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Risk- and Value-Based Management for Non-Life Insurers under Solvency Constraints

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RISK- AND VALUE-BASED MANAGEMENT FOR NON-LIFE INSURERS UNDER SOLVENCY CONSTRAINTS

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ABSTRACT

The aim of this paper is to propose an internal model for a non-life insurer and to apply this model for deriving optimal risk- and value-based management decisions regarding the insurer's investment strategy, which contribute to increasing shareholder value. We thereby considerably extend previous work by explicitly accounting for the policyholders' willingness to pay depending on their risk sensitivity based on the insurer's reported solvency status, which will be of great relevance under Solvency II. We further study the impact of the risk-free interest rate on attainable and admissible risk-return asset combinations, dependencies between assets and liabilities as well as the influence of reinsurance contracts, and we derive analytical solutions for maximizing shareholder value. One main finding is that the consideration of the policyholders' willingness to pay depending on their risk sensitivity towards the insurer's reported solvency status is vital for optimal management decisions and that in the present setting, reinsurance increases shareholder value only for non-risk sensitive policyholders. Our results further emphasize that low interest rates strongly restrict the insurer's investment opportunities.

Keywords: Risk- and value-based decision-making; non-life insurance; Solvency II; shareholder value optimization

JEL Classification: G22; G28; G31

1. INTRODUCTION

Pillar 1 of Solvency II requires that insurers derive solvency capital requirements either by means of a standard model provided by the regulatory authorities or based on a company-specific internal model that adequately reflects the firm's risks (see, e.g., Eling et al., 2009). An internal model should also be used in Pillar 2 for the firm's own risk and solvency assessment (ORSA) and should thus represent an integral part of an insurer's risk- and value-based management, i.e., to be applied for corporate risk management and asset allocation decisions, for instance. However, the development of internal models requires considerable financial and technical resources, which implies that the degree of complexity and the achieved

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benefits should be carefully balanced. Against this background, the aim of this paper is to establish a simplified internal model for a non-life insurer and to apply this model for deriving (optimal) risk- and value-based management decisions regarding the investment strategy, which contribute to increasing shareholder value. In particular, by adjusting the asset allocation to satisfy solvency capital requirements, the approach should be less costly than raising equity capital or adjusting the liability side (see Eling et al., 2009). Toward this end, we considerably extend the analyses and model frameworks in previous work (e.g., Eling et al., 2009; Braun et al., 2015; Zimmer et al., 2014) by studying the impact of several new key features and their impact on shareholder value with analytical solutions, including the policyholders' willingness to pay depending on the insurer's reported solvency status, which despite its great impact has not been studied to date in this context, the dependencies between assets and liabilities as well as the impact of reinsurance contracts.

In previous work, Eling et al. (2009) derive minimum requirements for a non-life insurer's capital investment strategy that satisfy solvency restrictions based on different risk measures. Using "solvency lines", i.e. isoquants of risk and return combinations of the asset allocation for a fixed safety level of the insurer, they determine admissible risk and return asset combinations given a certain liability structure, and then compare these to allocation opportunities actually available at the capital market based on portfolio theory. Similarly, but in a life insurance context, Braun et al. (2015) study optimal asset allocations taking into account restrictions from solvency capital requirements, thereby comparing the Solvency II standard formula with an internal model. They find that the standard formula cannot well differentiate investments on the basis of risk-return profiles due to an insufficient consideration of diversification effects and expected returns, and hence tends to promote inefficient portfolios. Furthermore, experimental and empirical research (Lorson et al., 2012; Wakker et al., 1997; Zimmer et al., 2009; Zimmer et al., 2014) shows that an insurer's default risk can have a strong influence on customer demand, where lower safety levels can lead to a considerable reduction of achievable premiums. In this context, Zimmer et al. (2014) develop a risk management model assuming that the insurer's default risk is fully known to consumers, and based on this derive the solvency level that maximizes shareholder value, which is the case for a shortfall probability of zero. These results emphasize that an insurer's safety level should be taken into account in risk- and value-based management as the reaction of customers to default risk (by way of the premium level) can considerably impact shareholder value. This will be even more relevant when insurers have to publicly report their solvency status under Solvency II.

Another aspect that should be considered in the context of optimal risk-return asset allocations under solvency constraints are potentially incorrect model specifications, which can lead to biased results and to misguided decisions based thereon (Schmeiser et al., 2012; Wagner,

2014). For instance, Schmeiser et al. (2012) show that, among others, dependencies between insurance asset classes have a considerable influence on risk measures in the context of capital requirements. In the context of deriving minimum requirements for the investment strategy, Fischer and Schlütter (2015) further criticize that the standard model leads to an incentive to avoid diversification between assets and liabilities, as dependencies are not adequately taken into account in the standard model, which is in line with the results from Braun et al. (2015). Overall, it is thus essential to consider the asset-liability relation for deriving minimum requirements for the investment performance.

The purpose of this paper is to contribute to the existing literature in various relevant ways. First, in contrast to Braun et al. (2015), we study a non-life insurer instead of a life insurer and directly consider an internal model. Moreover, we extend the analysis in Eling et al. (2009) and Braun et al. (2015) by focusing on the firm's shareholder value under risk- and value-based management decisions and by explicitly taking into account the policyholders' willingness to pay in our model framework when deriving admissible asset allocations under solvency constraints. To investigate the influence of the policyholders' risk sensitivity, we also extend the approach in Lorson et al. (2012) and Zimmer et al. (2014) and model the achievable premium as a function of the insurer's safety level and the policyholders' risk assessment. In this setting, we link the admissible risk-return combinations of the insurer's asset portfolio (i.e. those that are allowed under solvency constraints) to allocation opportunities actually attainable at the capital market using portfolio theory, and then additionally derive the maximum achievable shareholder value. Our model further accounts for dependencies between assets and liabilities, proportional reinsurance contracts as well as the influence of the risk-free interest rate. In a numerical analysis, we study the impact of these determinants on the set of attainable and admissible risk-return combinations for the asset allocation as well as on the maximum shareholder value under different policyholder risk sensitivity scenarios. This allows establishing a risk- and value-based management that can be used for an enterprise-wide decision making by balancing risk-taking, solvency levels, and shareholder value, thereby taking into account policyholder risk sensitivities. Given a certain level of equity capital, deriving a solvency line and optimal value-based management strategies regarding assets and liabilities allows the derivation of key drivers that increase shareholder value. One main finding is that the consideration of policyholders' willingness to pay is of great relevance when deriving optimal risk-return asset allocations under solvency constraints and that reinsurance can considerably impact these results, depending on the level of the policyholders' risk sensitivity.

The remainder of this paper is structured as follows. Section 2 presents the model framework for a non-life insurer, while Section 3 focuses on the derivation of attainable and admissible risk-return combinations as well as shareholder value maximization given several underlying

conditions. An application of the developed approach with sensitivity analyses for various policyholders' risk sensitivities, asset-liability correlations and proportional reinsurance contracts is provided in Section 4. The last section summarizes our main findings.

2. MODEL FRAMEWORK

Modeling assets and liabilities

In what follows, we present a simple one period-model for a non-life insurer, where at time $t = 0$, shareholders make an initial contribution of equity capital E_0 and policyholders pay a premium P_0 to insure possible claims C_1 at time $t = 1$. For simplification purposes and following Braun et al. (2015), the claims are assumed to be normally distributed with expected value μ_C and variance σ_C^2 , i.e.,¹

$$C_1 \sim N(\mu_C, \sigma_C^2).$$

The premium in case of a default-free insurer is given by the actuarial expected value principle based on the expected claims with a nonnegative percentage loading δ , i.e.,

$$P_0 = \mu_C \cdot (1 + \delta).$$

We further assume that the insurer may additionally purchase a proportional reinsurance contract (re) at time $t = 0$, which covers a fraction λ of the policyholders' claims,

$$X_1^{re}(\lambda) = \lambda \cdot C_1.$$

The corresponding reinsurance premium π^{re} is given by the expected payoff with a constant loading δ^{re} (reflecting costs for moral hazards, for instance), such that

$$\pi^{re} = E[X_1^{re}] \cdot (1 + \delta^{re}) = \lambda \cdot \mu_C \cdot (1 + \delta^{re}).$$

Taking into account that the reinsurer bears part of the claims, the (net) liabilities of the insurer at time $t = 1$ are thus given by

$$L_1 = C_1 - X_1^{re}.$$

The initial capital A_0 thus consists of equity E_0 and premiums P_0 less reinsurance premiums π^{re} ,

¹ This assumption was subject to robustness tests using lognormally distributed claims.

$$A_0 = E_0 + P_0 - \pi^{re},$$

which is invested at the capital market. The value of assets at time $t = 1$ is then given by

$$A_1 = A_0 \cdot (1 + r),$$

where r is the stochastic return on assets between $t = 0$ and $t = 1$, where we again assume a normal distribution with expected value μ_r and variance σ_r^2 (see also Braun et al., 2015), i.e.,

$$r \sim N(\mu_r, \sigma_r^2).$$

We further assume a linear correlation ρ_{rC} between assets A_1 and claims C_1 . The resulting equity capital at time $t = 1$ is then also normally distributed with

$$E_1 = A_1 - L_1 \sim N(E[E_1], V[E_1]),$$

where the expected value (taking into account proportional reinsurance) is given by

$$E[E_1] = A_0 \cdot (1 + \mu_r) - E[L_1] = A_0 \cdot (1 + \mu_r) - (1 - \lambda) \cdot \mu_C, \quad (1)$$

and the variance is

$$\begin{aligned} V[E_1] &= A_0^2 \cdot \sigma_r^2 + V[L_1] - 2 \cdot A_0 \cdot Cov(r, L_1) \\ &= A_0^2 \cdot \sigma_r^2 + (1 - \lambda)^2 \cdot \sigma_C^2 - 2 \cdot A_0 \cdot (1 - \lambda) \cdot \sigma_r \cdot \sigma_C \cdot \rho_{rC}. \end{aligned} \quad (2)$$

Policyholders' risk sensitivity

According to experimental and empirical research (Lorson et al., 2012; Wakker et al., 1997; Zimmer et al., 2009; Zimmer et al., 2014), an insurer's default risk can strongly impact customer demand, with lower safety levels leading to a considerable reduction of the achievable premiums. Whereas expected utility theory suggests that a small increase in the shortfall probability should only reduce the policyholders' willingness to pay in a marginal way, Wakker et al. (1997) observe that the actual willingness to pay decreases sharply in the context of "probabilistic insurance", i.e. insurance contracts with a small non-zero shortfall probability. They explain this phenomenon based on Kahneman and Tversky's (1979) prospect theory, according to which individuals tend to overweigh small (extreme) probability events and vice versa. The low probabilities of an insurer's default are thus assigned a weight higher than the objective probability, implying a considerable reduction of premiums below the actuarially fair premium, which also directly impacts shareholder value.

Hence, we adapt our model framework in order to take into account the policyholders' willingness to pay as a reaction to the reported solvency status depending on their risk sensitivity. Toward this end, we extend the approach in Lorson et al. (2012), who calculate the premium reduction compared to the premium offered by a default-free insurer as a function of the reported one-year shortfall probability SP , which is the probability that the insurer's assets are not sufficient to cover the liabilities, i.e., the equity capital in $t = 1$ becomes negative:

$$SP = P(A_1 < L_1) = P(E_1 < 0).$$

To derive the premium reduction depending on the shortfall probability, Lorson et al. (2012) perform a regression based on the empirical results of Zimmer et al. (2009), who conducted a survey with 719 responses about the willingness to pay for three non-life (household) insurance contracts differing only in the level of default risk (shortfall probabilities 0%, 0.3% and 4.9%). Their results show that the participants require a premium reduction of more than 26% in case of a shortfall probability of 4.9% as compared to the default-free insurer. The estimated premium reduction function PR (in comparison to the default-free premium P_0) as estimated by Lorson et al. (2012) is given by²

$$PR(SP) = 0.0419 \cdot \ln(SP) + 0.3855. \quad (3)$$

For an increasing safety level, i.e. a decreasing shortfall probability SP , the premium reduction function decreases until the default-free state is reached and the policyholders exhibit full willingness to pay; the highest premium reduction amounts to 38.55% for a shortfall probability converging to 1. However, since the policyholders' risk assessment is not known, i.e. how well-informed the policyholders are and if they can assess the numeric shortfall probability correctly, the actual premium reduction might differ. To take into account the policyholders' risk sensitivity, we thus extend the model in Lorson et al. (2012) and not only link the premiums paid in $t = 0$ to the insurer's reported shortfall probability $SP = \alpha$, but also to the policyholders' risk sensitivity represented by a scaling parameter s similar to Gatzert and Kellner (2014). The premium payments in $t = 0$, $P_0(SP, s)$, are then given by

$$P_0(SP, s) = P_0 \cdot \max(1 - s \cdot PR(SP), 0), \quad s \geq 0, \quad SP = \alpha \in (0, 1), \quad P_0 = \mu_c(1 + \delta), \quad (4)$$

where $SP = \alpha$, represents the reported one-year shortfall probability and PR the premium reduction function in Equation (3), which is chosen for illustration purposes and can also be adjusted (e.g. depending on the type of contract). For $s = 1$, we obtain the risk sensitivity described in Lorson et al. (2012).

² Since this estimate is based on very few data points, Lorson et al. (2012) also consider an upper and lower bound for the premium reduction to take into account the variability.

3. OPTIMAL ATTAINABLE AND ADMISSIBLE RISK-RETURN COMBINATIONS

To derive optimal risk- and value-based management decisions, we next consider minimum solvency requirements in a risk-return (asset) context (here: expected return and standard deviation of assets that are compatible with the solvency requirements) and link these “admissible” risk-return combinations of the insurer’s asset portfolio to allocation opportunities actually “attainable” at the capital market as is done in Eling et al. (2009). In contrast to previous work, however, we explicitly take into account the policyholders’ willingness to pay depending on the insurer’s solvency level, dependencies between assets and liabilities as well as the effect of reinsurance contracts, which impact the liability side of the insurer’s balance sheet.

Capital market line: “Attainable” risk-return combinations

To identify the risk-return profiles (σ_r, μ_r) that are actually attainable at the market, we follow the classical approach of Tobin (1958) and derive the capital market line (CML) representing the set of efficient investment portfolios given risk-free lending and borrowing.

Let $R = (R_0, R_1, \dots, R_N)'$ denote the stochastic $((N+1) \times 1)$ -vector containing the returns of N risky assets $R_i, i = 1, \dots, N$, and a risk-free investment with $R_0 = r_f$. In addition, M stands for the vector with the expected values of the returns and Σ for the respective variance-covariance matrix. The vector with the investment proportions of the different asset classes is given by $w = (w_0, w_1, \dots, w_N)'$. Hence, the expected value and the variance of the portfolio return $w'R$ is given by $\mu_r = w'M$ and $\sigma_r = w'\Sigma w$. Assuming that the market participants are risk-averse, they aim to minimize the variance of the portfolio return for a given expected value, resulting in the following optimization problem

$$\min_w w'\Sigma w \quad (5)$$

subject to the constraints

$$w'M = \text{const.}, \quad (6)$$

$$w'e = 1, \quad (7)$$

$$e' = (1, \dots, 1),$$

which ensure that the expected value of the assets equals the target expected value (6) and that the sum of the investment proportions is one (7) (budget constraint). Solving the numerical optimization problem for varying expected values of the portfolio return yields the efficient risk–return combinations (σ_r, μ_r) which describe the classical capital market line $\mu_r = CML(\sigma_r)$

derived from the Tobin separation theorem (Tobin, 1958). The solution of the optimization problem can also be determined by using the analytical expression in Merton (1972).

Solvency lines: “Admissible” risk-return combinations

Having identified the risk-return combinations that are actually attainable at the market as described by Equation (5), we next link them with the minimum requirements for the insurer’s investment performance based on a target safety level. To derive risk-return combinations (σ_r, μ_r) for the insurer’s investment strategy that are compatible with solvency requirements, we fix the insurer’s desired safety level. In what follows, we use the shortfall probability SP at time $t = 1$ and fix it to a prescribed maximum value α denoted “target shortfall probability” (note that the risk measure can as well be changed as is done in Eling et al. (2009), but closed-form solutions may not be derivable). The insurer’s fixed solvency level simultaneously impacts the achievable premiums as illustrated in Section 2 (Equation (4)). At time $t = 0$ the insurer sets the maximum value α as a target level, which is then communicated and revealed to the policyholders, who in turn adapt their willingness to pay based on this information. Based on the resulting amount of premium income (see Equation (4)), the insurer makes the actual investment decision (i.e., chooses a risk-return combination), which must be compatible with the announced target level α to preserve the policyholders’ trust and confidence. Thus, the real shortfall probability SP must not exceed the reported target shortfall probability α , i.e.,

$$SP = P(E_1 < 0) = \Phi \left(-\frac{E[E_1]}{\sqrt{V[E_1]}} \right) \leq \alpha,$$

where Φ stands for the distribution function of the standard normal distribution. With N_α denoting the α -quantile of the standard normal distribution, we obtain

$$N_\alpha = -\frac{E[E_1]}{\sqrt{V[E_1]}} = -\frac{A_0 \cdot (1 + \mu_r) - E[L_1]}{\sqrt{V[E_1]}}.$$

For a given σ_r (reflected in the variance of equity capital $V[E_1]$, see Section 2), we can solve for μ_r and obtain the so-called “solvency lines” $SolvL$ (the notion follows Eling et al., 2009) as closed-form solutions,

$$\begin{aligned}\mu_r = \text{SolvL}(\sigma_r) &= \frac{E[L_1] - N_\alpha \cdot \sqrt{V[E_1]}}{A_0} - 1 \\ &= \frac{(1-\lambda) \cdot \mu_C - N_\alpha \cdot \sqrt{A_0^2 \cdot \sigma_r^2 + (1-\lambda)^2 \cdot \sigma_C^2 - 2 \cdot A_0 \cdot (1-\lambda) \cdot \sigma_r \cdot \sigma_C \cdot \rho_{rC}}}{A_0} - 1,\end{aligned}\tag{8}$$

with $A_0 = E_0 + P_0(\alpha, s) - \pi^{re}$.

Thus, the solvency lines are (σ_r, μ_r) -combinations that satisfy Equation (8), which implies that for a given risk (here measured with the standard deviation σ_r of returns of the asset portfolio) the expected return μ_r needs to be at least as high as the right side of Equation (8) in order to ensure that the shortfall probability does not exceed the given target level α .

Maximizing shareholder value given attainable and admissible risk-return combinations

Among the typical firm objectives is the creation of shareholder value through risk- and value-based decision making regarding assets and liabilities. Toward this end, our model can be used for deriving the shareholders' maximum expected utility while maintaining a minimum (typically regulatory required) solvency level in order to protect the policyholders.

Let Ψ denote the shareholders' preference function to determine their expected utility dependent on the (σ_r, μ_r) -combination of the insurer's asset allocation. In its decisions regarding the investment portfolio, the insurer can take into account the shareholders' preferences within the limits of attainable and admissible investment opportunities, leading to the following optimization problem

$$SHV^{\max, \Psi} = \max_{(\sigma_r, \mu_r)} \Psi,\tag{9}$$

subject to the constraints

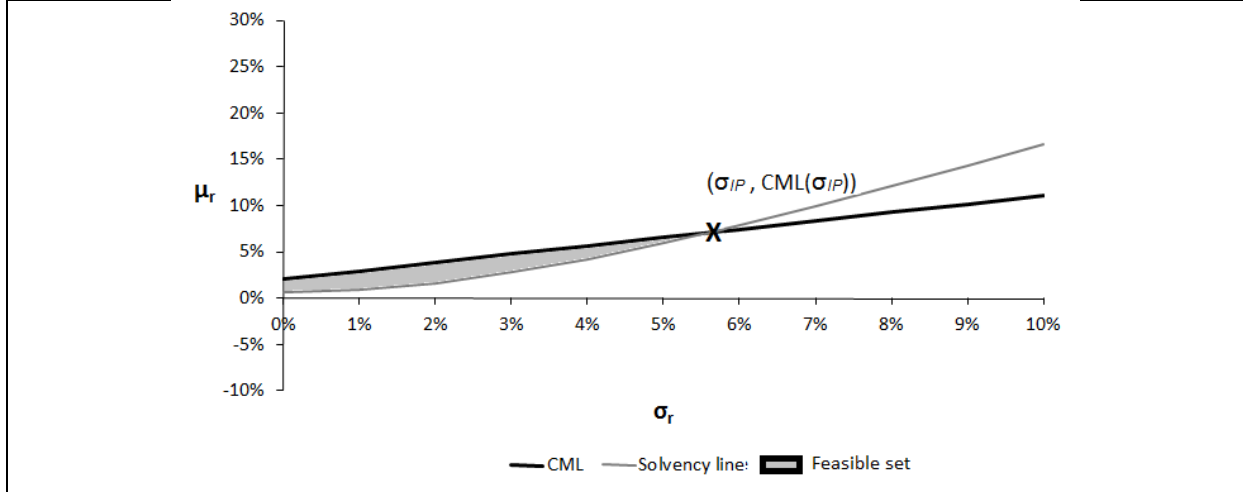
$$c = \left(\begin{array}{l} \sigma_r \geq 0 \\ \mu_r \leq CML(\sigma_r) \\ \mu_r \geq SolvL(\sigma_r) \end{array} \right).$$

The constraints ensure that only risk-return combinations are taken into consideration that are attainable at the market and that are admissible according to solvency restrictions. Figure 1 exemplarily displays the feasible set

$$\Upsilon = \{(\sigma, \mu(\sigma)) : \sigma \in [0, \sigma_{IP}], \mu(\sigma) \in [SolvL(\sigma), CML(\sigma)], SolvL(\sigma) \leq CML(\sigma)\},\tag{10}$$

with σ_{IP} denoting the σ -coordinate of the intersection point IP between the solvency line and the capital market line.

Figure 1: Illustration of a feasible set Υ of attainable (capital market line CML) and admissible (“solvency line”) risk-return asset combinations with intersection point IP (marked with “X”)



The optimal (σ_r, μ_r) -combination that solves the optimization problem (9) depends on the actual preference function Ψ . Since decisions based on any utility function can be well approximated by assuming mean-variance preferences (see Kroll et al., 1984), we assume that Ψ is based on the expected value and variance of the shareholders’ wealth at the end of the period (see, e.g., Braun et al., 2015; Gatzert et al., 2012).³ One should thereby take into account that shareholders generally have limited liability and thus at most lose their initial equity capital in case of insolvency.⁴ Due to limited liability, the shareholders’ wealth at $t = 1$ is given by $\max(0, E_1)$ and, hence, the preference function Ψ^1 is given by

$$\Psi_k^1 = E(\max(E_1, 0)) - \frac{k}{2} \cdot V(\max(E_1, 0)), \quad (11)$$

where k represents the risk aversion coefficient, with $k < 0$ implying a risk-seeking, $k = 0$ a risk-neutral, and $k > 0$ a risk-averse risk attitude.

³ Note that full compatibility of mean-variance analysis and expected utility theory is given in case of normally distributed returns or a quadratic utility function. Even if these conditions do not hold, decisions of any utility function can be well approximated by those based on mean-variance preferences as shown in Kroll et al. (1984).

⁴ In general, the premium level should thus correspond to the value of indemnity payments less the default put option arising from the shareholders’ limited liability, which we have not taken into account in pricing as we assume (in the sense of a behavioral-type approach) that policyholders are willing to pay a premium that depends on their risk sensitivity and that may thus exceed the expected payoff. These assumptions can also be changed, but closed-form solutions for the optimization problem as well as the solvency lines are no longer possible, such that one has to revert to numerical simulation approaches.

In general, the term $\max(0, E_1)$ can be interpreted as a normally distributed random variable censored at 0 from below. If Φ stands for the distribution function of the standard normal distribution and φ for the corresponding density function, the following holds for $Y \sim N(\mu, \sigma^2)$ (see, e.g., Greene, 2012):

$$E[\max(a, Y)] = \Phi(\alpha) \cdot a + (1 - \Phi(\alpha))(\mu + \sigma \cdot \lambda) \text{ and}$$

$$V[\max(a, Y)] = \sigma^2 \cdot (1 - \Phi(\alpha)) \cdot (1 - \delta + (\alpha - \lambda)^2 \cdot \Phi(\alpha)),$$

$$\text{with } \alpha = \frac{a - \mu}{\sigma}, \lambda = \frac{\varphi(\alpha)}{1 - \Phi(\alpha)} \text{ and } \delta = \lambda^2 - \lambda \cdot \alpha.$$

For the terms in Equation (11) we can thus derive the closed-form expressions

$$E[\max(0, E_1)] = E[E_1] \cdot \Phi(x) + \sqrt{V[E_1]} \cdot \varphi(x) \text{ and}$$

$$V[\max(0, E_1)] = V[E_1] \cdot \Phi(x) \cdot \left[1 - \left(\left(\frac{\varphi(x)}{\Phi(x)} \right)^2 + \frac{\varphi(x)}{\Phi(x)} \cdot x \right) + \left(x + \frac{\varphi(x)}{\Phi(x)} \right)^2 \cdot \Phi(-x) \right],$$

$$\text{with } x = \frac{E[E_1]}{\sqrt{V[E_1]}}.$$

The expected value $E[\max(0, E_1)]$ is monotonically increasing in $E[E_1]$ (and thus in μ_r) as well as in $V[E_1]$ (and thus in σ_r depending on the correlation, see also Equations (1) and (2)) as can be seen from the derivatives (see the Appendix for the detailed derivation)

$$\left(\begin{array}{c} \frac{\partial E[\max(0, E_1)]}{\partial E[E_1]} \\ \frac{\partial E[\max(0, E_1)]}{\partial V[E_1]} \end{array} \right) = \left(\begin{array}{c} \Phi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \\ = \frac{1}{2\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \end{array} \right),$$

and thus

$$\begin{aligned}
& \left(\frac{\partial E[\max(0, E_1)]}{\partial \mu_r} \right) = \left(\frac{\partial E[\max(0, E_1)]}{\partial E[E_1]} \cdot \frac{\partial E[E_1]}{\partial \mu_r} \right) \\
& \left(\frac{\partial E[\max(0, E_1)]}{\partial \sigma_r} \right) = \left(\frac{\partial E[\max(0, E_1)]}{\partial V[E_1]} \cdot \frac{\partial V[E_1]}{\partial \sigma_r} \right) \\
& = \left(\begin{array}{c} \Phi \left(\frac{E[E_1]}{\sqrt{V[E_1]}} \right) \cdot A_0 \\ \frac{1}{2\sqrt{V[E_1]}} \cdot \varphi \left(\frac{E[E_1]}{\sqrt{V[E_1]}} \right) \cdot (2 \cdot A_0^2 \cdot \sigma_r - 2 \cdot A_0 \cdot (1-\lambda) \cdot \sigma_c \cdot \rho_{rc}) \end{array} \right), \tag{12}
\end{aligned}$$

while the derivatives of the variance $V[\max(0, E_1)]$ cannot be expressed with a suitable closed-form expression.

Therefore, in case of risk-neutrality ($k = 0$) and non-positive correlation ρ_{rc} , $\Psi^1 (= E[\max(0, E_1)])$ is monotonically increasing in μ_r and σ_r . In addition, since the capital market line $CML(\cdot)$ as the upper bound of attainable risk-return combinations is strictly monotonically increasing under the common assumption of a positive Sharpe ratio, i.e. a positive risk premium per unit of standard deviation, it holds that

$$\max_{(\sigma, \mu) \in \Upsilon} \mu = CML(\sigma_{IP}).$$

Under these assumptions, the maximization problem in (9) is thus solved by the intersection point IP between the capital market line and the solvency line,

$$\left(\max_{(\sigma, \mu) \in \Upsilon} \sigma, \max_{(\sigma, \mu) \in \Upsilon} \mu \right) = (\sigma_{IP}, CML(\sigma_{IP})) \in \Upsilon,$$

which is an element of the feasible set Υ (see also Figure 1). Hence, in this case, maximizing the shareholder value while maintaining a minimum solvency status in order to protect the policyholders requires the insurer to invest in the risk-return combination given by the intersection point between the capital market line and the solvency line.

However, in case of a risk-averse or risk-seeking attitude, general statements about the impact of μ_r and σ_r on the preference function Ψ^1 cannot be derived due to a lack of knowledge about the gradient of the variance $V[\max(0, E_1)]$, which is why we later use numerical analyses to obtain more insight regarding the optimal risk-return combination that maximizes this preference function (in Section 4).

To simplify the optimization problem and in order to derive closed-form expressions, we next assume a preference function Ψ^2 that does not account for limited liability, i.e.,

$$\Psi_k^2 = E(E_1) - \frac{k}{2} \cdot V(E_1). \quad (13)$$

When considering the gradients of the preference function Ψ^2 ,

$$\begin{pmatrix} \frac{\partial \Psi_k^2}{\partial \mu_r} \\ \frac{\partial \Psi_k^2}{\partial \sigma_r} \end{pmatrix} = \begin{pmatrix} A_0 \\ -k \cdot A_0^2 \cdot \sigma_r + k \cdot A_0 \cdot (1-\lambda) \cdot \sigma_C \cdot \rho_{rC} \end{pmatrix}, \quad (14)$$

one can see that the function is monotonically increasing in μ_r , but that the effect of σ_r again depends on the correlation between assets and claims ρ_{rC} as well as the risk aversion parameter k . In particular, Ψ^2 is monotonically increasing in σ_r if and only if

$$-k \cdot A_0^2 \cdot \sigma_r + k \cdot A_0 \cdot (1-\lambda) \cdot \sigma_C \cdot \rho_{rC} \geq 0.$$

We first concentrate on the case where $k > 0$ (risk-aversion), which implies that this equation holds if the correlation satisfies

$$k \cdot A_0 \cdot (1-\lambda) \cdot \sigma_C \cdot \rho_{rC} \geq k \cdot A_0^2 \cdot \sigma_r \Leftrightarrow \rho_{rC} \geq A_0 \cdot \sigma_r / ((1-\lambda) \cdot \sigma_C). \quad (15)$$

To ensure this condition (and that $\rho_{rC} \leq 1$), the correlation must be either rather large (and positive, implying a good diversification between assets and liabilities), or the standard deviation of assets (in the nominator) should be rather small as compared to the standard deviation of liabilities, which can thus be rather restrictive. If $k \leq 0$ (risk-neutrality or risk-seeking behavior of shareholders), which is generally in line with the theoretical results in Gollier et al. (1997),⁵ there is either no restriction for the correlation (if $k = 0$) or in case of $k < 0$, the correlation must satisfy

$$\rho_{rC} \leq A_0 \cdot \sigma_r / ((1-\lambda) \cdot \sigma_C). \quad (16)$$

⁵ Gollier et al. (1997) theoretically show that (contrary to first intuition) a risk-neutral or risk-averse attitude of shareholders in case of limited liability (see preference function Ψ^1) corresponds to a risk-seeking or risk-neutral attitude when considering the preference function Ψ^2 in terms of unlimited equity capital E_1 . In particular, if shareholders are risk-neutral (or perfectly diversified) in case of limited liability, they will aim to maximize the expectation of a convex function of the equity capital E_1 (Gollier et al., 1997, p. 348), implying that shareholders exhibit risk-seeking behavior in investment decisions regarding the equity capital E_1 since they can only benefit from additional risk in E_1 . The shareholders' risk-averse attitude in case of limited liability leads to less extreme but similar results. Here, the optimal risk exposure of E_1 is also always higher than under full liability and often results in maximum risk taking.

Condition (16) holds in the special case where the correlation between assets and claims ρ_{rC} is zero or negative, thus implying an insufficient diversification in an asset-liability context (see Fischer and Schlütter, 2015). In particular, in this case low asset values are positively related to high liability values and hence risks are not well diversified, resulting in a risky asset-liability-profile (Fischer and Schlütter, 2015).

In both cases, i.e., in case of risk-neutrality or if Equations (15) or (16) are satisfied depending on k and the respective correlations, we can see that the preference function Ψ^2 is not only monotonically increasing in μ_r but also in σ_r . Identical argumentation as in case of Ψ^1 with risk-neutrality and non-positive correlation leads to the result that the intersection point IP between the capital market line and the solvency line solves the optimization problem (9) and thus maximizes shareholder value while maintaining a minimum solvency status to protect the policyholders. We conduct further analyses on the other cases in the following numerical analyses section.

4. NUMERICAL ANALYSES

In this section, we provide numerical examples based on the closed-form solutions derived in the previous section to illustrate the requirements for the insurer's capital investment strategy that satisfy the solvency rules as well as the associated maximum shareholder value under various conditions and to derive respective key drivers.

Input parameters

Input parameters are summarized in Tables 1 to 3. The parameters of the insurer in Table 1 are thereby based on the parameters of a German non-life insurer estimated in Eling et al. (2009) (except for the newly introduced parameters asset-claims correlation, premium loadings, fraction of proportional reinsurance, which were subject to robustness tests). Furthermore, the market portfolio and the capital market line are calibrated based on monthly time series from January 2004 to November 2015 of benchmark indices from the Datastream database that illustrate the available investment opportunities. Each benchmark measures the total investment returns for its asset on a Euro basis including coupons and dividends where applicable. As is done in Eling et al. (2009), we consider 11 indices with different regional focus in the four asset classes (stocks, bonds, real estate, and money market instruments) insurers typically invest in. The empirical risk-return profiles for all considered assets are given in Table 2, the associated variance-covariance matrix in Table 3. For the base case, the resulting capital market line CML is then given by the solution of the optimization problem (5):

$$\mu_r = 2.04\% + 0.34 \cdot \sigma_r. \quad (17)$$

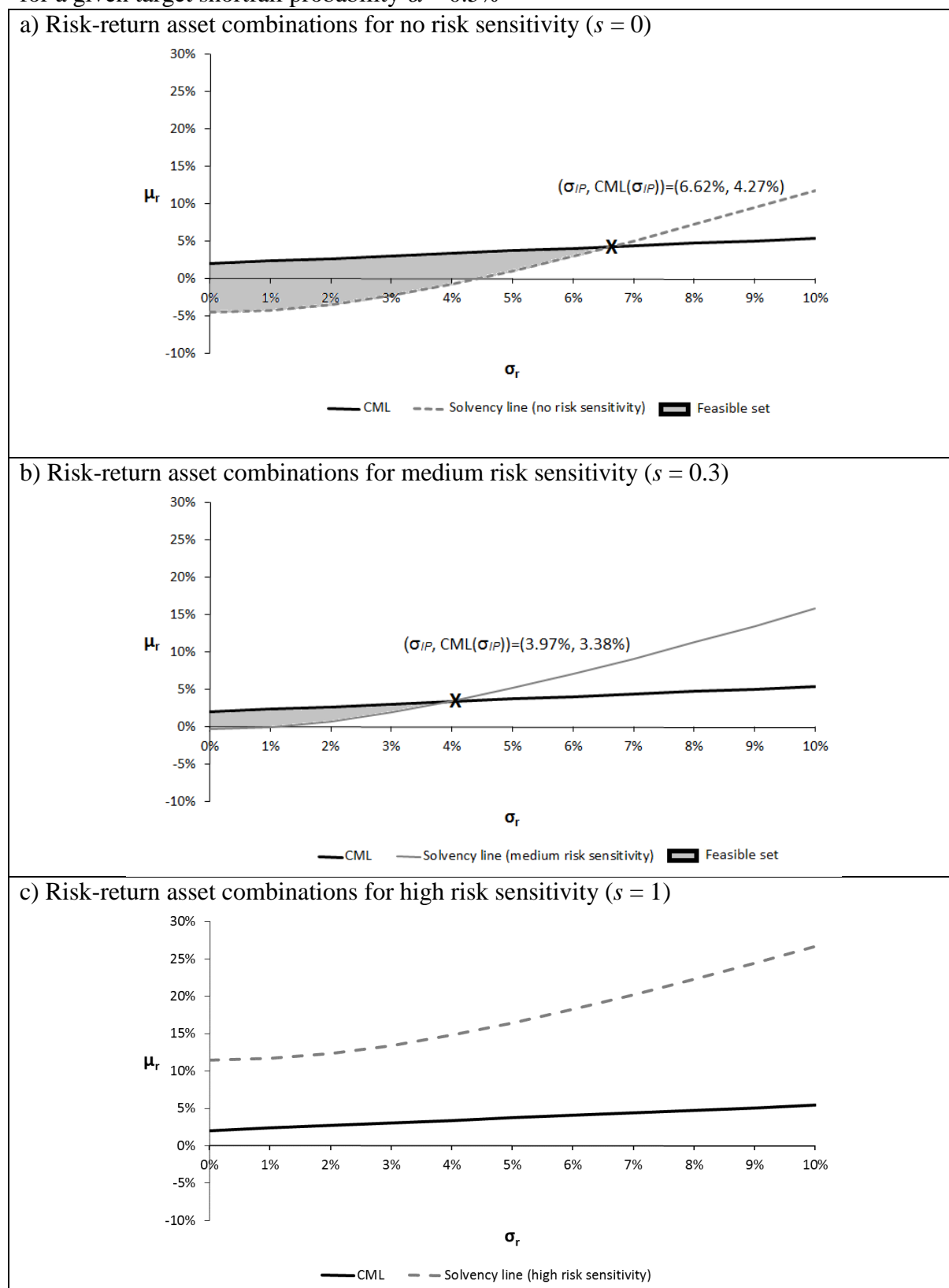
The impact of policyholder risk sensitivity on admissible risk-return asset profiles

Figure 2 displays *attainable* risk-return combinations based on market restrictions concerning the asset allocation as reflected by the capital market line (CML, solid black line, see also Equation (17)). Furthermore, *admissible* combinations are presented as defined by the solvency lines in Equation (8), which represent the minimum expected rate of return μ_r that the insurer has to achieve for a given standard deviation σ_r in order to satisfy the intended safety level given by the target shortfall probability α . Therefore, for its investment strategy the insurer has to choose risk-return combinations above the respective solvency line and below the capital market line, which represent the set Υ of attainable and admissible risk-return combinations (Equation (10)) as highlighted by the grey area in Figure 2.

Figure 2 shows that the admissible risk-return combinations (solvency lines) strongly depend on the policyholders' risk sensitivity. In particular, for higher risk sensitivities (going from $s = 0$ to 1 in the considered example, see Figures 2a) to c)), the solvency lines are shifted upwards for a given shortfall probability ($\alpha = 0.5\%$), until for $s = 1$ the solvency line in Figure 2c) lies above the CML, such that no possible allocation opportunities remain and the grey area disappears. Thus, the results emphasize that a more risk sensitive assessment of the solvency status reduces policyholder demand and hence the achievable premium income, implying that the insurer has considerably less flexibility for its asset allocation in order to fulfill the solvency requirements.

We also observe in Figure 2a) that in the case without policyholder risk sensitivity, expected asset returns may even be negative for low standard deviations, and the insurer would still satisfy the required safety level due to sufficient equity capital and premium loadings. Further sensitivity analysis emphasizes that reducing the initial equity capital or lowering the premium loading implies an upward shift of the solvency lines, such that negative expected returns are no longer permitted.

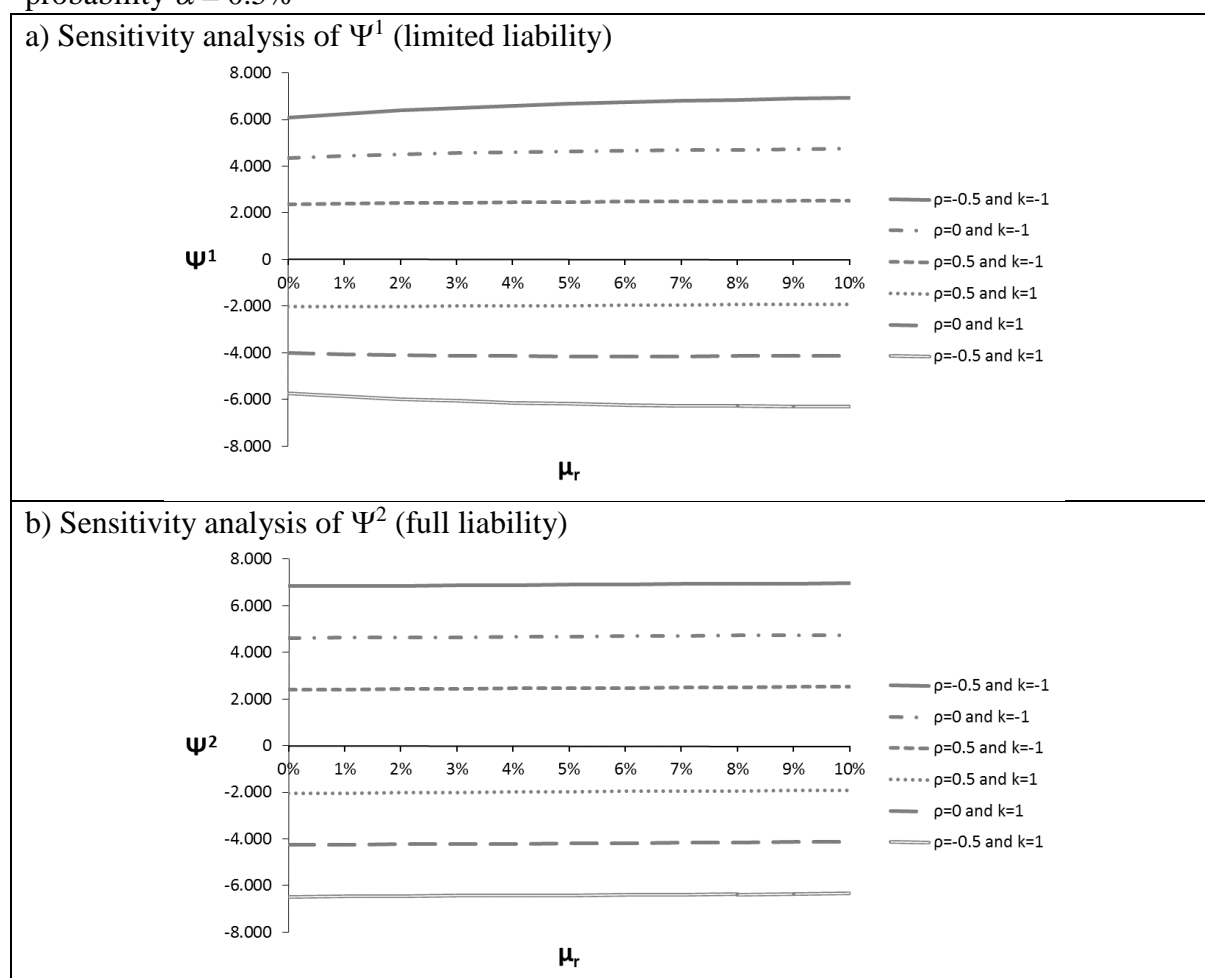
Figure 2: Attainable (capital market line CML) and admissible (“solvency line”) risk-return combinations given no ($s = 0$), medium ($s = 0.3$) and high ($s = 1$) policyholder risk sensitivity for a given target shortfall probability $\alpha = 0.5\%$



Maximizing shareholder value

The impact of the assets' risk-return combinations on the shareholders' risk preferences is not straightforward in all cases as already laid out in Section 3. However, in case of risk-neutral attitude, both preference functions Ψ^1 (in case of non-positive correlation) and Ψ^2 , i.e. $E[\max(0, E_1)]$ and $E[E_1]$, are monotonically increasing in μ_r and σ_r (see Equations (12) and (14)), implying that the intersection point between the capital market line and the solvency line IP solves the optimization problem (9) and thus maximizes the shareholders' expected utility as formally derived in Section 3. However, in case of risk-averse or risk-seeking attitude, further numerical analyses are needed. Therefore, Figures 3 and 4 as well as Table 4 exemplarily demonstrate the impact of μ_r and σ_r on the preference functions Ψ^1 and Ψ^2 for $k = -1$ (risk-seeking) and $k = 1$ (risk-aversion) and $\rho_{rC} = -0.5, 0, 0.5$ for medium policyholder risk sensitivity ($s = 0.3$) given a reported target shortfall probability of $\alpha = 0.5\%$. Both figures show that the results strongly depend on the shareholders' risk attitude as well as on the correlation between assets and liabilities (see also Table 4 regarding the sign of the impact).

Figure 3: Sensitivity analysis of Ψ^1 and Ψ^2 regarding μ_r for various correlations ρ_{rC} and risk attitudes k in case of medium policyholder risk sensitivity ($s = 0.3$) for a given target shortfall probability $\alpha = 0.5\%$



While the changes in the preference functions due to changes in μ_r in Figure 3 appear negligible, one can see in Table 4 that – consistent with what we formally showed in Section 3 – the impact of μ_r on Ψ^2 (full liability case) is positive in all considered examples, which implies that maximizing μ_r *ceteris paribus* also maximizes shareholder value. This is different for the preference function Ψ^1 , which assumes limited liability. One can observe that in case of risk-averse attitude ($k = 1$) along with a non-positive asset-liability correlation ($\rho_{rC} = -0.5, 0$), the preference function is first decreasing and then increasing again (see Figure 3a and Table 4), which stems from offsetting effects arising from the variance and expected value of the shareholders' wealth (see Equation (11)). While an increasing μ_r leads to an increase in $E[\max(0, E_1)]$ and $V[\max(0, E_1)]$, the latter increase is dampened for higher μ_r . Thus, the increase of $0.5 \cdot V[\max(0, E_1)]$ is weaker as compared to the increase in $E[\max(0, E_1)]$, thus implying that in case of risk-aversion with $k = 1$, $\Psi^1 = E[\max(0, E_1)] - 0.5 \cdot V[\max(0, E_1)]$ is first decreasing and then increasing for higher μ_r .

Table 4: The impact of μ_r and σ_r on the shareholders' preference functions Ψ^1 and Ψ^2 for various correlations ρ_{rC} and risk attitudes k in case of medium policyholder risk sensitivity ($s = 0.3$) for a given target shortfall probability $\alpha = 0.5\%$

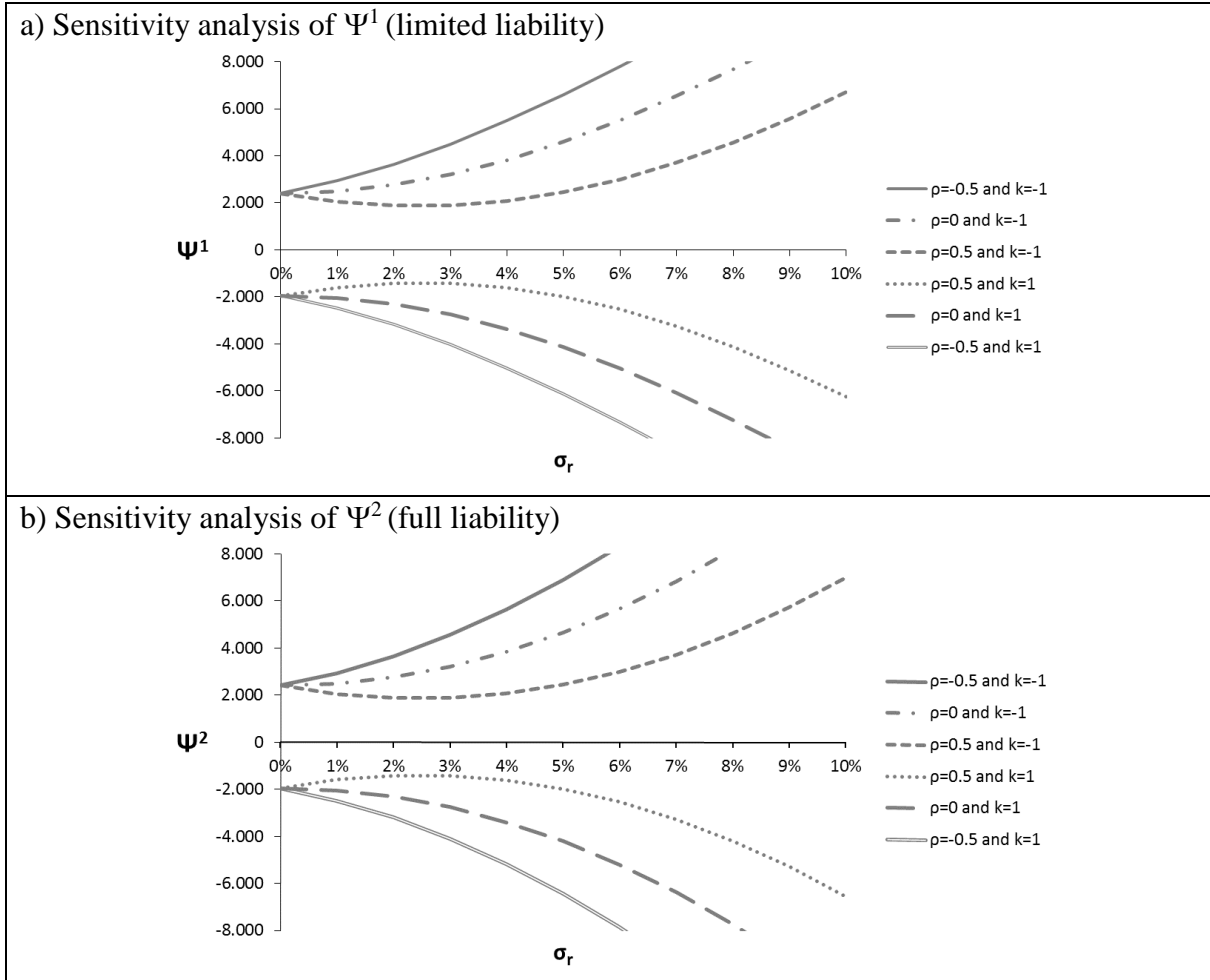
		Ψ^1 (given limited liability), impact of...		Ψ^2 (given full liability), impact of...	
		μ_r	σ_r	μ_r	σ_r
$\rho_{rC} = -0.5$	$k = -1$	↑	↑	↑	↑
	$k = 1$	↓↑	↓	↑	↓
$\rho_{rC} = 0$	$k = -1$	↑	↑	↑	↑
	$k = 1$	↓↑	↓	↑	↓
$\rho_{rC} = 0.5$	$k = -1$	↑	↓↑	↑	↓↑
	$k = 1$	↑	↑↓	↑	↑↓

Furthermore, Figure 4 shows that the impact of σ_r on Ψ^1 and Ψ^2 is very similar for both preference functions, and that it is exactly opposite for risk-averse and risk-seeking attitude (see also Table 4). For non-positive asset-liability correlations ($\rho_{rC} = -0.5, 0$), both Ψ^1 and Ψ^2 are monotonically increasing in σ_r for a risk-seeking attitude ($k = -1$) and decreasing in case of risk-aversion ($k = 1$). Hence, in this case the insurer *ceteris paribus* would have to choose the most ($k = -1$) or least ($k = 1$) risky asset allocations for maximizing shareholder value in the considered examples.

For a positive correlation $\rho_{rC} = 0.5$ and thus in the presence of positive diversification benefits, the results are not as straightforward as in case of a non-positive correlation, since the sign of the slope of both Ψ^1 and Ψ^2 changes, e.g. in case of a risk-seeking attitude ($k = -1$, see upper lines in Figure 4) from decreasing to increasing, and in case of risk-aversion ($k = 1$) vice versa, which is due to the non-monotonic behavior of $V[\max(0, E_1)]$ and $V[E_1]$ regarding σ_r . Due to the positive asset-liability correlations ρ_{rC} , both $V[\max(0, E_1)]$ and $V[E_1]$ are de-

creasing for low σ_r and increasing for high σ_r (in case of $V[E_1]$ this could be analytically derived in Equation (15)).

Figure 4: Sensitivity analysis of Ψ^1 and Ψ^2 regarding σ_r for various correlations ρ_{rC} and risk attitudes k in case of medium policyholder risk sensitivity ($s = 0.3$) for a given target shortfall probability $\alpha = 0.5\%$



In summary, the preference functions Ψ^2 is monotonically increasing in μ_r and σ_r in case of $k = 0$ (i.e. risk-neutral attitude), which is the same for Ψ^1 in case of $k = 0$ (risk-neutral attitude) and Ψ^2 in case of $k < 0$ (risk-seeking attitude) and non-positive correlations between assets and liabilities. This leads to the result formally shown in Section 3 that the intersection point between the capital market line and the solvency line IP solves the optimization problem for the present setting and thus maximizes the shareholders' preference functions under solvency constraints.⁶ The intersection point is marked with an "x" in Figure 2, which illustrates that a higher policyholder risk sensitivity leads to a lower intersection point IP and therefore to a reduction of maximum shareholder value in terms of the considered preference functions Ψ^1

⁶ The numerical analysis of Ψ^1 shows similar results, i.e. monotonically increasing behavior in μ_r and σ_r in case of non-positive correlation between assets and liabilities and $k \leq 0$, and hence implies that the intersection point also solves the shareholder maximization problem.

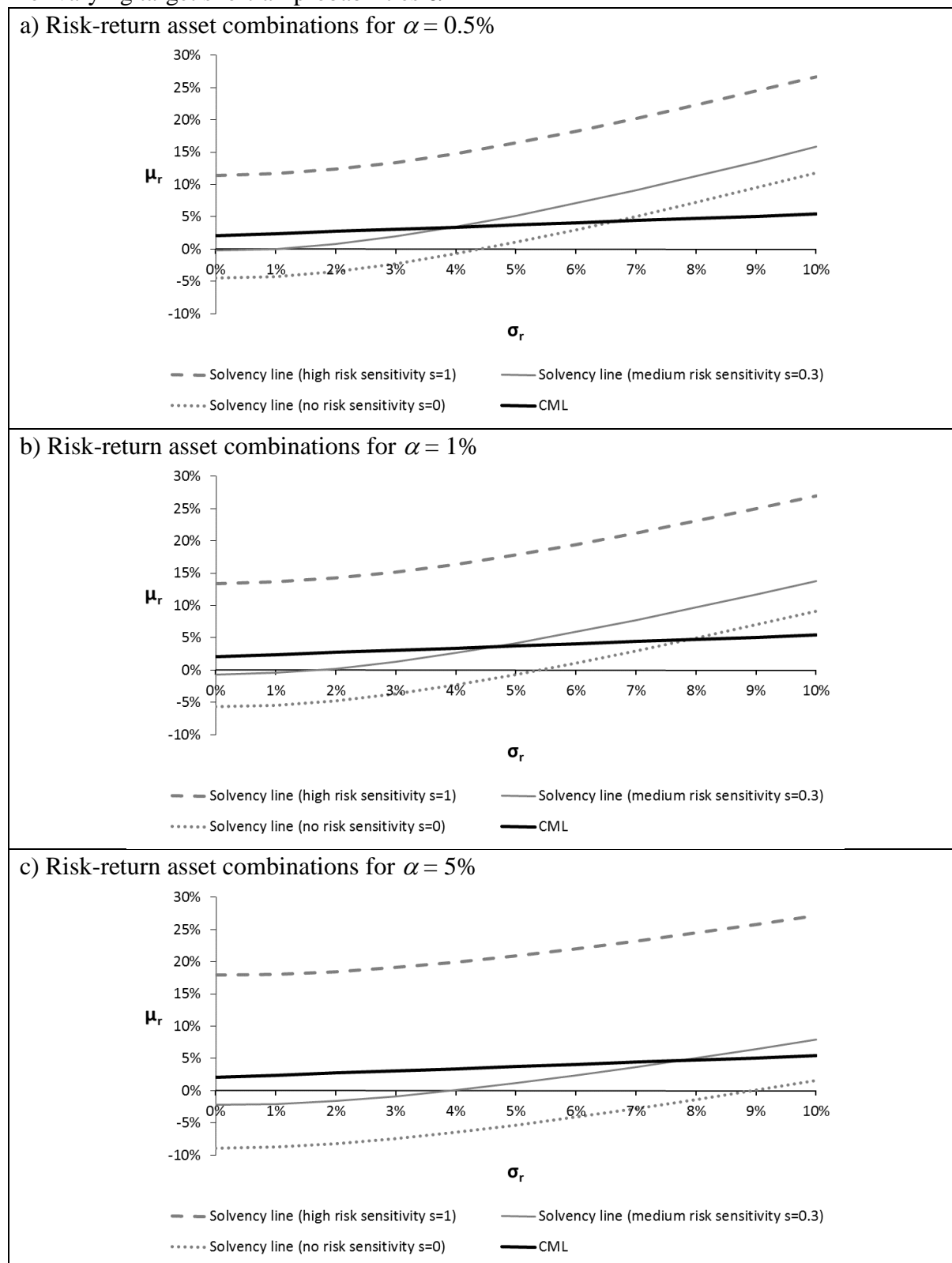
and Ψ^2 . In particular, for preference function Ψ^1 (limited liability) and risk-neutrality ($k = 0$), the maximum shareholder value decreases from $SHV^{\max, \Psi^1}(k = 0) = 294$ in case of no ($s = 0$) risk sensitivity (Figure 2a) to $SHV^{\max, \Psi^1}(k = 0) = 219$ for medium ($s = 0.3$) risk sensitivity (Figure 2b). In case of high ($s = 1$) risk sensitivity (Figure 2c), policyholders *ceteris paribus* pay considerably lower premiums, implying that no investment possibilities remain that fulfill the solvency requirements, and thus preventing the creation of shareholder value. For reasons of tractability regarding shareholder value maximization, in what follows we concentrate on these cases where the intersection point between the capital market line and the solvency line ($\sigma_{IP}, CML(\sigma_{IP})$) maximizes the shareholders' preferences.

The impact of the insurer's safety level

We next investigate the impact of the policyholders' risk sensitivity and thus their willingness to pay depending on the reported solvency levels by varying the target shortfall probability $\alpha = 0.5\%$, 1% , 5% in Figure 5. As can be seen when comparing Figures 5a) to c), the gap between the solvency lines for $s = 0$ and $s = 1$ increases considerably the higher the target shortfall probability. In particular, the solvency line neglecting the policyholders' willingness to pay (no risk sensitivity $s = 0$, lowest dotted line) shifts downward for a higher α , since it is easier for the insurer to fulfill the solvency requirements. In contrast, the solvency line for a high ($s = 1$) policyholder risk sensitivity shifts upward for a higher α , since the information of the higher shortfall probability reduces the achievable premium income (see Equation (4)). Therefore, it is increasingly difficult to satisfy the target solvency status communicated to the policyholders the weaker the target solvency standards are. In case of medium ($s = 0.3$) risk sensitivity, the solvency line remains nearly unchanged resulting from the sum of these two offsetting effects.

Overall, the results strongly emphasize that it is crucial to take into account the policyholders' risk sensitivity and thus the purchase behavior depending on the safety level, especially if insurers have to reveal their solvency status as required by Solvency II. For higher reported target shortfall probabilities, the willingness to pay by risk-sensitive policyholders declines and the premium income strongly decreases, which can lead to considerable difficulties in maintaining the desired solvency level and thus also in generating shareholder value.

Figure 5: Attainable (capital market line CML) and admissible (“solvency line”) risk-return combinations given no ($s = 0$), medium ($s = 0.3$) and high ($s = 1$) policyholder risk sensitivity for varying target shortfall probabilities α

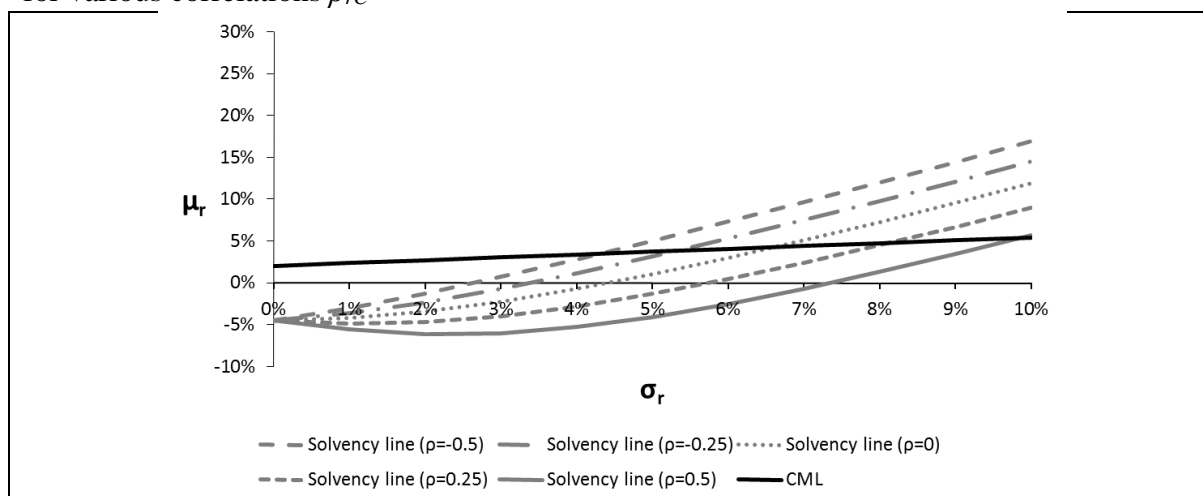


The impact of dependencies between assets and liabilities

To investigate the impact of dependencies between assets and liabilities on the requirements for the investment strategy and hence for maximizing shareholder value, we derive and compare the solvency lines under different correlations $\rho_{rC} = -0.5, -0.25, 0, 0.25, 0.5$, and thus different diversification levels in Figure 6. The base case is $\rho_{rC} = 0$, where asset and liability values are uncorrelated. In case of positive correlations, high asset values are positively related to high liability values and hence the risks are well diversified, resulting in a well-balanced asset-liability profile. Negative correlations, in contrast, represent an increasing riskiness of the asset-liability profile (in terms of the variance of equity capital) due to insufficient diversification, i.e. low asset values are positively related to high liability values (see Fischer and Schlütter, 2015).

Figure 6 shows that the convexity of the solvency line increases for higher correlations ρ_{rC} , while the intercept remains unchanged. A more negative linear dependence between assets and liabilities is thereby penalized by higher solvency capital requirements and reduces the area of acceptable and attainable risk-return combinations. It is thus crucial to take into account diversification effects between assets and the insurance portfolio, which considerably impact the range of admissible investment risk profiles. Furthermore, an insufficient diversification between in an asset-liability context in terms of a lower ρ_{rC} has negative consequences in regard to maximizing shareholder value, as the intersection point moves to the lower left, leading to restrictions in capital investment and a lower maximum shareholder value.

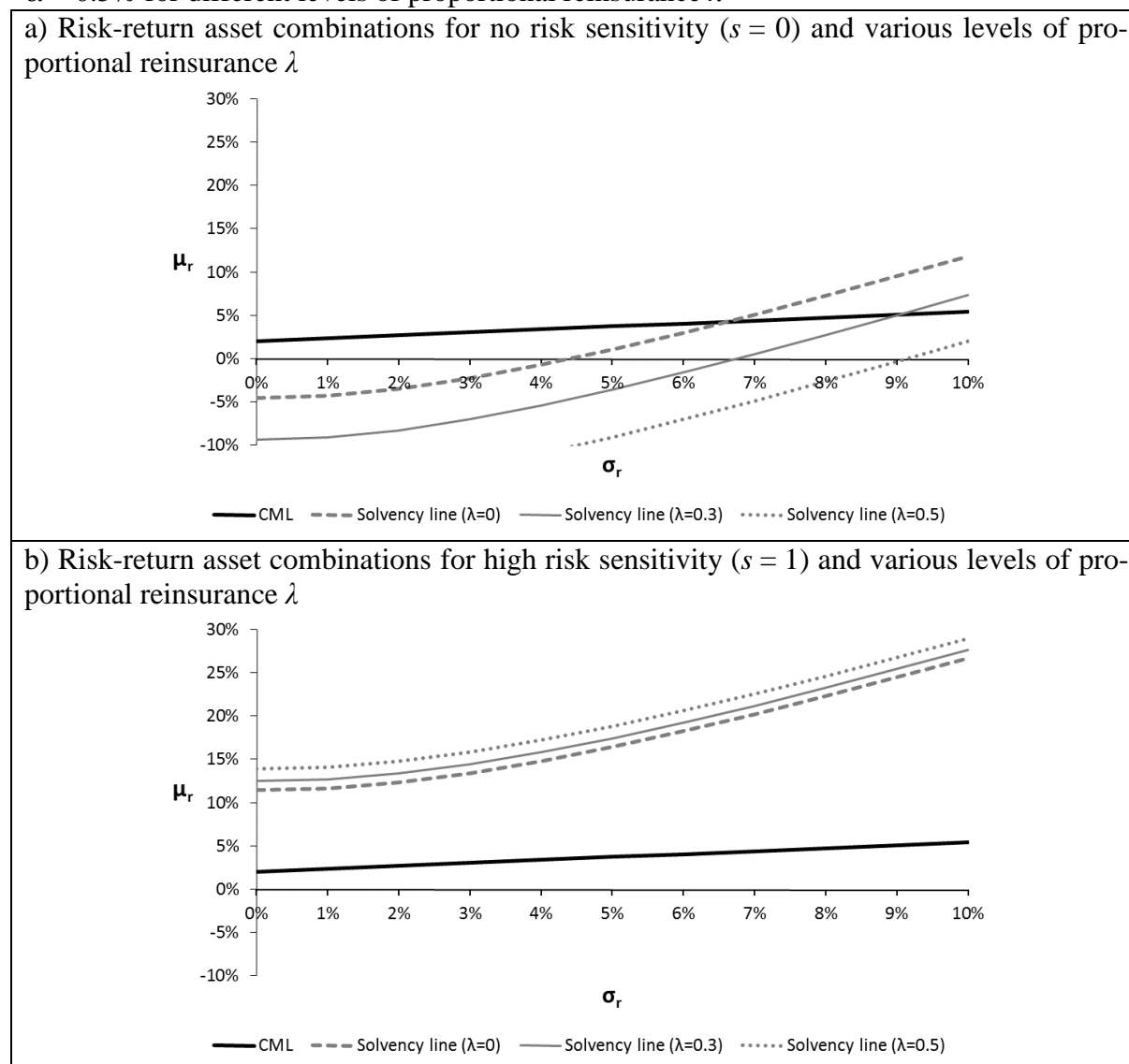
Figure 6: Attainable (CML) and admissible (solvency lines) risk-return combinations for a given target shortfall probability $\alpha = 0.5\%$ and medium ($s = 0.3$) policyholder risk sensitivity for various correlations ρ_{rC}



The impact of reinsurance decisions

We next focus on decisions regarding the liability side by studying the impact of reinsurance contracts on (optimal) asset portfolio combinations. Figure 7 exhibits risk-return asset combinations for different fractions of proportional reinsurance $\lambda = 0, 0.3, 0.5$.

Figure 7: Attainable (CML) and admissible (“solvency lines”) risk-return combinations for $\alpha = 0.5\%$ for different levels of proportional reinsurance λ



The results show that the solvency lines shift upward when increasing λ given a high ($s = 1$) risk sensitivity (Figure 7b), whereas in the case without ($s = 0$) risk sensitivity (Figure 7a) the solvency lines shift downward, thus allowing the insurer more flexibility in the asset allocation and creating opportunities for enhancing shareholder value. This opposite behavior can be explained by the fact that the reinsurance premium is fixed, whereas the premiums of the insurer vary depending on to the policyholders' risk sensitivity. In the case without ($s = 0$)

risk sensitivity, premiums and reinsurance prices are calculated in the same manner by using the actuarial expected value principle with the same loading. Hence, such decisions regarding the liability side generate more flexibility on the asset side if policyholders exhibit no (or a low) risk sensitivity. This is generally in line with Diasparra and Romera (2008) who study upper bounds for the shortfall probability in an insurance model where the risk process can be controlled by proportional reinsurance. They show i.a. (also assuming a fixed premium income) that a rising reinsurance level leads to decreasing upper bounds of the shortfall probability. However, our results show that taking into account the policyholders' willingness to pay given a high ($s = 1$) risk sensitivity, this effect is reversed. In particular, as in this case there are no admissible and attainable asset allocation opportunities, shareholder value cannot be created as the insurer has to pay the fixed reinsurance premiums from a much lower premium income caused by the policyholders' premium reduction (see Equation (4)). The higher relative costs for reinsurance thus lead to stronger restrictions on the asset side, and increasing the quota share of reinsurance λ further intensifies this effect.⁷ Overall, this illustrates the strong interaction between decisions on the asset and liability side and it emphasizes the relevance of taking into account policyholders' willingness to pay for insurance products also in the context of reinsurance contracts, for instance, given that solvency levels have to be reported.

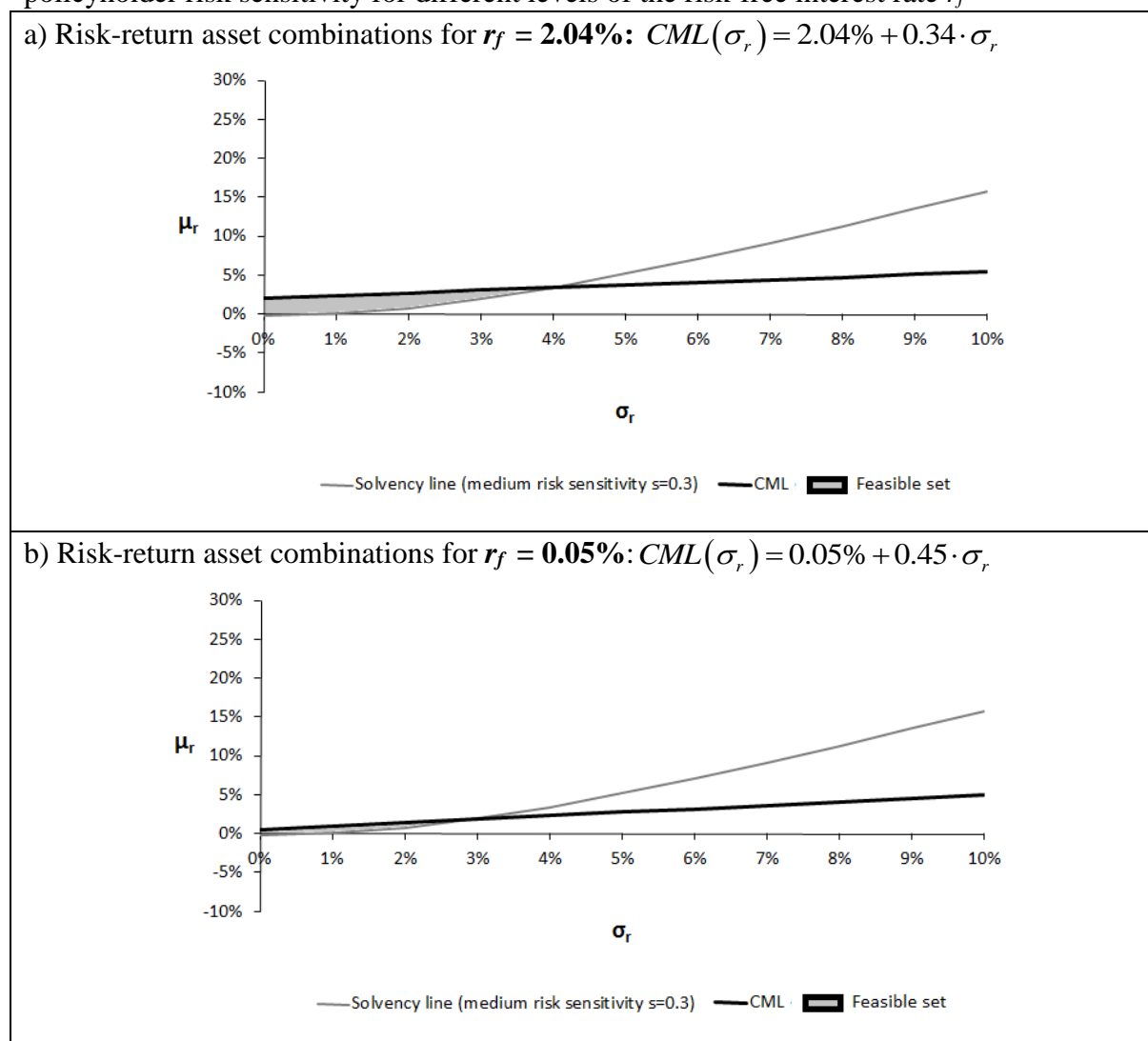
The impact of the risk-free interest rate

Lastly, since interest rates play an important role for solvency levels, especially against the background of currently very low interest rate levels, Figure 8 illustrates the attainable and admissible risk-return asset combinations for various levels of the risk-free rate.

When comparing Figures 8a) and 8b) it can be seen that the slope of the capital market line *ceteris paribus* increases for a decreasing risk-free interest rate from $r_f = 2.04\%$ (base case) to $r_f = 0.05\%$, i.e. the risk premium per unit of standard deviation increases, whereas the intercept of the capital market line decreases. Aggregating both offsetting effects leads to a reduction of the set of attainable and admissible risk-return combinations in the present setting and thus to a lower intersection point $(\sigma_{IP}, CML(\sigma_{IP}))$, implying a reduction of maximum shareholder value. These results emphasize that the current phase of low interest rates strongly restricts the insurer's range of admissible and attainable risk profiles regarding the asset investment, since the insurer cannot invest in riskier assets while maintaining the solvency level, resulting in a lower shareholder value.

⁷ Further analyses showed that, as expected, the solvency lines shift upward for an increasing reinsurance premium loading δ^{re} . Therefore, the insurer generally has to take into account that higher loadings δ^{re} increase the costs and thus reduce the area of attainable and admissible risk-return combinations and thus the maximum shareholder value.

Figure 8: Attainable (capital market line CML) and admissible (“solvency line”) risk-return combinations for a given target shortfall probability $\alpha = 0.5\%$ and given medium ($s = 0.3$) policyholder risk sensitivity for different levels of the risk-free interest rate r_f



5. SUMMARY

In this article, we provide a simplified internal model of a non-life insurer and use this model to study risk- and value-based management decisions regarding the capital investment strategy depending on various decisions regarding the liability side. We derive minimum capital standards for the insurer’s asset allocation based on a fixed solvency level, since adjusting the asset side to satisfy solvency capital requirements should generally be easier than short-term adaptations regarding equity capital or the liability side as pointed out by Eling et al. (2009). In this setting, we link the admissible (i.e. satisfying solvency requirements, following Eling et al. (2009)) risk-return combinations of the insurer’s asset portfolio to actually attainable allocation opportunities at the capital market using Tobin’s (1958) capital market line. We thereby extend previous work in several important ways: We explicitly include the policyholders’

willingness to pay, which to the best of our knowledge has not been done so far in this context, take into account the insurer's reported target safety level and the policyholders' risk sensitivity, and combine this model with a shareholder value maximization approach for risk and value-based management. We further study the impact of asset-liability dependencies, the influence of proportional reinsurance contracts and the impact of the risk-free interest rate, which considerably influence the set of attainable and admissible risk-return combinations as well as the maximum achievable shareholder value.

To study the impact of decisions with respect to assets and liabilities on shareholder value, we assume mean-variance preferences for shareholders and consider the cases with and without limited liability. To gain deeper insight, we conduct comprehensive analytical and numerical analyses and formally show that the intersection point between the capital market line and the solvency line maximizes shareholder value in certain scenarios (in case shareholders exhibit risk-neutral attitude or in case of unlimited liability with risk-seeking attitude with non-positive asset-liability correlations), while optimal solutions in other cases require numerical analyses.

Our results further show that the consideration of the policyholders' willingness to pay depending on their risk sensitivity is crucial. In case of high risk sensitivity, a low solvency status communicated to the policyholders can considerably decrease the premium income, which causes difficulties in maintaining the solvency level and ultimately reduces shareholder value due to an insufficient premium income and a reduced flexibility to invest in riskier assets. This is especially relevant in the future, where insurers have to report their solvency status according to Solvency II. To satisfy the interests of both shareholders and policyholders, our approach can thus be used for balancing shareholder value and risk taking. In particular, depending on the policyholders' risk sensitivity, it is advisable for firms to closely monitor the reactions of policyholders to safety levels and to possibly enhance the solvency level in order to generate more flexibility for the investment strategy and to increase shareholder value.

In addition, our numerical analyses reveal the impact of different key drivers on admissible risk-return combinations and hence shareholder value. We find that a negative correlation between assets and liabilities can considerably reduce the area of attainable and admissible risk-return combinations, emphasizing that diversification between assets and the underwriting portfolio can generate more flexibility regarding the risk profile of the capital investment, which also implies the potential of generating a higher shareholder value. We further find that the set of attainable and admissible investment opportunities and thus the maximum shareholder value only increases for higher reinsurance portions if policyholders are not risk sensitive. In particular, this effect is reversed and the shareholder value generally decreases when taking into account the policyholders' willingness to pay, as the insurer's premium income,

which must be used to pay the fixed reinsurance premiums (not subject to risk sensitivity influences), is reduced, implying a reduction of the set of available risk-return asset combinations. Furthermore, an analysis of the impact of the risk-free interest rate shows that the current phase of low interest rates strongly restricts the insurer's investment opportunities and considerably reduces shareholder value.

Overall, our results strongly emphasize the strong interaction between decisions regarding the asset and the liability side, and they underline the importance of considering the policyholders' demand for insurance products given that solvency levels have to be reported, which should be taken into account by insurers in the context of their risk- and value-based management decisions.

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APPENDIX

Derivatives of $E[\max(0, E_1)]$:

$$\begin{aligned}
& \frac{\partial}{\partial E[E_1]} E[\max(0, E_1)] \\
&= 1 \cdot \Phi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) + E[E_1] \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \cdot \frac{1}{\sqrt{V[E_1]}} + \sqrt{V[E_1]} \cdot \frac{\partial}{\partial \frac{E[E_1]}{\sqrt{V[E_1]}}} \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \cdot \frac{1}{\sqrt{V[E_1]}} \\
&= \Phi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) + \frac{E[E_1]}{\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) - \frac{E[E_1]}{\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) = \Phi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right),
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\partial}{\partial V[E_1]} E[\max(0, E_1)] \\
&= E[E_1] \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \cdot \left(-\frac{E[E_1]}{2(V[E_1])^{1.5}}\right) + \frac{1}{2\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \\
&\quad + \sqrt{V[E_1]} \cdot \frac{\partial}{\partial x} \varphi(x) \Big|_{x=\frac{E[E_1]}{\sqrt{V[E_1]}}} \cdot \left(-\frac{E[E_1]}{2(V[E_1])^{1.5}}\right) \\
&= E[E_1] \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \cdot \left(-\frac{E[E_1]}{2(V[E_1])^{1.5}}\right) + \sqrt{V[E_1]} \cdot \left(-\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \cdot \left(-\frac{E[E_1]}{2(V[E_1])^{1.5}}\right) \\
&\quad + \frac{1}{2\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right) \\
&= \frac{1}{2\sqrt{V[E_1]}} \cdot \varphi\left(\frac{E[E_1]}{\sqrt{V[E_1]}}\right),
\end{aligned}$$

where we use $\frac{\partial}{\partial x} \varphi(x) = -x \cdot \varphi(x)$, $x \in \mathbb{R}$.