The Effect of Risk Aversion and Loss Aversion on Equity-Linked Life Insurance with Surrender Guarantees[1]

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Abstract

This paper values equity-linked life insurance contracts with surrender guarantees for risk averse and loss averse policyholders in continuous time. With increasing risk aversion, policyholders surrender their insurances for higher values of the underlying equity fund, compared to an optimally stopped insurance contract, leading to substantial losses. Moreover, high discounting amplifies suboptimal surrender behavior. Loss averse policyholders display a different surrender behavior: Such policyholders surrender only policies for which the surrender benefit represents a large gain, while holding on to less successful contracts, so that the disposition effect increases the contract value relative to a contract stopped by a risk averse policyholder.

Keywords: Equity-linked life insurance, behavioral insurance, surrender, loss aversion
JEL Codes: G11, G22

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1 Introduction

Equity-linked life insurance products combine a classical term life insurance and a savings component, which invests the policyholder's premiums in equity. Equity-linked life insurance contracts form a large market in many developed countries, for example, the United States market has a volume of over $100 billion annual sales, c.f. Hardy (2003). Commonly, equity-linked life insurance products include a capital guarantee plus a minimum interest rate guarantee, and the option to surrender the contract.[1] This investment guarantee is risky for insurers because, unlike mortality risk, the equity risk cannot be diversified across cohorts as it is identical for a large number of insurance contracts (Hardy (2003)). In particular, in combination with the equity risk, the surrender of insurance policies is an important risk for insurers as identified by European Union regulators (Eling and Kochanski (2013)).

Empirically, the policyholder's surrender behavior depends on the economic situation of the policyholder in general, and the financial performance of the insurance contract, which depends on the financial markets through an underlying equity-fund. A cointegration analysis of the surrender rate of United States policyholders by Kuo, Tsai, and Chen (2003) links the surrender rate to both the unemployment rate and the interest rate. High unemployment increases surrenders, because policyholders cannot afford premiums and need access to their savings. In situations with high interest rates, the policyholder's surrender rate increases, because alternative investments perform better compared to the equity-linked life insurance contract. Eling and Kiesenbauer (2013) empirically assess drivers of surrender rates for german policyholders, for which they find both policyholders' and insurance contract characteristics to be significant drivers.

This paper analyzes the influence of policyholder's risk preferences on the valuation and surrender behavior of equity-linked life insurance contracts with surrender guarantees. Our contribution is twofold: Firstly, we provide a model of discounted expected utility for loss averse utility functions in continuous time with an embedded stopping problem, which we then apply to value equity-linked life insurance contracts with surrender guarantees for

[1] There are different technical ways of terminating a life insurance contract. One option is to hand the contract back to the insurance company and collect a surrender benefit, another is to keep the insurance that the already invested capital provides, but set all future premiums to zero. In this paper, we consider the first version. The second version is sometimes referred to as 'lapse of the contract'.

policyholders who's surrender behavior roots in the contract's expected utility. Then, we derive the implied exercise boundary, that is, the combination of the underlying fund's value and time to maturity, for which a policyholder surrenders his policy. Secondly, we show that policyholder's risk preferences have a strong impact on the surrender value and subsequently the insurance policy's value. Purely risk averse policyholders who are not loss averse surrender their insurance policy earlier and for higher fund values. This effect is strengthened by high discounting, representing a higher impatience of the policyholder. Furthermore, we show that policyholders who are loss averse have a drastically changed surrender behavior compared to purely risk averse policyholders. On the one hand, they surrender only insurance policies if the surrender benefit exceeds their reference point, which is a benchmark value the policyholder hopes to achieve. On the other hand, loss averse policyholders do not surrender their contracts if the surrender benefit is is either classified as a loss or not enough of a gain relative to the reference point. This behavior is known as the disposition effect and introduced by Shefrin and Statman (1985). The surrender behavior implied by the policyholder's risk preferences in turn influences the insurance policy's value: While purely risk averse policyholders may reduce their contract's value by up to twenty percent because of early termination, policyholders who are loss averse will achieve higher contract values, but also fall short of optimally stopped contracts.

Our representative policyholder in spirit of Albizzati and Geman (1994) assesses his insurance policy based on realized gains and losses. The policyholder considers his life insurance contract in isolation of his other investments as he frames narrowly through a mental account for his life insurance contract. This mental account consists of the value of the life insurance from the policyholder's perspective as it moves over time. Mental accounting is introduced by Thaler (1980, 1985) and applied to finance by Shefrin and Statman (1985), who explain how investors use isolated mental accounts for each stock investment they own. The policyholder considers his insurance value in his mental account over time, from the signing of the policy up to the maturity date. If the policyholder surrenders his insurance policy or the policy reaches maturity, he receives the respective benefit from the insurance company. Then, the policyholder closes his mental account 'life insurance' and evaluates the payoff he receives. The policyholder only draws utility from realizing payments as gains and losses, but not from observing the insurance contract's performance during the lifetime of the contract, in line with the model of Barberis (2012). The policyholder makes his surrender choice based on how his utility of the realized payment develops: If the utility of immediate surrender exceeds the expected utility for later times, he surrenders.

Our policyholder follows loss averse preferences given by Kahneman and Tversky (1979) to analyze the payoff he receives when he closes his mental account 'life insurance'. He considers his life insurance policy in comparison to a reference point, for example his initial investment or expectation of the contract's performance. This reference point splits the outcome of the equity-linked life insurance contract into gains and losses. Empirically, losses loom larger than gains. For example, Kahneman and Tversky (1979) estimate the pain of a loss to be 2.25 times larger than the joy over a gain of the same size. Furthermore, the preferences exhibit diminishing sensitivity of returns, which produce in combination with loss aversion the S-shape of the policyholder's utility function. As a special case, we also consider standard expected utility theory in form of the power utility function.

In economic terms, we endogenize the surrender behavior of the policyholder and explain the exogenously assumed increased surrender behavior of policyholders in the literature, e.g. by Li and Szimayer (2014) by loss averse preferences: In our model, the policyholder has a different surrender behavior compared to the optimal surrender behavior in terms of the option pricing literature because the policyholder evaluates his mental account *life insurance* instead of optimally stopping an American style option. Compared to the optimal surrender behavior, we show that the policyholder deviates from this surrender behavior in two ways: Firstly, the policyholder surrenders contracts earlier, if he is sufficiently risk-averse. The life insurance's embedded guarantee usually offers a lower return compared to the risk-free return. If the underlying fund of the equity-linked contract performs poorly, the policyholder surrenders earlier because he wants to secure the risk-free return for his premium, but the potential chance to receive gains from a recovery of the fund plays a small role. This effect is also present for the optimal stopping in the Black and Scholes (1973) sense, but it is more pronounced for risk-averse policyholders.

Furthermore, we show the importance of the reference point in combination with loss aversion to the policyholder's surrender behavior. The reference point controls an important part of the surrender behavior: By steering how large the difference between the reference point and the surrender benefit is, the loss averse preferences imply a strong disposition effect. If the reference point lies substantially above the surrender benefit, the policyholder displays the disposition effect by surrendering the contract. Delaying surrender is not attractive in this case, because further gains in the contract's value suffer heavily from a diminishing sensitivity of utility towards further gains, while a decrease in value causes a strong disutility. On the other hand, a policyholder with a high reference point has an incentive to not surrender his policy, because further gains will benefit from being close to the reference point through a high sensitivity of utility.

Finally, we quantify the loss of the policyholder due to the surrender following his risk preferences. In particular, we show that the policyholder loses up to a twenty percent of his insurance value because he surrenders sub-optimally. A policyholder who surrenders according to classical power utility preferences will suffer strongly from suboptimal surrenders. Such a policyholder will exit his insurance policy already if the underlying equity fund drops slightly in value because of the risk aversion. This is countered by the disposition effect for a loss averse policyholder, as this type of policyholder will stay with the contract even though it suffers temporary losses. This surrender behavior is overall value enhancing compared to a pure risk averse policyholder without loss aversion, as staying with the insurance policy longer than optimal decerases the contract's value less than early surrender. In total, both styles of risk preferences decrease the policyholder's contract value, but less for a loss averse policyholder.

This has also a substantial impact for insurers. In this paper, we show that a misspecification of the insurance policyholder's risk preferences leads to potentially large errors in the pricing of the insurance policy.

This paper contributes to several strands of literature. Firstly, a large insurance literature considers how to price and hedge equity-linked life insurance contracts based on the Black and Scholes (1973) option pricing theory. This literature considers the equity-linked life insurance contract in isolation of the policyholder's general economic situation. The embedded surrender guarantee of the insurance contract takes the form of an American option, which Bacinello (2005) prices in a Binomial model. Many other works price and hedge life insurance contracts with and without surrender guarantees in continuous time: Grosen and Jorgensen (1997) price early exercise interest guarantees and apply it to life insurance contracts in Grosen and Jorgensen (2000). Shen and Xu (2005) use a partial differential equation approach to value equity-linked life insurance contracts with surrender guarantees. All these previous works assume a policyholder who has unlimited access to the perfect financial market. In particular, the policyholder can follow any financial strategy and replicate any derivative. To this literature, we contribute the pricing of an insurance policy in continuous time with a loss averse utility by deriving a pricing partial differential equation and solving the subsequent optimal stopping problem. Because of the structural form of the utility function with loss aversion, we compute a viscosity solution for the Hamilton-Jacobi-Bellman equation implied by our optimal stopping problem.

Secondly, we contribute to the literature on including boundedly rational policyholders to the pricing and hedging of insurance contracts. Recently, Li and Szimayer (2014) and De Giovanni (2010) include bounded rationality into the pricing of the equity-linked life insurance, but do not endogenize this bounded rationality. Instead, they describe the policyholders surrender behavior by a doubly stochastic Poisson process. This Cox process produces financially suboptimal surrender behavior. These studies extend Albizzati and Geman (1994), who analyze the surrender guarantee from the perspective of a representative policyholder and derive the dependence of the surrender decision on stochastic interest rates. Ebert, Koos, and Schneider (2013) and Doskeland and Nordahl (2008) consider equity-linked life insurance and pension insurance contracts, respectively, in combination with loss averse preferences. These papers explain the demand for products with investment guarantees by cumulative prospect theory. However, expected utility with standard preferences alone cannot explain the demand for guarantees in life and pension insurance, neither can cumulative prospect theory for complex contracts. By extending cumulative prospect theory to a version with multiple reference points, Russ and Schelling (2015) explain the policyholders demand for insurance policies with complex guarantees. However, the above papers either do not analyze surrender guarantees, or do not explain the policyholder's decision making endogenously.

The remainder of this paper is organized as follows: Firstly, we introduce the model of the financial market underlying the asset linked to the insurance by the equity-linked contract and the policyholders preferences in Section 2. Secondly, in Section 3 we evaluate the contract from the perspective of the policyholder both fully rational and under risk preferences. Section 4 studies the differences in the contract valuation due to the different exercise strategies. Section 5 studies how the surrender behavior implied by the different types of risk preferences influences the insurance policy's contract value. Finally, Section 6 concludes.

2 Model Setup

2.1 Equity-Linked Life Insurance Contract

The policyholder considers a typical equity linked life insurance contract with an embedded option to surrender the insurance. Hardy (2003) provides an overview for the different popular variations of this type of insurance.

The policy consists of a capital guarantee and a participation in an underlying equity fund S at maturity T, leading to a terminal payoff $\Phi(S_T, T)$, which has the functional form

$$\Phi(S_T, T) = P \cdot \max(\left\{ (1+g)^T, \left(\frac{S_T}{S_0} \right)^k \right\}, \tag{1}$$

where P denotes the initial premium, g is the minimal guaranteed rate, and k represents a participation in the underlying fund's performance. The insurance offers a contribution guarantee scheme in spirit of Nielsen, Sandmann, and Schlögl (2011), which guarantees the initial premium without any discount or fee. The higher the participation coefficient is, the more the policyholder participates in the underlying's performance. Typically, as the participation coefficient satisfies $0 < k < 1$, the policyholder profits from the funds performance, but not fully. The minimal guaranteed rate g usually lies below the risk free interest rate as the insurer finances the downside protection with the difference.

In case the policyholder surrenders his policy prematurely, the insurance pays a surrender benefit $L(t)$, taking the form

$$L(t) = P \cdot (1 + g_{surr})^t \cdot (1 - \beta(t)), \tag{2}$$

where g_{surr} represents the minimal guaranteed surrender rate, and $\beta(t)$ is a penalty term for early surrender. In general, this penalty is decreasing in time. Regulation on the minimal guaranteed surrender rate requires it to at least equal the minimal guaranteed rate to not penalize prior surrender too much, as Bernard and Lemieux (2008) point out.

For simplicity, we ignore the impact of mortality. The policyholder's mortality has limited impact in our setting, as the policyholder needs to assess his own mortality. In an empirical study, Andersson and Lundborg (2007) show that people systematically underestimate their own death risk, both in general and through specific causes of death. The typical investor in equity-linked products has a relatively low mortality rate to begin with, as this type of insurance is a vehicle for retirement saving, and not used primarily as a life insurance (compared to term life insurance). Hence, we can safely ignore the effect of mortality in our framework, cf. Doskeland and Nordahl (2008).

The insurance contract is traded on a Black and Scholes (1973)-type financial market, which we define on a filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^{T}, \mathbb{P}\right)$. The financial market consists of a risky fund S, which provides the underlying equity for our insurance contract in Equation (1), and a risk-free bond. The two assest have the dynamics

$$dS_t = \mu S_t dt + \sigma S_t dW_t^{\mathbb{P}}, \qquad S_0 > 0, \tag{3}$$

$$dB_t = r B_t dt, \qquad B_0 = 1. \tag{4}$$

The fund depends on the drift μ, volatility σ, and is driven by a standard Brownian motion $W^{\mathbb{P}}$ under the real world probability measure $\mathbb{P}$. The financial market is complete and arbitrage-free.

2.2 Policyholder

Our investor considers his life insurance in isolation of his other investment activities. This way of evaluation is consistent with the concept of *narrow framing* and *mental accounting*. People do not mentally combine risky choices, but consider each decision in isolation, which Tversky and Kahneman (1981) label as narrow framing. The similar concept of mental accounting (Thaler (1980, 1985)) explains how people use mental accounts for separate activities to keep their finances in order. Furthermore, people use mental accounting as a tool to avoid self-control problems. Thaler (1985) gives the example of a household financing decision: A couple does not repay a credit with their savings for a holiday home, because they are afraid they will never replace the savings although the credit has a higher interest rate as the couples' saving account. Shefrin and Statman (1985) explain how investors use isolated mental accounts for each stock investment they own. The investor does not mentally connect his stock investments, in particular, the investor does not analyze interdependencies. In our context, the policyholder has a mental account "life insurance", which consists of the running value of his insurance. If the policyholder receives the terminal value Φ or surrenders the contract for L, he closes his mental account and evaluates the outcome. Hence, he derives utility from realizing the value of his mental account. This modeling is similar to the model of Barberis (2012)'s casino gambling model, which Ebert and Hilpert (2016) apply to a financial setup.

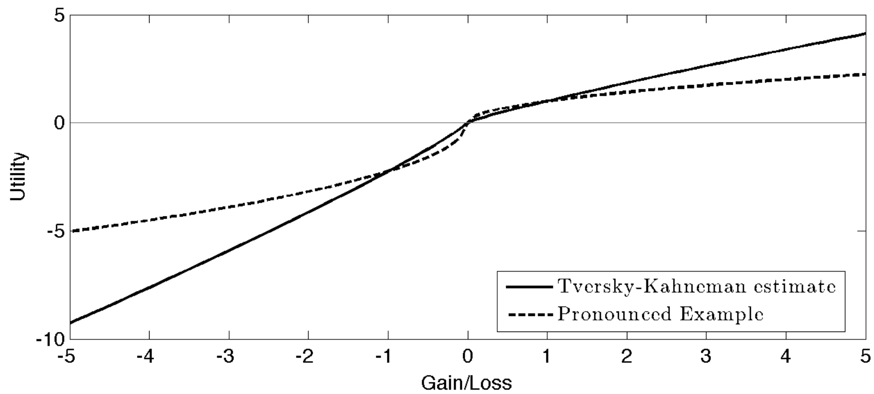

Figure 1: *Example of a loss averse utility function dependent on the outcome of a gamble relative to the policyholder's reference point. The utility function has a loss aversion level of* $\lambda = 2.25$*. The S-shape parameter is* $\gamma = 0.5$ *(dashed line) and* $\gamma = 0.88$ *(solid line).*

The next part of our model for the policyholder's perception of the life insurance is *how* he judges his mental account for his insurance. We describe this part of the policyholder's decision making by his risk preferences. While mainstream economics has been using the concept of risk aversion for a long time, the descriptive power of a purely risk-averse decision maker in experimental and real world data is limited. A popular alternative to model risk preferences is to include loss aversion. Hence, according to Kahneman and Tversky (1979)'s theory of loss aversion, the policyholder has a utility function $U : \mathbb{R}^+ \longrightarrow \mathbb{R}$, which has the following form:[2]

$$U(x) = \begin{cases} (x-R)^\gamma, & x \geq R \\ -\lambda\left(-(x-R)\right)^\gamma, & x < R \end{cases} \tag{5}$$

Hereby, the policyholder's perception of a random outcome x of a lottery depends on a reference point R. The reference point captures that people

[2] We use the notion of utility function instead of Kahneman and Tversky (1979)'s value function to distinguish it from the value function in the optimal control problem in the main part of this paper.

do not consider absolute wealth levels, but isolate the the risky choice and analyze the outcome relative to some reference value, say their expected outcome or the status quo.

Furthermore, losses loom larger than gains: the policyholder feels the pain of losses stronger than the joy over gains. The loss aversion parameter λ measures how much more pain from losses the policyholder feels compared to joy over gains. Finally, the utility function displays diminishing marginal sensitivity to both gains and losses. The parameter γ captures this diminishing sensitivity of returns and creates the S-shape of prospect theory. Figure 1 illustrates the features of the loss averse utility function: It plots a stylized utility function for a sample parametrization which delivers a pronounced S-shape with a S-shape parameter of $\gamma = 0.5$ and a loss aversion by $\lambda = 2$ (dashed line). Furthermore, Figure 1 displays the Kahneman and Tversky (1979) estimate of the value function (solid line) with a S-shape parameter of $\gamma = 0.88$ from experimental data, which is going to serve us as set of example parameters in the subsequent analysis.

3 Insurance Policy Valuation

The first part of the problem of evaluating the contract is the rational valuation with full access to the financial market in the Black and Scholes (1973) model to establish a benchmark.[3] This rational valuation refers to the optimal exercise strategy of the American option embedded in the insurance contract. It is a worst case scenario value for the insurer, as this strategy maximizes the monetary value of the embedded option, and hence minimizes the contract value from the insurer's perspective. The second subsection derives the policyholder's valuation under loss averse preferences and the behavioral exercise strategy. Here, the policyholder cannot access the financial market directly except through the equity-linked life insurance contract. Consequently, the policyholder cannot replicate the insurance contract and use risk-neutral pricing. As shown by Ahn and Wilmott (1998), the policyholder then does not want to stop optimally according to the Black-Scholes strategy, and also

[3] Alternatively, a risk neutral policyholder who maximizes his expected utility from the insurance policy forms a reasonable benchmark case. This case is included in the following analysis as a special case. However, using it as the benchmark strategy does not affect this paper's results.

cannot hedge potential profits due to his limited access to the underlying.[4]

3.1 American Option Approach

The standard approach to price this equity-linked life insurance with surrender guarantee is to derive the pricing partial differential equation (PDE) for the contract using risk-neutral pricing approach, for example as in Shen and Xu (2005). Because the financial market is complete and arbitrage free by assumption, there exists a unique risk-neutral probability measure $\mathbb{Q}$, under which the policyholder prices the contract as an American style contingent claim in the Black-Scholes framework. Hence, the policyholder considers the optimal stopping problem

$$V(t,S_t) = \sup_{\tau\in[t,T]} \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau}\phi(\tau,S_\tau)\right], \tag{6}$$

The optimal exercise time is τ^{Am}, which is the first time where the surrender benefit exceeds the value of continuing the contract. In this case, the policyholder applies the risk-neutral pricing methodology and hence calculates the expectation and subsequently the optimal stopping time under the risk-neutral pricing measure $\mathbb{Q}$. The payoff function $\phi(t,S_t)$ depends on time and satisfies

$$\phi(t,S_t) = \begin{cases} \Phi(t,S_t), & t = T \\ L(t), & t < T \end{cases} \tag{7}$$

Formally, we have for a continuation value $V(t,S_t)$ of the insurance value the following representation of the optimal stopping time τ^{Am}:

$$\tau^{Am} := \inf\left\{t\in(0,T] : L(t) > V(t,S_t)\right\} \tag{8}$$

which allows us to write the contract value of the insurance contract as the solution to the free boundary problem on $t < \tau^{Am}\wedge T$ with $x\wedge y$ denoting the minimum of x and y:

Proposition 1 (Pricing PDE)**.** *The value of an American-style option describes the maximal possible value of the contract and assumes no financial*

[4] In this paper, the policyholder cannot trade the risky asset to replicate his own contract. Alternatively, the policyholder is not smart enough to replicate.

constraints in and unlimited access to an efficient financial market. On $\{t < \tau^{Am} \wedge T\}$ the value of the American option is $V_t = v(t, S_t)$ where $v : [0,T] \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ satisfies the free boundary value problem given by the PDE

$$\frac{\partial v}{\partial t}(t,s) + rs \cdot \frac{\partial v}{\partial s}(t,s) + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2}(t,s) - rv(t,s) = 0 \tag{9}$$

with constraint $v(t,s) \geq L(t)$ on $[0,T) \times \mathbb{R}_+$ and terminal condition $v(T,s) = \Phi(s)$, for $s \in \mathbb{R}_+$.

Since the value of this contract cannot be established in closed form, we use the finite difference methodology to price this contract. The Crank-Nicoloson Scheme, derived in Crank and Nicolson (1947), allows us to evaluate this free boundary problem efficiently. We use the Crank-Nicolson Scheme to solve the grid of 2,000 price steps and 2,000 time steps. The Crank-Nicolson method is unconditionally stable and under some regularity assumptions, the order of the method is $O(\Delta s^2) + O(\Delta t^2)$, with Δs and Δt being the step sizes of the mesh in the s and t direction, respectively. In our study, we use $\Delta s = 0.0025$ and $\Delta t = 0.0050$. A detailed discussion of the Crank-Nicolson method is beyond the scope of our paper. For an extensive discussion c.f. for example Seydel (2012).

3.2 Risk Preference Approach

We now turn to a policyholder who cannot apply the risk-neutral pricing approach anymore, but instead aims at maximizing his expected utility with an utility function from Section 2.2. In this case, the policyholder maximizes his expected utility, which takes the form

$$V^U(t, S_t) = \sup_{\tau \in [0,T]} \mathbb{E}^{\mathbb{P}} \left[e^{-\delta\tau} U\left(\phi(\tau, S_\tau)\right)\right], \tag{10}$$

where the expected value now is under the real world probability measure $\mathbb{P}$ instead of $\mathbb{Q}$, and the policyholder discounts any payments with his personal discount rate δ. This personal discount rate captures the time preference of the policyholder and thus measures his impatience. Furthermore, the policyholder does not consider the plain payoff at any given time, but instead considers the utility burst he receives upon closing his mental account at this point, which gives us

$$\tau^U := \inf\left\{t \in (0,T] : U(L(t)) > V^U(t, S_t)\right\} \tag{11}$$

for the stopping time τ^U.

Although we pose the optimal stopping problem in Equation (10) in a similar fashion to the optimal stopping problem in Equation (6), the stopping problem in hand offers a technical challenge. Whereas the optimal stopping problem for the American case in Equation (6) allows for the derivation of a standard Hamiltion-Jacobi-Bellman equation (Pham (2009)), the new problem violates the necessary differentiability condition required. In particular, the loss averse utility function specified in Equation (5) is not continuously differentiable in the reference point, so that we need to extend our classical approach. Hence, we use the concept of a viscosity solution, which relaxes the assumptions on continuity. The function in the expectation of Equation (10) is continuous, and the objective function of the optimal stopping problem is locally bounded. According to Touzi (2013), we have the dynamic programming equation in the following proposition:

Proposition 2 (Utility PDE)**.** *On $\{t < \tau^P \wedge T\}$, the expected utility of the equity-linked life insurance with surrender benefit is $u_t = u(t, S_t)$ where $u : [0,T] \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the obstacle problem*

$$\frac{\partial u}{\partial t}(t,s) + \mu s \cdot \frac{\partial u}{\partial s}(t,s) + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial s^2}(t,s) - \delta u(t,s) = 0 \tag{12}$$

with terminal condition $u(T,s) = U(\Phi(s))$, for $s \in \mathbb{R}_+$.

Again, we cannot solve this partial differential equation in closed form. We apply the finite difference method again, but as we now handle a viscosity solution, the standard Crank-Nicolson scheme becomes infeasible. Forsyth and Labahn (2007) provide a remedy by establishing general conditions under which the implicit scheme converges, hence we apply the implicit scheme to handle the above partial differential equation.

4 The Surrender Behavior

We now turn to the analysis of the policyholder's surrender behavior. The optimal stopping under the risk neutral distribution again serves as our benchmark because it maximizes the policyholder's contract value. Then, the section presents the exercise behavior of risk- and loss averse policyholders, respectively. The main results are (i) pure risk aversion leads to earlier surrenders, (ii) loss aversion deters surrender, and (iii) impatience leads to earlier surrender.

To study the policyholder's surrender behavior, we employ the exercise boundary for a contract to characterize the policyholder's surrender behavior. The exercise boundary depends on the set of (s, t)-tuples for which the exercise of the surrender guarantee is beneficial to the investor, which we label early exercise region. Formally, we define the early exercise region to be

$$H := \left\{(s,t) \in \mathbb{R}^+ \times [0,T] : L(t) \geq V(s,t)\right\}. \tag{13}$$

The exercise boundary is the upper contour of the early exercise region.

We consider a sample contract with the following specifications: The initial premium of the policyholder is $P = 10,000$, on which the insurance guarantees a rate of $g = 0.02$. The participation rate is $k = 0.9$, and the contract matures in twenty years. We abstract from a penalty on early surrender by setting $\beta(t) = 0 \,\forall\, t$. The financial market offers a risk-free asset with a rate of $r = 0.035$, whereas the equity fund has an drift of $\mu = 0.06$, and a volatility of $\sigma = 0.3$. For simplicity, the fund has an initial value of 10,000.

4.1 American Option Solution

Because of risk-neutral valuation, this surrender behavior is a worst case scenario in terms of hedging. The price of the hedging strategy is the value of the contract derived in Proposition 1. Because this strategy gives the maximal contract value, any deviation from this exercise for example due to bounded rationality or risk aversion lowers the price of the contract.

Figure 2 shows the exercise boundary of the insurance contract for the above parametrization. Firstly, the guaranteed rate of the contract implies an embedded put option for the equity-linked part of the insurance contract. Figure 2 shows that it is rational to surrender the contract if the underlying fund drops far enough because the contract offers a lower guaranteed rate than the financial market offers on the risk-free account. Hence, if it is unlikely for the equity-component to outperform the guarantee, the contract value is lower than the surrender benefit.

Secondly, the exercise boundary is rising in time. If the contract is closer to maturity, the policyholder surrenders the insurance contract for higher values of the underlying fund. This effect occurs because of the guaranteed rate of the insurance contract is lower than the risk-free rate. Hence, if the underlying fund of the insurance is substantially below the initial investment value, it's probability to end up above the guaranteed rate during the contract's remaining lifetime decreases in time. If the probability of this event is low enough, it is preferable for the investor to surrender the contract and invest his money for the risk-free rate.

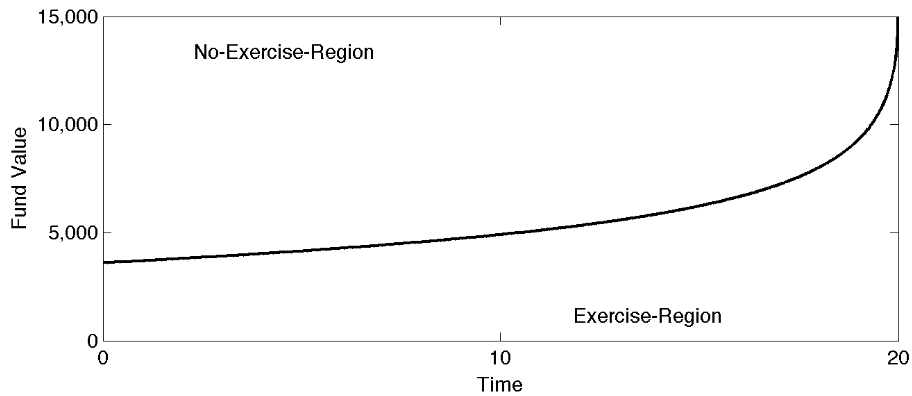

Figure 2: Exercise boundary for the rational pricing approach. *The graph displays the optimal exercise boundary, separating the exercise region H and the non-exercise region $\overline{H}$: Above the graph are the time-fund value combinations for which the insurance contract's surrender is not optimal. Below the graph are the time-fund value combinations for which surrender gives higher value than continuation. The initial fund value is 10,000.*

4.2 Classical Expected Utility Solution

We now turn to the surrender behavior of a policyholder with classical power utility preferences. This section shows that the policyholder's risk aversion has substantial impact on the surrender behavior as higher risk aversion induces early surrender at higher fund values. Furthermore, we observe a strong impact of the patience or discount rate of the policyholder to his perceived contract value. While a patient policyholder will surrender only for low fund values, higher levels of discounting can quickly reduce the contract's value to a level that is unattractive to the policyholder.

Figure 3 displays the exercise boundary for the American optimal solution (bold line), the expected-utility solution for a risk-neutral policyholder (dashed line), and for moderate and high risk-averse policyholders (dashed-dotted line and dotted line, respectively.)

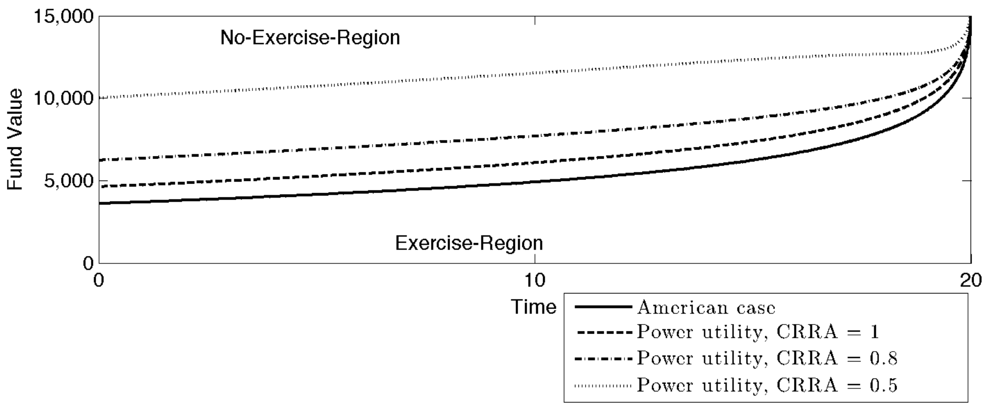

Figure 3: Exercise boundary for different levels of risk aversion. *The graph displays the exercise and no-exercise regions for a policyholder with constant relative risk aversion for the risk aversion levels of $\gamma = 0.5$, $\gamma = 0.8$, $\gamma = 1$, and as a benchmark, the exercise region for the optimal surrender. For each line, the time-fund value combinations above the line have a higher continuation value than surrender value, while below the lines, it is optimal to surrender the insurance for the policyholder with the respective preferences.*

Firstly, the policyholder in this section cannot access the financial market as in the previous section. This limited access to the market increases the underlying fund's value for which the policyholder terminates his contract. The reason for this behavior lies in the valuation of the insurance policy: The policyholder's expected value depends on the real world probability measure and the real world drift of the underling fund. As the policyholder does not lock in profits by dynamically hedging his positions, early termination at higher fund values provides an alternative way to save already existent profits to the policyholder.

If we turn to the risk averse policyholder, this effect increases. Not only can the policyholder lock in profits anymore, but is also risk averse which increases his desire to reduce risk from the policy. Figure 3 demonstrates the further increase of fund values for which the policyholder surrenders the policy early to secure the policy from further value drops. This effect continues up to a point at which the policyholder prefers to keep his cash instead of investing in the insurance policy, because he prefers to receive the risk-free interest rate rather than a investment policy which may provide a lower payoff.

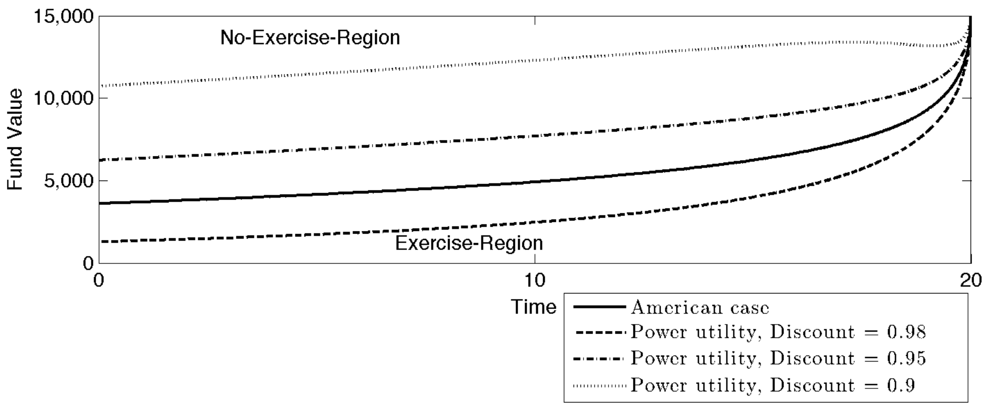

Figure 4: Exercise boundary for different time preferences. *The graph displays the exercise and no-exercise regions for a policyholder with constant relative risk aversion for a risk aversion level of $\gamma = 0.8$, and three levels for the personal discount rate δ, with values in $\delta \in \{0.98, 0.95, 0.9\}$, and as a benchmark, the exercise region for the optimal surrender. For each line, the time-fund value combinations above the line have a higher continuation value than surrender value, while below the lines, it is optimal to surrender the insurance for the policyholder with the respective preferences.*

On top of that, the policyholder's time preference δ plays a crucial role in controlling the surrender behavior. Figure 4 shows how the exercise boundary changes if the policyholder's time preference changes. It displays three cases: No time preference (dashed line), an intermediate (dashed-dotted line), and a low level of patience (dotted-line). If the policyholder is patient enough, as in the case of $\delta = 0.98$, there will be later surrender, as the policyholder waits for future returns. Having the surrender benefit earlier is not attractive to the policyholder, as he is happy to wait for the higher payoffs later on. However, if the fund value drops too low, the chance of recovery is so small that even a patient policyholder takes the surrender benefit. On the other hand, if the policyholder's patience is low, this can reduce the insurance policy's value to a level such that the policyholder does not want to buy the insurance anymore in the first place. We also observe that the policy's value from the policyholder's perspective is quite sensitive to changes in patience.

4.3 Expected Utility with Loss Aversion Solution

In this section, we study the effect of loss aversion on the surrender behavior. The policyholder will not terminate the insurance policy if the surrender benefit is so low that early termination would realize a loss. This is another realization of the disposition effect: The policyholder will stay with losses for a long time, but if the insurance policy's surrender benefit exceeds the reference point substantially, the policyholder surrenders his contract. Again, the discounting strongly influences the surrender decision, with high patience leading to surrender at a reduced level of the underlying fund's value. The loss aversion also impacts how the policyholder's risk aversion influences the exercise boundary: A stronger S-shape leads to stronger diminishing sensitivity of returns which increases the impact of the disposition effect.

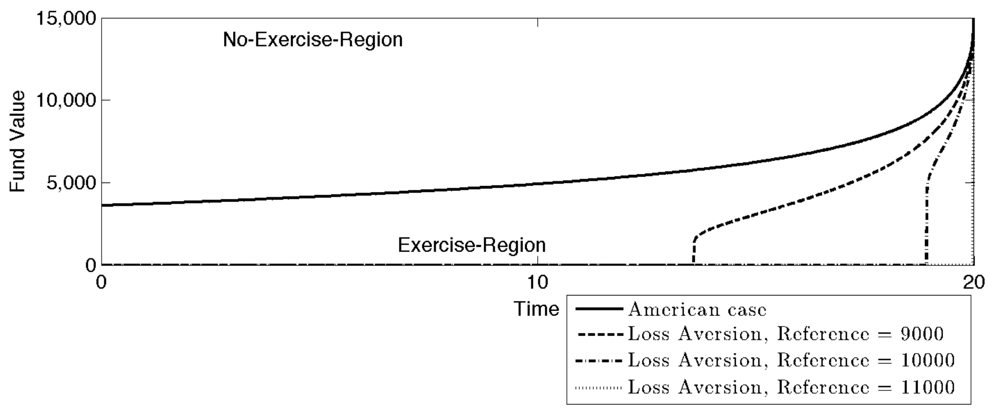

Figure 5: Exercise boundary for different reference points. *The graph displays the exercise and no-exercise regions for a policyholder with constant relative risk aversion* $\gamma = 0.8$ *and a loss aversion level of* $\lambda = 2.25$*, and as a benchmark, the exercise region for the optimal surrender (bold). The reference points equals* $R = 9,000$*,* $R = 10,000$*, and* $R = 11,000$*. For each line, the time-fund value combinations above the line have a higher continuation value than the surrender value, while below the lines, it is optimal to surrender the insurance for the policyholder with the respective preferences.*

Figure 5 displays the exercise boundary for the American optimal stopping case (solid line), and for a loss averse policyholder with the reference points $R = 9,000$ (dashed line), $R = 10,000$ (dashed-dotted line), and $R = 11,000$ (dotted line), that is, a reference point which is below the initial investment, at the initial investment, and above the initial investment, respectively. With loss averse preferences, the policyholder does not surrender prematurely up to a specific cutoff time, after which the policyholder's exercise boundary quickly increases in the fund value. If the policyholder perceives the insurance policy's surrender value as a loss, premature surrender is unattractive. Our policyholder displays the disposition effect and surrenders insurances only if he realizes a gain which is sufficiently in the gain area. Is the reference point relatively high, the surrender benefit cannot reach a high enough level to make early termination attractive.

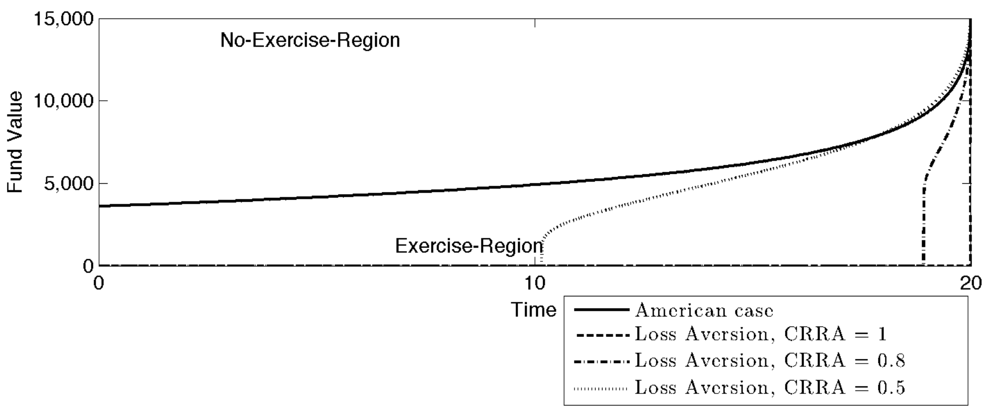

Figure 6: Exercise boundary for different levels of risk aversion. *The graph displays the exercise and no-exercise regions for a policyholder with different levels constant relative risk aversion of* $\gamma = 0.5$*,* $\gamma = 0.8$*,* $\gamma = 1$*, and a loss aversion level of* $\lambda = 2.25$*, and as a benchmark, the exercise region for the optimal surrender (bold). The reference point equals* $R = 10,000$*. For each line, the time-fund value combinations above the line have a higher continuation value than the surrender value, while below the lines, it is optimal to surrender the insurance for the policyholder with the respective preferences.*

Figure 6 shows the exercise boundary for a loss averse policyholder with a reference point of $R = 10,000$, with levels of relative risk aversion of $\gamma = 0.5$ (dotted line), $\gamma = 0.8$ (dashed-dotted line), and $\gamma = 1$ (dashed line), respectively. Again, the benchmark is the rational American exercise boundary (solid line). Higher risk aversion strongly affects the cutoff time, before which the policyholder does not surrender the insurance contract. The risk aversion parameter controls the S-shape of the utility function, with a high value leading to a strong diminishing sensitivity of gains and losses. If the S-shape is pronounced, the policyholder surrenders earlier, because the disposition effect becomes more pronounced: A policyholder will experience the diminishing sensitivity of gains and losses much stronger. Is this diminishing sensitivity strong, then the policyholder will quickly reach a level at which further gains do not provide substantial additional value, leading to earlier premature termination. Does the policyholders' utility function not display strongly diminishing sensitivity, waiting for higher payoffs offsets the disposition effect.

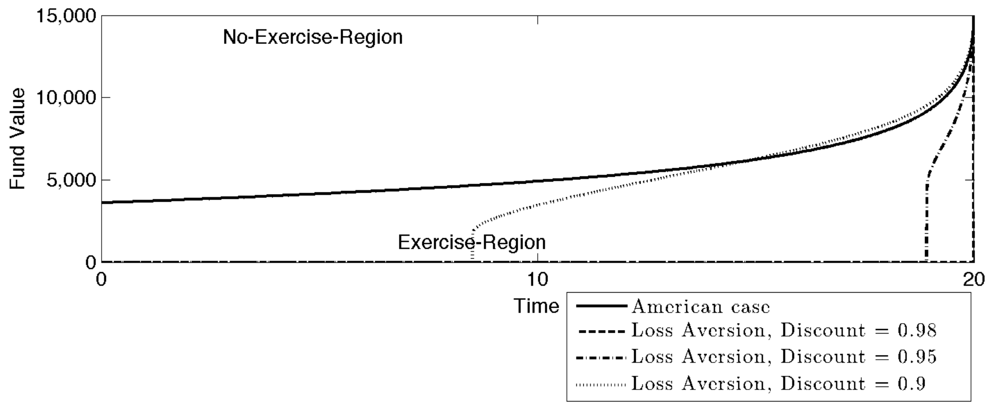

Figure 7: Exercise boundary for different levels of time preferences. *The graph displays the exercise and no-exercise regions for a policyholder with constant relative risk aversion of* $\gamma = 0.8$ *and loss aversion level of* $\lambda = 2.25$*, and as a benchmark, the exercise region for the optimal surrender (bold). The reference point equals 10,000. The discount rate ranges from* $\delta = 0.98$*,* $\delta = 0.95$*, and* $\delta = 0.9$*. For each line, the time-fund value combinations above the line have a higher continuation value than the surrender value, while below the lines, it is optimal to surrender the insurance for the policyholder with the respective preferences.*

Figure 7 presents the impact of time preference on the policyholder's exercise boundary. The figure presents the American optimal exercise boundary as the benchmark (solid line), as well as three levels of discount rates: $\delta = 0.98$ (dashed line), $\delta = 0.95$ (dashed-dotted line), and $\delta = 0.9$ (dotted ra). If the policyholder is impatient, as in the case of $\delta = 0.9$, the policyholder surrenders substatially earlier and at lower fund values. On the other hand a more patient policyholder will surrender later, which also impacts the disposition effect. Very patient policyholders will resist their urge to surrender their policy, as they are patient to wait for higher future returns.

5 The Impact of Surrender Behavior on the Contract Value

Having derived how the exercise behavior of risk averse and loss averse policyholders deviates from the optimal exercise implied by treating the insurance contract's embedded option as an American option, we now assess how much this deviation influences the contract value from the policyholder's and insurers perspective. In this section, we show that in particular strong risk aversion implicates a surrender behavior which decreases the insurance contract's value substantially. Moreover, we show that the disposition effect has a high impact on the contract value to the policyholder. While the disposition effect decreases the policyholder's contract value relative to the optimally stopped version, compared to a pure risk averse surrender behavior it is value enhancing.

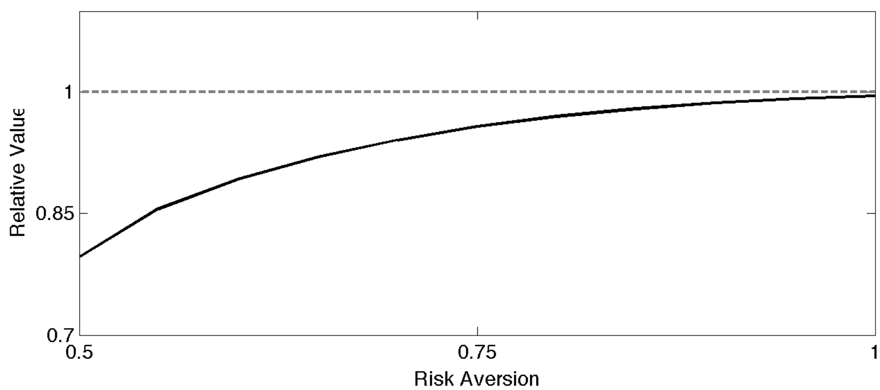

Figure 8: Relative monetary loss of power utility policyholder. *The graph displays the relative inital contract value for a policyholder with constant relative risk aversion perferences for different levels of risk aversion. A level of one indicates the same value as an optimally exercised insurance policy.*

Figure 8 displays the contract value as a function of the policyholder's coefficient of relative risk aversion, relative to the maximal value available through optimal exercise. The figure shows a decreasing value of the insurance contract if the exercise is based on the risk averse utility. While the loss of a risk-neutral policyholder is marginal, the value loss of a intermediate to strongly risk averse policyholder is high and can reach up to twenty percent of the contract's initial value.

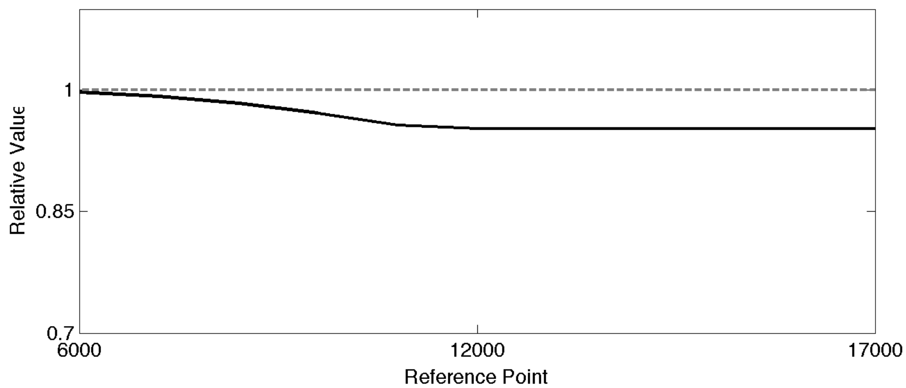

Figure 9: Relative monetary loss of loss-averse policyholder. *The graph displays the relative initial contract value for a policyholder with risk aversion of* $\gamma = 0.8$ *who exhibits loss aversion. A level of one indicates the same value as an optimally exercised insurance policy.*

Figure 9 presents the impact of the reference point on the insurance policy's value with exercise according to the loss averse utility function relative to the value implied by the optimal exercise behavior. The policyholder's risk aversion is fixed at a moderate level of $\gamma = 0.8$. We see that the reference point has a substantial impact on the contract's value. For a high reference point the policyholder's disposition effect prevents early termination, which renders the value of the insurance contract if the contract is considered as a European style contract. For low reference points the policyholder potentially disposes of the contract through early exercise of the surrender option. However, in this case the policyholder's loss is limited relative to the pure risk aversion case, as the value is limited from below through the european value.

6 Conclusion

In this paper, we have derived the impact of risk and loss aversion on the surrender behavior of policyholders of equity-linked life insurance contracts. To this end, we have derived a viscosity solution to the Hamilton-Jacobi-Bellman equation to an optimal stopping problem implied by a policyholder with a potentially loss averse utility function. While risk averse policyholders surrender their contracts earlier and at higher fund values, loss averse policyholders display the disposition effect which leads to delayed surrenders. Furthermore, the policyholder's risk preferences imply an exercise behavior which is costly to policyholders. As the policyholder's risk aversion and loss aversion deviate from risk-neutrality, the surrender behavior induces loss of a substantial fraction of the contract's value under the Black-Scholes approach. In comparison, pure risk averse policyholders lose substantially more compared to loss averse policyholders.

A further interesting aspect to study for the surrender behavior is the aspect of probability weighting to analyze the impact of the Tversky and Kahneman (1992)'s full cumulative prospect theory. However, the incorporation of probability weighting is technically challenging. While the cumulative prospect theory of Kahneman and Tversky is the most prominent descriptive theory of decision making under risk, further behavioral biases, for example policyholder's regret and emotional aspects of the policyholder, are left out in this paper, but are potentially interesting to study.

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