAC) estimator for $V_{\boldsymbol{\alpha}_z}$:

$$\hat{V}_{\boldsymbol{\alpha}_z} = \sum_{j=1-T}^{T-1} \kappa\left(\frac{j}{\tau}\right) \hat{\Gamma}(j),$$

with the sample autocovariance matrix:

$$\hat{\Gamma}(j) := \begin{cases} \frac{1}{T}\sum_{t=j+1}^{T}\left(\hat{\psi}_{\boldsymbol{\alpha}_z,t} - \frac{1}{T}\sum_{t=1}^{T}\hat{\psi}_{\boldsymbol{\alpha}_z,t}\right)\left(\hat{\psi}_{\boldsymbol{\alpha}_z,t-j} - \frac{1}{T}\sum_{t=1}^{T}\hat{\psi}_{\boldsymbol{\alpha}_z,t}\right)^{\top}, & j \geq 0, \\ \frac{1}{T}\sum_{t=-j+1}^{T}\left(\hat{\psi}_{\boldsymbol{\alpha}_z,t+j} - \frac{1}{T}\sum_{t=1}^{T}\hat{\psi}_{\boldsymbol{\alpha}_z,t}\right)\left(\hat{\psi}_{\boldsymbol{\alpha}_z,t} - \frac{1}{T}\sum_{t=1}^{T}\hat{\psi}_{\boldsymbol{\alpha}_z,t}\right)^{\top}, & j < 0, \end{cases}$$

the bandwidth $\tau > 0$, the kernel function $\kappa(\cdot)$, and $\hat{\psi}_{\boldsymbol{\alpha}_z,t} := \left(\hat{\psi}^{\top}_{\alpha_{z1,t}}, \ldots, \hat{\psi}^{\top}_{\alpha_{zN,t}}\right)^{\top}$ with

$$\hat{\psi}_{\alpha_{zi},t} := z_t\left(x_{it} - \hat{\beta}_i x_{mt}\right) - \left(T^{-1}\sum_{t=1}^{T} z_t x_{mt}\right)\hat{\psi}_{\beta_i,t},$$

in which

$$\hat{\psi}_{\beta_i,t} := \frac{1}{\widehat{\mathrm{cov}}[x_{mt}, \hat{k}_{mt}]}\hat{\psi}^{A}_{\beta_i,t} - \frac{\widehat{\mathrm{cov}}[x_{it}, \hat{k}_{mt}]}{\widehat{\mathrm{cov}}[x_{mt}, \hat{k}_{mt}]^2}\hat{\psi}^{B}_{\beta_i,t}, \tag{A.29}$$

with

$$\begin{aligned} \hat{\psi}^{A}_{\beta_i,t} \quad : \quad = & \left(x_{it}\hat{k}_{mt} - \frac{1}{T}\sum_{t=1}^{T} x_{it}\hat{k}_{mt}\right) - \bar{x}_i(\hat{k}_{mt} - \bar{k}_m) - \bar{k}_m(x_{it} - \bar{x}_i) \\ & + \left(\frac{1}{T}\sum_{t=1}^{T}(x_{it} - \bar{x}_i)\frac{\partial}{\partial R}\hat{k}_{mt}\right)\hat{\psi}_{R,t}, \end{aligned} \tag{A.30}$$

and

$$\begin{aligned} \hat{\psi}^{B}_{\beta_i,t} \quad : \quad = & \left(x_{mt}\hat{k}_{mt} - \frac{1}{T}\sum_{t=1}^{T} x_{mt}\hat{k}_{mt}\right) - \bar{x}_m(\hat{k}_{mt} - \bar{k}_m) - \bar{k}_m(x_{mt} - \bar{x}_m) \\ & + \left(\frac{1}{T}\sum_{t=1}^{T}(x_{mt} - \bar{x}_m)\frac{\partial}{\partial R}\hat{k}_{mt}\right)\hat{\psi}_{R,t}; \end{aligned} \tag{A.31}$$

$\hat{\psi}_{R,t}$ is the $\hat{R}$-based sample analogue of $\psi_{R,t}$:

$$\hat{\psi}_{R,t} = \begin{cases} -\hat{R}^2 \left(T^{-1}\sum_{t=1}^{T} x_{mt}\exp(-x_{mt}/\hat{R})\right)^{-1}\left(\exp(-x_{mt}/\hat{R})-1\right), & AS, \\ \hat{R}^2 \left(T^{-1}\sum_{t=1}^{T} x_{mt}(1+x_{mt}/\hat{R})^{-1}\right)^{-1}\ln(1+x_{mt}/\hat{R}), & FH, \\ (\hat{R}^2/\gamma)\left(T^{-1}\sum_{t=1}^{T} x_{mt}(1+x_{mt}/\hat{R})^{\gamma-1}\right)^{-1}\left((1+x_{mt}/\hat{R})^{\gamma}-1\right), & BCCY, \\ (\hat{R}^2(1-\gamma)/(a\gamma))\left(T^{-1}\sum_{t=1}^{T} x_{mt}(1+(x_{mt}/\hat{R})(a/(1-\gamma)))^{\gamma-1}\right)^{-1} & \\ \quad \times\left(\left(1+(x_{mt}/\hat{R})(a/(1-\gamma))\right)^{\gamma}-1\right), & GR; \end{cases} \tag{A.32}$$

$\hat{k}_{mt} := k(x_{mt}, \hat{R})$ and $\frac{\partial}{\partial R}\hat{k}_{mt} := \frac{\partial}{\partial R}k(x_{mt}, \hat{R})$ such that:

$$\frac{\partial}{\partial R}\hat{k}_{mt} = \begin{cases} (x_{mt}/\hat{R}^2)\exp(-x_{mt}/\hat{R}), & AS, \\ (x_{mt}/\hat{R}^2)(1+x_{mt}/\hat{R})^{-2}, & FH, \\ (1-\gamma)(x_{mt}/\hat{R}^2)(1+x_{mt}/\hat{R})^{\gamma-2}, & BCCY, \\ (ax_{mt}/\hat{R}^2)(1+(x_{mt}/\hat{R})(a/(1-\gamma)))^{\gamma-2}, & GR. \end{cases}$$

See, e.g., Andrews (1991) and Hall (2000) for a general discussion on the consistency and positive definiteness of the HAC estimator. For simplicity, we make use of the Bartlett kernel: $\kappa(j/\tau) = 1 - |j/\tau|$ if $|j| < \tau$ (otherwise, zero) with $\tau = 3$ as in Hong, Tu and Zhou (2007, p.1553).

D.4 The proposed method encompasses the OLS method

To show that our estimation and testing method encompasses the OLS method, note that, for the MV-CAPM, $k(x_{mt}, R) = x_{mt}$ is free of R; thus, $\beta_i(R) = \beta_i^{MV}$, and $\hat{\lambda}_i$ degenerates to the OLS estimator:

$$\hat{\lambda}_i = (\hat{\alpha}_i, \hat{\beta}_i)^{\top} = \left(\hat{\alpha}_{i,OLS}, \hat{\beta}_{i,OLS}\right)^{\top}. \tag{A.33}$$

Denote $v_{it} := x_{it} - \beta_i^{MV} x_{mt}$, $\boldsymbol{v}_t := (v_{1t}, \ldots, v_{Nt})^{\top}$, $\sigma_{v,i}^2 := \mathbb{E}[v_{it}^2]$, $\Sigma_{\boldsymbol{v}} := \mathbb{E}[\boldsymbol{v}_t \boldsymbol{v}_t^{\top}]$ and $V_{\boldsymbol{\alpha}} := V_{\boldsymbol{\alpha}_z}|_{\{z_t=1\}}$. In the following, we prove that, if $\{x_{it}, x_{mt}\}$ is an IID sequence, then V_{λ_i} and $V_{\boldsymbol{\alpha}}$

can be, respectively, simplified as:

$$V_{MV,\lambda_i} = \sigma_i^2 \begin{bmatrix} 1 + \frac{\mathbf{E}[x_{mt}]^2}{\text{var}[x_{mt}]} & -\frac{\mathbf{E}[x_{mt}]}{\text{var}[x_{mt}]} \\ -\frac{\mathbf{E}[x_{mt}]}{\text{var}[x_{mt}]} & \frac{1}{\text{var}[x_{mt}]} \end{bmatrix} \tag{A.34}$$

and

$$V_{MV,\boldsymbol{\alpha}} = \left(1 + \frac{\mathbb{E}[x_{mt}]^2}{\text{var}[x_{mt}]}\right) \Sigma_{\boldsymbol{\nu}} \tag{A.35}$$

for the MV-CAPM. Given $V_{\lambda_i} = V_{MV,\lambda_i}$, we can see that the asymptotic distribution of $\hat{\lambda}$ shown in Proposition 2 degenerates to that of (A.33). Given $V_{\boldsymbol{\alpha}} = V_{MV,\boldsymbol{\alpha}}$, we can further see that the proposed zero-alpha test reduces to the χ^2 test shown in Cochrane (2000, Equation 168), which is based on the OLS estimator and is widely applied in testing the MV-CAPM or other one-factor asset pricing models.

To show (A.34) and (A.35), note that, given $k(x_{mt}, R) = x_{mt}$, $\psi_{\lambda_i,t}$ degenerates to

$$\psi_{\lambda_i,t} = \left(\psi_{\alpha_i,t}, \psi_{\beta_i,t}\right)^\top, \tag{A.36}$$

in which $\psi_{\alpha_i,t}$ follows (A.12), and $\psi_{\beta_i,t}$ reduces to the form:

$$\psi_{\beta_i,t} = \frac{1}{\text{var}[x_{mt}]}\psi^A_{\beta_i,t} - \frac{\text{cov}[x_{it}, x_{mt}]}{\text{var}[x_{mt}]^2}\psi^B_{\beta_i,t} = \frac{1}{\text{var}[x_{mt}]}\left(\psi^A_{\beta_i,t} - \beta_i^{MV}\psi^B_{\beta_i,t}\right), \tag{A.37}$$

with

$$\psi^A_{\beta_i,t} = (x_{it}x_{mt} - \mathbb{E}[x_{it}x_{mt}]) - \mathbb{E}[x_{it}](x_{mt} - \mathbb{E}[x_{mt}]) - \mathbb{E}[x_{mt}](x_{it} - \mathbb{E}[x_{it}]) \tag{A.38}$$

and

$$\psi^B_{\beta_i,t} = \left(x_{mt}^2 - \mathbb{E}[x_{mt}^2]\right) - 2\mathbb{E}[x_{mt}](x_{mt} - \mathbb{E}[x_{mt}]). \tag{A.39}$$

In addition, the MV-CAPM model implies that $x_{it} = \beta_i^{MV}x_{mt} + v_{it}$, $\mathbb{E}[x_{it}] = \beta_i^{MV}\mathbb{E}[x_{mt}]$,

$x_{it} - \mathbb{E}[x_{it}] = \beta_i^{MV}(x_{mt} - \mathbb{E}[x_{mt}]) + v_{it}$ and $\mathbb{E}[v_{it}x_{mt}] = 0$. According to these implications, we can rewrite (A.38) as:

$$\begin{aligned}
\psi^A_{\beta_i,t} &= \left(\beta_i^{MV} x_{mt}^2 + v_{it}x_{mt} - \beta_i^{MV}\mathbb{E}[x_{mt}^2]\right) - \beta_i^{MV}\mathbb{E}[x_{mt}](x_{mt} - \mathbb{E}[x_{mt}]) \\
&- \beta_i^{MV}\mathbb{E}[x_{mt}](x_{mt} - \mathbb{E}[x_{mt}]) - v_{it}\mathbb{E}[x_{mt}] \\
&= \beta_i^{MV}\left(x_{mt}^2 - \mathbb{E}[x_{mt}^2]\right) + v_{it}\left(x_{mt} - \mathbb{E}[x_{mt}]\right) - 2\beta_i^{MV}\mathbb{E}[x_{mt}](x_{mt} - \mathbb{E}[x_{mt}]) \\
&= \beta_i^{MV}\psi^B_{\beta_i,t} + v_{it}\left(x_{mt} - \mathbb{E}[x_{mt}]\right),
\end{aligned} \tag{A.40}$$

with $\psi^B_{\beta_i,t}$ following (A.39). By plugging (A.40) in (A.37), we obtain that

$$\psi_{\beta_i,t} = \left(\frac{x_{mt} - \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right) v_{it}. \tag{A.41}$$

By introducing (A.41) in (A.12), we further obtain that

$$\psi_{\alpha_i,t} = v_{it} - \mathbb{E}[x_{mt}]\psi_{\beta_i,t} = \left(1 - \frac{(x_{mt} - \mathbb{E}[x_{mt}])\ \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right) v_{it}. \tag{A.42}$$

Thus,

$$\psi_{\lambda_i,t} = \begin{bmatrix} \left(1 - \frac{(x_{mt} - \mathbb{E}[x_{mt}])\ \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right) \\ \left(\frac{x_{mt} - \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right) \end{bmatrix} v_{it} \tag{A.43}$$

and

$$\psi_{\boldsymbol{\alpha},t} = \left(1 - \frac{(x_{mt} - \mathbb{E}[x_{mt}])\ \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right) \boldsymbol{v}_t \tag{A.44}$$

Note that

$$\mathbb{E}\left[\left(1 - \frac{(x_{mt} - \mathbb{E}[x_{mt}])\ \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right)^2\right] = 1 + \frac{\mathbb{E}[x_{mt}]^2}{\text{var}[x_{mt}]},$$

$$\mathbb{E}\left[\left(\frac{x_{mt} - \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right)^2\right] = \frac{1}{\text{var}[x_{mt}]}$$

and

$$\mathbb{E}\left[\left(\frac{x_{mt} - \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right)\left(1 - \frac{(x_{mt} - \mathbb{E}[x_{mt}])\ \mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}\right)\right] = -\frac{\mathbb{E}[x_{mt}]}{\text{var}[x_{mt}]}.$$

If $\{x_{it}, x_{mt}\}$ is an IID sequence, then $\boldsymbol{v}_t$ is conditionally homoskedastic in the sense that $\mathbb{E}[\boldsymbol{v}_t \boldsymbol{v}_t^\top | x_{mt}] = \Sigma_{\boldsymbol{v}}$; meanwhile, we can simplify V_{λ_i} and $V_{\boldsymbol{\alpha}}$ as:

$$V_{MV,\lambda_i} = \mathbb{E}[\psi_{\lambda_i,t}\psi_{\lambda_i,t}^\top] \quad \text{and} \quad V_{MV,\boldsymbol{\alpha}} = \mathbb{E}[\psi_{\boldsymbol{\alpha},t}\psi_{\boldsymbol{\alpha},t}^\top]. \tag{A.45}$$

The results in (A.34) and (A.35) are obtained by introducing (A.36) and (A.44) in (A.45) and using the conditional homoskedasticity of $\boldsymbol{v}_t$.

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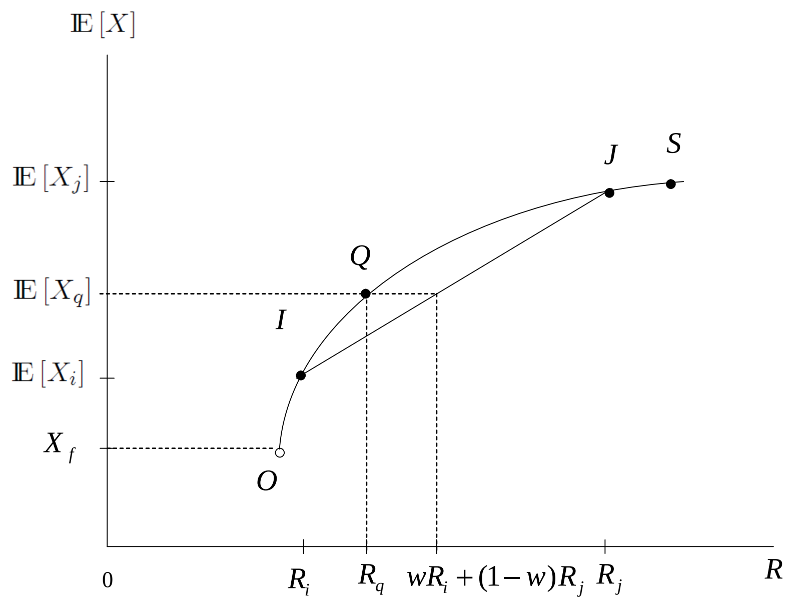

Figure 1 The possible values of $\mathbb{E}[X_q]$ and R_q obtainable with different combinations of assets i and j.

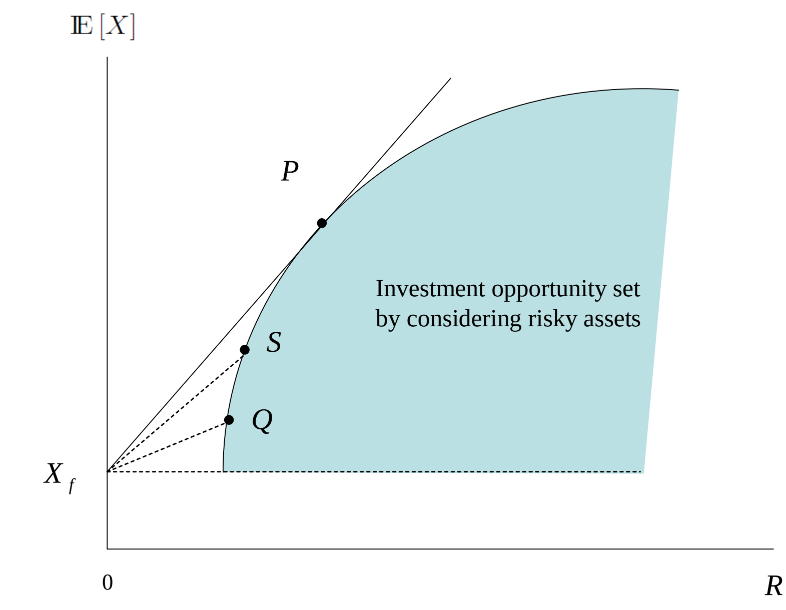

Figure 2 Efficient frontier.

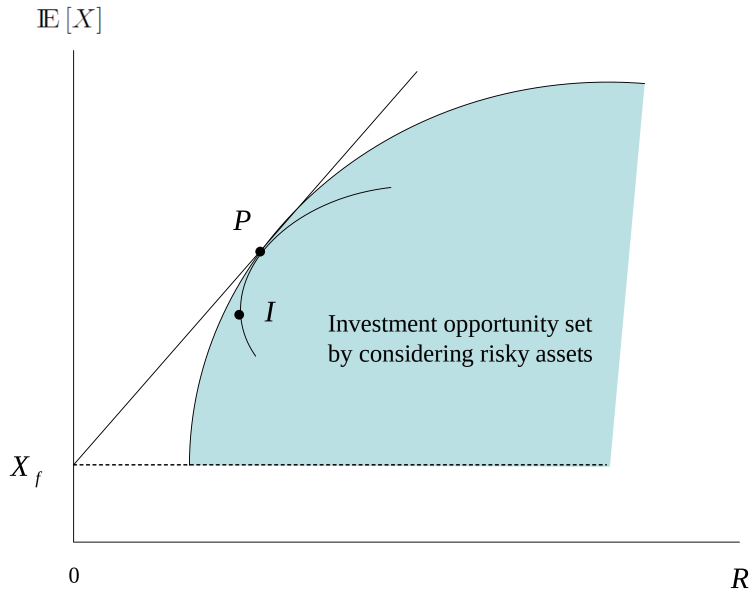

Figure 3 The linear combination of a risky asset and an efficient portfolio.

Table 1: Riskiness index estimates of x_{mt}

Period	AS	FH	BCCY	GR(a,γ) (1,-10)	(1,-0.1)	(10,-10)	(10,-0.1)
1981-2014	17.349	23.615	54.792	17.555	22.198	175.554	221.981
1961-2014	20.089	24.117	62.178	20.235	23.187	202.346	231.873

Note: This table shows the riskiness index estimator $\hat{R}$ in (22) with the $\phi(x_{mt}, R_m)$ following (18) and x_{mt} denoting the market return (the value-weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ) obtained from the data library of Kenneth French. In estimating the BCCY riskiness index, we set $\gamma = -2$. The $\hat{R}$ is numerically solved from (22) by using the OPTMUM of GAUSS$^{\text{TM}}$ with the Newton-Raphson method. The GAUSS code is available upon request.

Table 2: The $\hat{\alpha}_i$'s and the zero-alpha test statistics for the SB portfolios: 1981-2014

		GR					min		max	
portfolio	MV	AS	FH	BCCY	min	max	a	γ	a	γ
S1B1	-0.870^{**}	-0.885^{**}	-0.923^{**}	-0.885^{**}	-0.886^{**}	-0.914^{**}	9	-10.0	1	-0.1
	(11.940)	(12.543)	(14.141)	(12.698)	(12.603)	(13.844)				
S1B2	0.087	0.070	-0.014	0.069	0.008	0.068	1	-0.1	9	-10.0
	(0.172)	(0.113)	(0.003)	(0.109)	(0.001)	(0.107)				
S1B3	0.241	0.224	0.115	0.222	0.144	0.222	1	-0.1	9	-10.0
	(1.923)	(1.687)	(0.262)	(1.665)	(0.464)	(1.655)				
S1B4	0.444^{*}	0.422^{*}	0.272	0.419^{*}	0.312	0.419^{*}	1	-0.1	9	-10.0
	(6.085)	(5.535)	(0.982)	(5.414)	(1.530)	(5.454)				
S1B5	0.484^{*}	0.456^{*}	0.332	0.453^{*}	0.361	0.452^{*}	1	-0.1	9	-10.0
	(5.481)	(5.028)	(1.897)	(5.084)	(2.479)	(4.972)				
S2B1	-0.420^{*}	-0.424^{*}	-0.451^{**}	-0.425^{*}	-0.425^{*}	-0.444^{**}	9	-10.0	1	-0.1
	(5.865)	(6.016)	(6.906)	(6.047)	(6.038)	(6.680)				
S2B2	0.063	0.047	-0.071	0.045	0.005	0.045	5	-0.4	9	-10.0
	(0.177)	(0.101)	(0.099)	(0.090)	(0.001)	(0.091)				
S2B3	0.382^{**}	0.368^{**}	0.263	0.366^{**}	0.290	0.365^{**}	1	-0.1	9	-10.0
	(7.510)	(6.991)	(1.686)	(6.840)	(2.400)	(6.915)				
S2B4	0.372^{*}	0.352^{*}	0.266	0.350^{*}	0.285	0.350^{*}	1	-0.1	9	-10.0
	(6.124)	(5.502)	(2.197)	(5.301)	(2.710)	(5.421)				
S2B5	0.299	0.281	0.188	0.280	0.212	0.279	1	-0.1	9	-10.0
	(2.338)	(2.078)	(0.686)	(2.057)	(0.953)	(2.049)				
S3B1	-0.240	-0.244	-0.244	-0.244	-0.244	-0.245	9	-10.0	1	-0.1
	(2.646)	(2.748)	(2.940)	(2.758)	(2.758)	(2.917)				
S3B2	0.177	0.169	0.083	0.167	0.106	0.167	1	-0.1	9	-10.0
	(2.167)	(1.975)	(0.226)	(1.913)	(0.432)	(1.939)				
S3B3	0.251	0.244	0.188	0.242	0.202	0.242	1	-0.1	9	-10.0
	(3.720)	(3.479)	(1.614)	(3.414)	(1.965)	(3.445)				
S3B4	0.328^{*}	0.323^{*}	0.281^{*}	0.322^{*}	0.291^{*}	0.322^{*}	1	-0.1	9	-10.0
	(5.390)	(5.198)	(4.019)	(5.213)	(4.300)	(5.173)				
S3B5	0.569^{**}	0.556^{**}	0.460^{*}	0.554^{**}	0.485^{*}	0.553^{**}	1	-0.1	9	-10.0
	(11.271)	(10.587)	(4.537)	(10.268)	(5.513)	(10.496)				
S4B1	-0.005	-0.001	0.027	-0.001	0.000	0.019	8	-5.0	1	-0.1
	(0.002)	(0.000)	(0.044)	(0.000)	(0.000)	(0.023)				
S4B2	0.065	0.056	-0.036	0.054	0.004	0.054	5	-0.2	9	-10.0
	(0.297)	(0.215)	(0.040)	(0.191)	(0.001)	(0.201)				
S4B3	0.135	0.126	0.093	0.124	0.098	0.124	1	-0.1	9	-10.0
	(0.937)	(0.792)	(0.502)	(0.778)	(0.525)	(0.773)				
S4B4	0.298^{*}	0.300^{*}	0.339^{*}	0.300^{*}	0.300^{*}	0.318^{*}	1	-10.0	5	-0.2
	(5.105)	(5.006)	(5.562)	(5.013)	(5.007)	(5.328)				
S4B5	0.301	0.296	0.252	0.295	0.264	0.295	1	-0.1	9	-10.0
	(3.271)	(3.111)	(2.312)	(3.118)	(2.545)	(3.096)				
S5B1	0.014	0.020	0.036	0.021	0.021	0.033	9	-10.0	1	-0.1
	(0.029)	(0.060)	(0.194)	(0.063)	(0.065)	(0.164)				
S5B2	0.113	0.112	0.087	0.112	0.095	0.112	1	-0.1	5	-10.0
	(1.610)	(1.550)	(0.803)	(1.527)	(0.964)	(1.539)				
S5B3	0.022	0.020	-0.006	0.019	0.000	0.019	1	-0.1	9	-10.0
	(0.041)	(0.034)	(0.003)	(0.031)	(0.000)	(0.032)				
S5B4	0.127	0.131	0.187	0.132	0.132	0.169	9	-10.0	1	-0.1
	(0.870)	(0.899)	(1.272)	(0.902)	(0.905)	(1.122)				
S5B5	0.223	0.223	0.237	0.223	0.223	0.232	10	-10.0	1	-0.1
	(1.776)	(1.767)	(2.208)	(1.775)	(1.769)	(2.057)				

Note: The entries in parentheses are the zero-alpha test statistics associated with the $\hat{\alpha}_i$'s, in which $\hat{\alpha}_i$ follows (24) with x_{it} denoting an SB portfolio, and the zero-alpha test statistic follows (36) with $z_t = 1$, $N = 1$ and $r = 1$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The (a, γ) of GR-min and that of GR-max are, respectively, reported in the blocks "min" and "max." The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(1)$ are, respectively, 3.841 and 6.635. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 3: The $\hat{\alpha}_i$'s and the zero-alpha test statistics for the SM portfolios: 1981-2014

		GR					min		max	
portfolio	MV	AS	FH	BCCY	min	max	a	γ	a	γ
S1M1	-0.892^{**}	-0.896^{**}	-0.844^{**}	-0.895^{**}	-0.862^{**}	-0.896^{**}	1	-0.1	9	-10.0
	(11.964)	(11.724)	(8.075)	(11.674)	(8.861)	(11.683)				
S1M2	0.014	-0.006	-0.087	-0.008	-0.009	-0.070	9	-10.0	1	-0.1
	(0.006)	(0.001)	(0.216)	(0.002)	(0.003)	(0.145)				
S1M3	0.315	0.290	0.148	0.287	0.183	0.286	1	-0.1	9	-10.0
	(3.729)	(3.257)	(0.393)	(3.231)	(0.711)	(3.191)				
S1M4	0.459^{**}	0.433^{**}	0.250	0.430^{**}	0.298	0.429^{**}	1	-0.1	9	-10.0
	(7.492)	(6.832)	(0.686)	(6.731)	(1.215)	(6.727)				
S1M5	0.608^{**}	0.583^{*}	0.447	0.580^{*}	0.482	0.580^{*}	1	-0.1	9	-10.0
	(6.794)	(6.377)	(2.473)	(6.385)	(3.207)	(6.324)				
S2M1	-0.686^{**}	-0.678^{**}	-0.553	-0.675^{**}	-0.589	-0.676^{**}	1	-0.1	9	-10.0
	(8.999)	(8.285)	(2.906)	(8.092)	(3.738)	(8.189)				
S2M2	0.045	0.037	-0.014	0.035	-0.003	0.036	4	-0.1	5	-10.0
	(0.080)	(0.054)	(0.007)	(0.050)	(0.000)	(0.050)				
S2M3	0.280^{*}	0.264^{*}	0.121	0.260	0.158	0.261	1	-0.1	9	-10.0
	(4.454)	(3.874)	(0.238)	(3.630)	(0.497)	(3.780)				
S2M4	0.420^{**}	0.401^{**}	0.238	0.398^{**}	0.282	0.398^{**}	1	-0.1	9	-10.0
	(10.327)	(9.277)	(0.747)	(8.807)	(1.323)	(9.102)				
S2M5	0.426^{*}	0.410^{*}	0.332	0.409^{*}	0.352	0.408^{*}	1	-0.1	9	-10.0
	(5.777)	(5.401)	(2.589)	(5.341)	(3.144)	(5.356)				
S3M1	-0.426	-0.402	-0.192	-0.398	-0.249	-0.398	1	-0.1	9	-10.0
	(3.799)	(3.213)	(0.237)	(3.031)	(0.483)	(3.129)				
S3M2	0.037	0.036	0.064	0.036	0.036	0.053	6	-2.0	1	-0.1
	(0.068)	(0.063)	(0.168)	(0.063)	(0.061)	(0.119)				
S3M3	0.242	0.236	0.162	0.234	0.181	0.234	1	-0.1	9	-10.0
	(3.586)	(3.296)	(1.013)	(3.233)	(1.378)	(3.250)				
S3M4	0.266^{*}	0.246^{*}	0.059	0.242	0.109	0.243^{*}	1	-0.1	9	-10.0
	(5.187)	(4.241)	(0.035)	(3.723)	(0.158)	(4.082)				
S3M5	0.337^{*}	0.324^{*}	0.214	0.322^{*}	0.244	0.322^{*}	1	-0.1	9	-10.0
	(4.566)	(4.217)	(0.881)	(4.123)	(1.341)	(4.167)				
S4M1	-0.499^{*}	-0.470	-0.120	-0.464	-0.218	-0.465	1	-0.1	9	-10.0
	(4.597)	(3.661)	(0.038)	(3.246)	(0.159)	(3.514)				
S4M2	0.071	0.075	0.079	0.074	0.074	0.077	1	-0.8	1	-0.1
	(0.266)	(0.283)	(0.323)	(0.280)	(0.280)	(0.303)				
S4M3	0.259^{*}	0.255^{*}	0.193	0.253^{*}	0.208	0.254^{*}	1	-0.1	9	-10.0
	(4.838)	(4.539)	(1.773)	(4.439)	(2.226)	(4.487)				
S4M4	0.270^{**}	0.262^{**}	0.172	0.260^{**}	0.195	0.260^{**}	1	-0.1	9	-10.0
	(7.863)	(7.193)	(1.042)	(6.710)	(1.600)	(7.067)				
S4M5	0.255	0.240	0.160	0.238	0.180	0.238	1	-0.1	9	-10.0
	(3.093)	(2.751)	(0.785)	(2.695)	(1.104)	(2.708)				
S5M1	-0.359	-0.331	-0.046	-0.326	-0.126	-0.327	1	-0.1	9	-10.0
	(2.464)	(1.923)	(0.008)	(1.746)	(0.075)	(1.845)				
S5M2	0.103	0.122	0.291	0.125	0.125	0.178	9	-10.0	5	-0.4
	(0.539)	(0.705)	(0.910)	(0.702)	(0.730)	(1.004)				
S5M3	-0.032	-0.032	-0.056	-0.032	-0.032	-0.049	9	-10.0	1	-0.1
	(0.130)	(0.122)	(0.345)	(0.129)	(0.125)	(0.275)				
S5M4	0.146	0.147	0.119	0.146	0.128	0.147	1	-0.1	9	-10.0
	(3.028)	(3.037)	(1.473)	(2.987)	(1.770)	(3.023)				
S5M5	0.158	0.156	0.115	0.155	0.128	0.156	1	-0.1	9	-10.0
	(1.589)	(1.526)	(0.670)	(1.523)	(0.888)	(1.521)				

Note: The entries in parentheses are the zero-alpha test statistics associated with the $\hat{\alpha}_i$'s, in which $\hat{\alpha}_i$ follows (24) with x_{it} denoting an SM portfolio, and the zero-alpha test statistic follows (36) with $z_t = 1$, $N = 1$ and $r = 1$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The (a, γ) of GR-min and that of GR-max are, respectively, reported in the blocks "min" and "max." The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(1)$ are, respectively, 3.841 and 6.635. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 4: The $\hat{\beta}_i$'s for the SB and SM portfolios: 1981-2014

portfolio	MV	GR				
		AS	FH	BCCY	min	max
S1B1	1.357	1.380	1.442	1.381	1.383	1.428
S1B2	1.153	1.180	1.316	1.183	1.281	1.184
S1B3	1.005	1.032	1.209	1.035	1.162	1.036
S1B4	0.927	0.962	1.204	0.967	1.140	0.967
S1B5	0.980	1.026	1.226	1.030	1.179	1.032
S2B1	1.350	1.356	1.400	1.357	1.357	1.388
S2B2	1.108	1.132	1.324	1.136	1.202	1.136
S2B3	0.987	1.009	1.179	1.012	1.134	1.012
S2B4	0.950	0.982	1.121	0.985	1.090	0.986
S2B5	1.048	1.077	1.228	1.079	1.188	1.080
S3B1	1.286	1.292	1.292	1.292	1.293	1.294
S3B2	1.087	1.100	1.239	1.103	1.202	1.102
S3B3	0.960	0.972	1.061	0.974	1.039	0.974
S3B4	0.926	0.933	1.001	0.935	0.986	0.935
S3B5	0.949	0.972	1.125	0.975	1.086	0.975
S4B1	1.210	1.203	1.158	1.202	1.201	1.170
S4B2	1.057	1.072	1.220	1.075	1.156	1.074
S4B3	1.029	1.044	1.097	1.046	1.089	1.046
S4B4	0.917	0.914	0.851	0.913	0.913	0.885
S4B5	0.960	0.969	1.040	0.970	1.020	0.970
S5B1	0.999	0.989	0.964	0.988	0.987	0.968
S5B2	0.938	0.939	0.980	0.940	0.968	0.940
S5B3	0.899	0.903	0.945	0.904	0.934	0.903
S5B4	0.806	0.799	0.708	0.798	0.798	0.737
S5B5	0.886	0.886	0.863	0.886	0.886	0.871
S1M1	1.351	1.356	1.274	1.355	1.302	1.356
S1M2	0.977	1.010	1.141	1.013	1.014	1.113
S1M3	0.903	0.944	1.173	0.949	1.116	0.950
S1M4	0.908	0.950	1.246	0.956	1.168	0.956
S1M5	1.127	1.168	1.387	1.172	1.330	1.173
S2M1	1.472	1.459	1.258	1.456	1.316	1.457
S2M2	1.086	1.099	1.182	1.101	1.164	1.101
S2M3	0.980	1.006	1.237	1.011	1.177	1.011
S2M4	0.981	1.013	1.276	1.017	1.204	1.017
S2M5	1.233	1.259	1.384	1.261	1.352	1.262
S3M1	1.397	1.357	1.018	1.351	1.110	1.351
S3M2	1.077	1.078	1.033	1.078	1.079	1.051
S3M3	0.969	0.979	1.099	0.983	1.069	0.982
S3M4	0.953	0.985	1.288	0.991	1.206	0.990
S3M5	1.196	1.218	1.395	1.221	1.346	1.221
S4M1	1.388	1.341	0.774	1.330	0.933	1.332
S4M2	1.094	1.089	1.082	1.089	1.089	1.085
S4M3	0.961	0.967	1.067	0.970	1.042	0.969
S4M4	0.934	0.947	1.093	0.950	1.055	0.949
S4M5	1.106	1.130	1.259	1.133	1.227	1.133
S5M1	1.256	1.211	0.749	1.202	0.879	1.203
S5M2	0.927	0.897	0.623	0.892	0.892	0.807
S5M3	0.883	0.882	0.921	0.883	0.883	0.911
S5M4	0.854	0.852	0.897	0.853	0.884	0.853
S5M5	1.006	1.009	1.075	1.010	1.054	1.010

Note: $\hat{\beta}_i$ follows (23) with x_{it} denoting an SB portfolio or an SM portfolio. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 5: The $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s in the case where the MV-CAPM is rejected but the FH-CAPM is accepted: 1981-2014

$\hat{\alpha}_i$		GR				
portfolio	MV	AS	FH	BCCY	min	max
S1B4	0.444	0.422	0.272	0.419	0.312	0.419
S1B5	0.484	0.456	0.332	0.453	0.361	0.452
S2B3	0.382	0.368	0.263	0.366	0.290	0.365
S2B4	0.372	0.352	0.266	0.350	0.285	0.350
S1M4	0.459	0.433	0.250	0.430	0.298	0.429
S1M5	0.608	0.583	0.447	0.580	0.482	0.580
S2M1	-0.686	-0.678	-0.553	-0.675	-0.589	-0.676
S2M3	0.280	0.264	0.121	0.260	0.158	0.261
S2M4	0.420	0.401	0.238	0.398	0.282	0.398
S2M5	0.426	0.410	0.332	0.409	0.352	0.408
S3M4	0.266	0.246	0.059	0.242	0.109	0.243
S3M5	0.337	0.324	0.214	0.322	0.244	0.322
S4M1	-0.499	-0.470	-0.120	-0.464	-0.218	-0.465
S4M2	0.071	0.075	0.079	0.074	0.074	0.077
S4M3	0.259	0.255	0.193	0.253	0.208	0.254
S4M4	0.270	0.262	0.172	0.260	0.195	0.260
$\hat{\beta}_i$		GR				
portfolio	MV	AS	FH	BCCY	min	max
S1B4	0.927	0.962	1.204	0.967	1.140	0.967
S1B5	0.980	1.026	1.226	1.030	1.179	1.032
S2B3	0.987	1.009	1.179	1.012	1.134	1.012
S2B4	0.950	0.982	1.121	0.985	1.090	0.986
S2B5	1.048	1.077	1.228	1.079	1.188	1.080
S1M4	0.908	0.950	1.246	0.956	1.168	0.956
S1M5	1.127	1.168	1.387	1.172	1.330	1.173
S2M1	1.472	1.459	1.258	1.456	1.316	1.457
S2M3	0.980	1.006	1.237	1.011	1.177	1.011
S2M4	0.981	1.013	1.276	1.017	1.204	1.017
S2M5	1.233	1.259	1.384	1.261	1.352	1.262
S3M4	0.953	0.985	1.288	0.991	1.206	0.990
S3M5	1.196	1.218	1.395	1.221	1.346	1.221
S4M1	1.388	1.341	0.774	1.330	0.933	1.332
S4M3	0.961	0.967	1.067	0.970	1.042	0.969
S4M4	0.934	0.947	1.093	0.950	1.055	0.949

Note: The MV-CAPM is rejected but the FH-CAPM is accepted for S1B4, S1B5, S2B3, S2B4, S2B5, S1M4, S1M5, S2M1, S2M3, S2M4, S2M5, S3M4, S3M5, S4M1, S4M3 and S4M4. The $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s shown in this table are the same as those shown in Tables 2, 3 and 4 for these portfolios. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$.

Table 6: The omitted variable test statistics for various CAPMs: 1981-2014

		GR				
portfolio	MV	AS	FH	BCCY	min	max
S1B1	18.022**	17.967**	17.801**	17.967**	17.961**	17.841**
S1B2	38.028**	37.740**	34.519**	37.749**	35.353**	37.714**
S1B3	73.690**	74.065**	59.999**	75.517**	64.204**	74.134**
S1B4	74.073**	74.478**	65.928**	75.807**	69.491**	74.563**
S1B5	85.537**	86.279**	85.397**	87.392**	87.464**	86.398**
S2B1	19.541**	19.561**	18.707**	19.525**	19.556**	18.906**
S2B2	58.783**	58.093**	50.150**	58.524**	56.194**	57.989**
S2B3	96.194**	95.339**	86.044**	95.011**	89.548**	95.190**
S2B4	97.595**	96.917**	92.012**	97.679**	94.652**	96.827**
S2B5	90.099**	89.551**	89.012**	89.023**	89.987**	89.473**
S3B1	21.143**	20.987**	20.623**	20.883**	20.967**	20.738**
S3B2	69.117**	68.513**	56.447**	67.348**	58.871**	68.337**
S3B3	64.481**	63.989**	59.558**	63.786**	60.862**	63.905**
S3B4	67.232**	67.299**	65.014**	67.570**	66.064**	67.298**
S3B5	67.327**	68.635**	65.760**	70.646**	68.575**	68.852**
S4B1	20.709**	20.878**	21.033**	21.088**	20.944**	21.298**
S4B2	22.066**	21.931**	18.839**	22.067**	20.657**	21.902**
S4B3	32.835**	33.309**	30.292**	33.849**	31.369**	33.339**
S4B4	47.471**	47.456**	44.401**	47.101**	47.403**	44.931**
S4B5	43.872**	44.771**	46.543**	45.652**	47.638**	44.895**
S5B1	112.201**	112.968**	115.083**	114.622**	113.126**	115.606**
S5B2	10.016**	10.005**	10.111**	10.003**	10.026**	10.007**
S5B3	18.827**	18.916**	17.068**	19.054**	17.729**	18.926**
S5B4	28.201**	27.493**	16.265**	26.777**	27.342**	17.675**
S5B5	42.957**	42.938**	35.432**	42.702**	42.836**	37.011**
S1M1	55.871**	55.674**	54.097**	55.703**	54.495**	55.665**
S1M2	87.344**	85.418**	74.725**	82.477**	85.083**	75.444**
S1M3	95.937**	93.553**	78.117**	90.329**	81.770**	93.090**
S1M4	72.810**	71.760**	55.836**	72.443**	60.353**	71.603**
S1M5	41.125**	40.912**	36.203**	40.961**	37.508**	40.872**
S2M1	61.563**	61.687**	46.405**	61.658**	48.554**	61.773**
S2M2	79.635**	77.409**	66.255**	75.549**	67.039**	77.033**
S2M3	93.877**	91.551**	70.948**	86.841**	75.774**	90.969**
S2M4	91.655**	90.809**	73.510**	91.564**	80.114**	90.667**
S2M5	39.741**	39.654**	36.516**	39.826**	37.493**	39.637**
S3M1	22.359**	22.756**	18.317**	22.105**	18.946**	22.819**
S3M2	39.484**	39.714**	37.896**	39.707**	39.718**	38.218**
S3M3	56.432**	56.077**	54.337**	55.746**	55.201**	56.018**
S3M4	63.239**	62.428**	44.427**	62.127**	50.469**	62.238**
S3M5	23.977**	23.802**	21.371**	23.840**	22.123**	23.774**
S4M1	7.433*	7.181*	5.384	7.166*	5.771	7.142*
S4M2	24.385**	24.734**	23.820**	24.711**	24.821**	24.156**
S4M3	30.142**	30.461**	25.778**	31.115**	27.535**	30.523**
S4M4	21.112**	21.369**	14.834**	21.912**	16.204**	21.412**
S4M5	8.550*	8.432*	7.831*	8.409*	7.970*	8.416*
S5M1	2.863	2.407	0.091	2.156	0.163	2.325
S5M2	9.358**	8.402*	0.966	7.710*	8.232*	4.504
S5M3	9.758**	9.744**	9.554**	9.766**	9.756**	9.607**
S5M4	14.698**	14.567**	13.933**	14.585**	13.958**	14.562**
S5M5	6.357*	6.282*	4.090	6.232*	4.462	6.258*

Note: The omitted variable test statistic follows (36) with z_t denoting a vector composed of the Fama-French size and book-to-market factors, $N = 1$ and $r = 2$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(2)$ are, respectively, 5.991 and 9.210.

Table 7: The $\hat{\alpha}_i$'s and the zero-alpha test statistics for the SB portfolios: 1961-2014

		GR					min		max	
portfolio	MV	AS	FH	BCCY	min	max	a	γ	a	γ
S1B1	−0.424*	−0.434*	−0.447*	−0.434*	−0.434*	−0.445*	8	-10.0	4	-0.1
	(4.340)	(4.579)	(5.043)	(4.613)	(4.600)	(4.928)				
S1B2	0.161	0.152	0.122	0.151	0.132	0.151	4	-0.1	8	-10.0
	(0.932)	(0.832)	(0.501)	(0.835)	(0.627)	(0.823)				
S1B3	0.251	0.244	0.205	0.243	0.219	0.243	4	-0.1	4	-10.0
	(3.128)	(2.941)	(1.630)	(2.953)	(2.274)	(2.921)				
S1B4	0.476**	0.467**	0.406*	0.466**	0.429**	0.465**	4	-0.1	4	-10.0
	(11.182)	(10.744)	(4.201)	(10.780)	(7.466)	(10.691)				
S1B5	0.577**	0.565**	0.514**	0.564**	0.531**	0.564**	4	-0.1	4	-10.0
	(12.797)	(12.353)	(8.163)	(12.508)	(10.596)	(12.307)				
S2B1	−0.262	−0.267	−0.272	−0.267	−0.267	−0.271	8	-10.0	4	-0.1
	(3.181)	(3.300)	(3.550)	(3.309)	(3.310)	(3.478)				
S2B2	0.113	0.106	0.056	0.105	0.074	0.105	4	-0.1	8	-10.0
	(0.905)	(0.796)	(0.112)	(0.788)	(0.318)	(0.782)				
S2B3	0.364**	0.357**	0.316*	0.356**	0.331**	0.356**	4	-0.1	4	-10.0
	(10.687)	(10.227)	(4.952)	(10.250)	(7.719)	(10.177)				
S2B4	0.418**	0.410**	0.375**	0.409**	0.386**	0.409**	4	-0.1	8	-10.0
	(13.262)	(12.687)	(8.045)	(12.653)	(10.379)	(12.624)				
S2B5	0.445**	0.438**	0.400*	0.437**	0.414**	0.437**	4	-0.1	8	-10.0
	(9.540)	(9.141)	(5.703)	(9.136)	(7.625)	(9.105)				
S3B1	−0.191	−0.194	−0.186	−0.194	−0.190	−0.194	4	-0.1	9	-2.0
	(2.718)	(2.789)	(2.434)	(2.788)	(2.657)	(2.800)				
S3B2	0.197*	0.191*	0.152	0.191*	0.166	0.191*	4	-0.1	8	-10.0
	(4.435)	(4.200)	(1.373)	(4.198)	(2.614)	(4.169)				
S3B3	0.242*	0.237*	0.218*	0.237*	0.224*	0.236*	4	-0.1	4	-10.0
	(5.982)	(5.697)	(4.365)	(5.694)	(5.036)	(5.669)				
S3B4	0.378**	0.377**	0.362**	0.376**	0.367**	0.377**	4	-0.1	8	-10.0
	(13.314)	(13.068)	(12.171)	(13.106)	(12.745)	(13.046)				
S3B5	0.530**	0.525**	0.491**	0.525**	0.503**	0.525**	4	-0.1	8	-10.0
	(15.735)	(15.230)	(9.838)	(15.126)	(12.652)	(15.171)				
S4B1	−0.041	−0.039	−0.020	−0.039	−0.028	−0.039	4	-0.1	4	-10.0
	(0.185)	(0.168)	(0.036)	(0.165)	(0.078)	(0.165)				
S4B2	0.024	0.020	−0.024	0.019	0.000	0.019	4	-0.2	8	-10.0
	(0.078)	(0.051)	(0.032)	(0.047)	(0.000)	(0.048)				
S4B3	0.201*	0.197*	0.175	0.197*	0.181	0.197*	4	-0.1	8	-10.0
	(4.428)	(4.141)	(2.984)	(4.110)	(3.371)	(4.104)				
S4B4	0.352**	0.354**	0.382**	0.355**	0.371**	0.360**	4	-0.1	7	-0.5
	(12.540)	(12.391)	(8.815)	(12.365)	(11.454)	(12.441)				
S4B5	0.331**	0.332**	0.326**	0.332**	0.332**	0.329**	9	-2.0	4	-0.1
	(6.942)	(6.858)	(6.928)	(6.863)	(6.842)	(6.933)				
S5B1	−0.044	−0.040	−0.031	−0.040	−0.033	−0.039	4	-0.1	4	-10.0
	(0.415)	(0.340)	(0.202)	(0.336)	(0.231)	(0.333)				
S5B2	0.060	0.060	0.046	0.060	0.052	0.060	4	-0.1	8	-10.0
	(0.874)	(0.862)	(0.362)	(0.854)	(0.574)	(0.858)				
S5B3	0.065	0.065	0.044	0.065	0.052	0.065	4	-0.1	8	-10.0
	(0.627)	(0.615)	(0.181)	(0.602)	(0.331)	(0.609)				
S5B4	0.143	0.147	0.181	0.148	0.148	0.161	4	-10.0	5	-0.2
	(2.188)	(2.276)	(1.838)	(2.281)	(2.286)	(2.508)				
S5B5	0.202	0.200	0.209	0.201	0.200	0.205	5	-6.0	4	-0.1
	(2.815)	(2.754)	(2.926)	(2.763)	(2.753)	(2.889)				

Note: The entries in parentheses are the zero-alpha test statistics associated with the $\hat{\alpha}_i$'s, in which $\hat{\alpha}_i$ follows (24) with x_{it} denoting an SB portfolio, and the zero-alpha test statistic follows (36) with $z_t = 1$, $N = 1$ and $r = 1$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The (a, γ) of GR-min and that of GR-max are, respectively, reported in the blocks "min" and "max." The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(1)$ are, respectively, 3.841 and 6.635. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 8: The $\hat{\alpha}_i$'s and the zero-alpha test statistics for the SM portfolios: 1961-2014

		GR					min		max	
portfolio	MV	AS	FH	BCCY	min	max	a	γ	a	γ
S1M1	-0.662^{**}	-0.663^{**}	-0.637^{**}	-0.662^{**}	-0.649^{**}	-0.663^{**}	4	-0.1	4	-10.0
	(11.373)	(11.154)	(7.781)	(11.136)	(9.494)	(11.131)				
S1M2	0.139	0.131	0.098	0.130	0.108	0.130	4	-0.1	4	-10.0
	(0.933)	(0.831)	(0.441)	(0.836)	(0.571)	(0.819)				
S1M3	0.437^{**}	0.426^{**}	0.364^{*}	0.425^{**}	0.385^{**}	0.425^{**}	4	-0.1	8	-10.0
	(10.364)	(9.918)	(4.076)	(10.054)	(7.047)	(9.866)				
S1M4	0.558^{**}	0.544^{**}	0.465^{*}	0.543^{**}	0.494^{**}	0.542^{**}	4	-0.1	8	-10.0
	(15.920)	(15.328)	(4.457)	(15.515)	(9.427)	(15.256)				
S1M5	0.751^{**}	0.735^{**}	0.678^{**}	0.735^{**}	0.697^{**}	0.734^{**}	4	-0.1	8	-10.0
	(17.299)	(16.771)	(10.560)	(16.916)	(14.267)	(16.715)				
S2M1	-0.598^{**}	-0.593^{**}	-0.533^{*}	-0.592^{**}	-0.558^{**}	-0.592^{**}	4	-0.1	4	-10.0
	(12.693)	(12.044)	(4.596)	(11.898)	(7.797)	(11.965)				
S2M2	0.077	0.074	0.052	0.073	0.058	0.074	4	-0.1	8	-10.0
	(0.400)	(0.365)	(0.171)	(0.362)	(0.228)	(0.359)				
S2M3	0.295^{**}	0.289^{**}	0.222	0.287^{**}	0.247	0.287^{**}	4	-0.1	4	-10.0
	(7.753)	(7.332)	(1.377)	(7.306)	(3.446)	(7.273)				
S2M4	0.478^{**}	0.468^{**}	0.396	0.467^{**}	0.423^{**}	0.467^{**}	4	-0.1	4	-10.0
	(20.004)	(19.027)	(3.750)	(18.958)	(9.224)	(18.896)				
S2M5	0.571^{**}	0.560^{**}	0.526^{**}	0.560^{**}	0.537^{**}	0.559^{**}	4	-0.1	8	-10.0
	(16.634)	(16.010)	(11.329)	(15.960)	(13.845)	(15.948)				
S3M1	-0.435^{**}	-0.423^{**}	-0.316	-0.422^{*}	-0.357	-0.421^{**}	4	-0.1	4	-10.0
	(7.419)	(6.758)	(0.934)	(6.545)	(2.503)	(6.665)				
S3M2	0.048	0.050	0.067	0.050	0.050	0.058	6	-10.0	4	-0.1
	(0.197)	(0.206)	(0.270)	(0.205)	(0.206)	(0.252)				
S3M3	0.194^{*}	0.191^{*}	0.162	0.190^{*}	0.172	0.190^{*}	4	-0.1	8	-10.0
	(4.103)	(3.887)	(2.044)	(3.881)	(2.905)	(3.859)				
S3M4	0.254^{**}	0.244^{**}	0.155	0.243^{**}	0.190	0.243^{**}	4	-0.1	4	-10.0
	(8.168)	(7.400)	(0.409)	(7.182)	(1.622)	(7.285)				
S3M5	0.559^{**}	0.549^{**}	0.492^{**}	0.548^{**}	0.513^{**}	0.548^{**}	4	-0.1	8	-10.0
	(19.515)	(18.789)	(6.902)	(18.661)	(12.454)	(18.701)				
S4M1	-0.496^{**}	-0.481^{**}	-0.308	-0.478^{**}	-0.376	-0.478^{**}	4	-0.1	4	-10.0
	(9.304)	(8.166)	(0.382)	(7.740)	(1.527)	(7.996)				
S4M2	0.011	0.014	0.020	0.014	0.015	0.018	4	-10.0	4	-0.1
	(0.011)	(0.019)	(0.037)	(0.018)	(0.019)	(0.029)				
S4M3	0.150	0.149	0.126	0.149	0.135	0.149	4	-0.1	8	-10.0
	(3.072)	(2.999)	(1.567)	(2.986)	(2.242)	(2.985)				
S4M4	0.292^{**}	0.289^{**}	0.254^{*}	0.288^{**}	0.267^{**}	0.288^{**}	4	-0.1	4	-10.0
	(15.487)	(15.007)	(4.846)	(14.860)	(9.202)	(14.933)				
S4M5	0.445^{**}	0.436^{**}	0.398^{**}	0.436^{**}	0.411^{**}	0.435^{**}	4	-0.1	4	-10.0
	(16.452)	(15.710)	(8.415)	(15.594)	(12.070)	(15.629)				
S5M1	-0.469^{**}	-0.453^{**}	-0.305	-0.451^{**}	-0.363	-0.451^{**}	4	-0.1	4	-10.0
	(8.764)	(7.712)	(0.506)	(7.413)	(1.823)	(7.567)				
S5M2	-0.037	-0.025	0.070	-0.024	0.000	-0.023	4	-0.4	4	-10.0
	(0.118)	(0.053)	(0.067)	(0.047)	(0.000)	(0.046)				
S5M3	-0.058	-0.058	-0.065	-0.058	-0.058	-0.062	8	-10.0	4	-0.1
	(0.683)	(0.673)	(0.807)	(0.676)	(0.674)	(0.763)				
S5M4	0.099	0.101	0.095	0.101	0.098	0.101	4	-0.1	3	-5.0
	(2.347)	(2.381)	(1.853)	(2.383)	(2.135)	(2.382)				
S5M5	0.272^{**}	0.269^{**}	0.251^{*}	0.269^{**}	0.259^{*}	0.269^{**}	4	-0.1	8	-10.0
	(6.895)	(6.696)	(4.686)	(6.692)	(5.870)	(6.680)				

Note: The entries in parentheses are the zero-alpha test statistics associated with the $\hat{\alpha}_i$'s, in which $\hat{\alpha}_i$ follows (24) with x_{it} denoting an SM portfolio, and the zero-alpha test statistic follows (36) with $z_t = 1$, $N = 1$ and $r = 1$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The (a, γ) of GR-min and that of GR-max are, respectively, reported in the blocks "min" and "max." The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(1)$ are, respectively, 3.841 and 6.635. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 9: The $\hat{\beta}_i$'s for the SB and SM portfolios: 1981-2014

		GR				
portfolio	MV	AS	FH	BCCY	min	max
S1B1	1.419	1.438	1.465	1.438	1.440	1.460
S1B2	1.233	1.250	1.308	1.251	1.289	1.252
S1B3	1.100	1.115	1.190	1.116	1.163	1.117
S1B4	1.023	1.040	1.158	1.042	1.114	1.043
S1B5	1.080	1.103	1.202	1.105	1.170	1.106
S2B1	1.399	1.408	1.417	1.408	1.408	1.416
S2B2	1.172	1.186	1.284	1.187	1.247	1.188
S2B3	1.059	1.073	1.152	1.074	1.123	1.074
S2B4	1.020	1.035	1.103	1.036	1.082	1.037
S2B5	1.118	1.132	1.205	1.133	1.178	1.133
S3B1	1.329	1.334	1.318	1.333	1.327	1.334
S3B2	1.117	1.127	1.205	1.129	1.176	1.129
S3B3	1.008	1.017	1.055	1.018	1.043	1.018
S3B4	0.968	0.970	0.997	0.970	0.988	0.970
S3B5	1.031	1.039	1.106	1.041	1.082	1.041
S4B1	1.223	1.219	1.183	1.219	1.197	1.219
S4B2	1.081	1.089	1.175	1.091	1.129	1.091
S4B3	1.028	1.036	1.079	1.037	1.067	1.037
S4B4	0.955	0.950	0.896	0.949	0.919	0.939
S4B5	1.042	1.040	1.050	1.040	1.040	1.045
S5B1	0.991	0.984	0.966	0.983	0.970	0.983
S5B2	0.932	0.932	0.959	0.933	0.947	0.932
S5B3	0.871	0.871	0.913	0.872	0.897	0.872
S5B4	0.828	0.819	0.754	0.819	0.818	0.793
S5B5	0.890	0.892	0.876	0.892	0.892	0.883
S1M1	1.372	1.372	1.322	1.371	1.346	1.372
S1M2	1.053	1.067	1.131	1.069	1.112	1.069
S1M3	0.990	1.012	1.131	1.014	1.090	1.014
S1M4	1.008	1.035	1.187	1.037	1.132	1.038
S1M5	1.213	1.243	1.355	1.245	1.317	1.247
S2M1	1.456	1.447	1.330	1.446	1.378	1.445
S2M2	1.122	1.128	1.171	1.129	1.158	1.129
S2M3	1.033	1.046	1.174	1.048	1.127	1.048
S2M4	1.055	1.074	1.213	1.076	1.160	1.077
S2M5	1.280	1.302	1.369	1.303	1.348	1.305
S3M1	1.368	1.344	1.136	1.341	1.216	1.341
S3M2	1.102	1.099	1.065	1.099	1.098	1.082
S3M3	1.015	1.022	1.078	1.023	1.059	1.023
S3M4	1.005	1.024	1.196	1.027	1.130	1.027
S3M5	1.215	1.234	1.345	1.235	1.304	1.236
S4M1	1.333	1.303	0.967	1.298	1.100	1.298
S4M2	1.108	1.101	1.090	1.101	1.101	1.094
S4M3	1.000	1.001	1.046	1.002	1.029	1.001
S4M4	0.994	0.999	1.067	1.001	1.042	1.000
S4M5	1.146	1.164	1.237	1.165	1.213	1.166
S5M1	1.236	1.206	0.918	1.202	1.032	1.202
S5M2	0.941	0.919	0.735	0.916	0.870	0.916
S5M3	0.904	0.904	0.918	0.904	0.904	0.913
S5M4	0.887	0.885	0.895	0.885	0.890	0.884
S5M5	1.021	1.026	1.060	1.026	1.045	1.026

Note: $\hat{\beta}_i$ follows (23) with x_{it} denoting an SB portfolio or an SM portfolio. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. Recall that the AS-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to -\infty$, the FH-CAPM is a limiting case of the GR-CAPM where $a = 1$ and $\gamma \to 0$, and the BCCY-CAPM is a subclass of the GR-CAPM where $a = 1 - \gamma$.

Table 10: The $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s in the case where the MV-CAPM is rejected but the FH-CAPM is accepted: 1961-2014

$\hat{\alpha}_i$		GR				
portfolio	MV	AS	FH	BCCY	min	max
S3B2	0.197	0.191	0.152	0.191	0.166	0.191
S4B3	0.201	0.197	0.175	0.197	0.181	0.197
S2M3	0.295	0.289	0.222	0.287	0.247	0.287
S2M4	0.478	0.468	0.396	0.467	0.423	0.467
S3M1	-0.435	-0.423	-0.316	-0.422	-0.357	-0.421
S3M3	0.194	0.191	0.162	0.190	0.172	0.190
S3M4	0.254	0.244	0.155	0.243	0.190	0.243
S4M1	-0.496	-0.481	-0.308	-0.478	-0.376	-0.478
S5M1	-0.469	-0.453	-0.305	-0.451	-0.363	-0.451

$\hat{\beta}_i$		GR				
portfolio	MV	AS	FH	BCCY	min	max
S3B2	1.117	1.127	1.205	1.129	1.176	1.129
S4B3	1.028	1.036	1.079	1.037	1.067	1.037
S2M3	1.033	1.046	1.174	1.048	1.127	1.048
S2M4	1.055	1.074	1.213	1.076	1.160	1.077
S3M1	1.368	1.344	1.136	1.341	1.216	1.341
S3M3	1.015	1.022	1.078	1.023	1.059	1.023
S3M4	1.005	1.024	1.196	1.027	1.130	1.027
S4M1	1.333	1.303	0.967	1.298	1.100	1.298
S5M1	1.236	1.206	0.918	1.202	1.032	1.202

Note: The MV-CAPM is rejected but the FH-CAPM is accepted for S3B2, S4B3, S2M3, S2M4, S3M1, S3M3, S3M4, S4M1 and S5M1. The $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s shown in this table are the same as those shown in Tables 7, 8 and 9 for these portfolios. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$.

Table 11: The omitted variable test statistics for various CAPMs: 1961-2014

		GR				
portfolio	MV	AS	FH	BCCY	min	max
S1B1	50.343**	50.256**	50.066**	50.323**	50.117**	50.247**
S1B2	70.708**	70.469**	69.353**	70.461**	69.774**	70.443**
S1B3	96.352**	96.111**	93.965**	96.069**	94.894**	96.082**
S1B4	110.103**	109.551**	102.567**	109.395**	105.174**	109.483**
S1B5	113.725**	112.473**	105.684**	112.141**	107.790**	112.358**
S2B1	41.603**	41.557**	41.332**	41.545**	41.428**	41.553**
S2B2	97.267**	96.250**	76.885**	95.440**	84.177**	96.090**
S2B3	135.138**	133.349**	103.741**	131.479**	112.965**	133.100**
S2B4	126.744**	124.753**	112.008**	123.476**	115.653**	124.524**
S2B5	122.729**	121.174**	108.898**	120.288**	112.067**	121.010**
S3B1	30.021**	29.982**	29.674**	29.976**	29.984**	30.211**
S3B2	120.619**	118.980**	69.664**	116.257**	82.050**	118.650**
S3B3	93.932**	92.992**	82.476**	91.964**	86.793**	92.862**
S3B4	95.940**	95.344**	93.134**	95.247**	93.902**	95.278**
S3B5	95.836**	95.767**	96.077**	95.761**	95.771**	96.097**
S4B1	31.820**	31.940**	30.012**	32.074**	31.964**	32.741**
S4B2	35.328**	34.499**	27.031**	33.893**	28.748**	34.360**
S4B3	46.647**	46.770**	45.694**	46.830**	46.244**	46.773**
S4B4	70.311**	70.435**	71.356**	70.407**	70.444**	71.065**
S4B5	76.697**	77.274**	77.796**	77.243**	77.314**	77.753**
S5B1	140.942**	141.121**	141.840**	141.602**	141.148**	141.778**
S5B2	15.908**	15.887**	12.858**	15.871**	15.063**	15.918**
S5B3	25.135**	25.074**	18.979**	25.159**	23.590**	25.332**
S5B4	46.556**	46.208**	32.224**	45.664**	38.241**	46.116**
S5B5	72.702**	72.202**	65.123**	72.119**	68.004**	72.115**
S1M1	91.670**	91.013**	72.415**	91.012**	84.305**	90.952**
S1M2	121.617**	117.880**	91.466**	117.302**	102.195**	117.511**
S1M3	132.357**	128.702**	91.778**	125.345**	97.480**	128.225**
S1M4	113.349**	112.007**	94.069**	110.851**	99.040**	111.805**
S1M5	82.559**	81.555**	74.403**	81.319**	77.458**	81.440**
S2M1	101.927**	102.133**	52.511**	101.368**	71.856**	102.151**
S2M2	115.927**	112.930**	88.657**	112.270**	98.109**	112.574**
S2M3	137.757**	134.270**	83.706**	127.776**	88.557**	133.535**
S2M4	136.255**	134.292**	100.673**	131.432**	106.302**	133.923**
S2M5	76.866**	75.634**	70.143**	75.394**	72.204**	75.505**
S3M1	45.668**	46.850**	27.521**	45.212**	32.438**	47.188**
S3M2	62.767**	62.432**	60.779**	62.432**	61.836**	62.412**
S3M3	92.078**	90.044**	75.176**	89.079**	79.070**	89.786**
S3M4	114.044**	111.845**	73.157**	106.627**	80.963**	111.259**
S3M5	47.554**	46.704**	37.408**	46.387**	41.191**	46.595**
S4M1	15.997**	16.260**	11.053**	15.770**	12.767**	16.372**
S4M2	36.754**	36.777**	36.521**	36.777**	36.641**	36.776**
S4M3	46.295**	46.402**	44.885**	46.410**	45.812**	46.402**
S4M4	35.764**	35.753**	33.956**	35.758**	34.943**	35.744**
S4M5	17.686**	17.235**	13.992**	17.107**	15.359**	17.184**
S5M1	2.830	2.344	0.010	2.162	0.120	2.270
S5M2	9.666**	8.762*	0.533	8.324*	1.861	8.627*
S5M3	11.134**	11.140**	10.648**	11.135**	11.038**	11.157**
S5M4	20.441**	20.244**	18.328**	20.269**	19.632**	20.225**
S5M5	8.214*	8.013*	6.425*	8.013*	6.998*	7.989*

Note: The omitted variable test statistic follows (36) with z_t denoting a vector composed of the Fama-French size and book-to-market factors, $N = 1$ and $r = 2$ for all t's. GR-min and GR-max are, respectively, the model with the minimum zero-alpha test statistic and that with the maximum zero-alpha test statistic in the class of GR-CAPMs with $a = 1, 2, \ldots, 10$ and $\gamma = -10, -9, \ldots, -1, -0.9, -0.8, \ldots, -0.1$. The symbols "**" and "*" indicate that the associated test statistics are, respectively, significant at the 1% level and the 5% level. The 95% and 99% critical values of $\chi^2(2)$ are, respectively, 5.991 and 9.210.