Variable Annuities with fees tied to VIX

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Abstract

A typical variable annuity (VA) consists of two phases. First, the policyholder makes regular payments into a fund managed by the insurer (accumulation phase). Then, she receives income from the insurance company with some minimum guarantees for some given period (payment phase).

As Solvency II regulation requests a market consistent value of liabilities, VAs issuers face new challenges. Reserve requirements for VA products are highly "scenario dependent" and are not known at time of issue: technical provisions and solvency capital requirements change over time depending on market conditions (e.g., interest rates, volatility, equity market). The issuer's income on most VAs is often computed as a fixed percentage fee of the fund under management which means that it is high (resp. low) when the market goes up (resp. down). The market value of embedded guarantees moves in opposite direction, thus it becomes smaller when it is most needed. When equity goes down, guarantees become expensive. Moreover, the volatility typically increases so that delta hedging programs need then to be rebalanced more often (more costly), and hedging programs that require to purchase options become very expensive as option prices increase when volatility increases.

We propose a new product design that allows to better align the guarantees' values and hedging costs when the market environment is changing. The proposed design has a state-dependent fee linked to the traded VIX index and is inspired from designs of recent VAs issued in the U.S. insurance market.

Key words: Variable annuities, Solvency II, Market valuation, hedging costs, VIX, state dependent fees.

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Introduction

A variable annuity (VA) is a tax-deferred unit-linked insurance product that provides various forms of guarantee riders to investors via equity participation in a collective investment selected by the policyholder. Typically, variable annuities' investors make payments until their retirement and then begin receiving regular retirement income from the insurance company. VA guarantees can be classified into two broad types: guaranteed minimum death benefits (GMDBs) and guaranteed minimum living benefits (GMLBs). The GMLBs include guaranteed minimum accumulation benefits (GMABs), guaranteed minimum income benefits (GMIBs) and guaranteed minimum withdrawal benefits (GMWBs).

VAs issuers face new challenges as Solvency II regulation requests a market consistent valuation of liabilities. Reserve requirements for VA products are then highly "scenario dependent" and are not known at time of issue. Technical provisions and solvency capital requirements change over time depending on market conditions (e.g., interest rates, volatility, equity market). The issuer's income on most VAs is typically computed as a fixed percentage fee of the fund under management and fluctuate as a function of the underlying value. VAs are long term contracts and market conditions will automatically change throughout the term of the contract. For example, the market value of guarantees goes up when equity goes down and volatility goes up. Moreover, when equity goes down, the volatility typically increases. Delta hedging programs need then to be rebalanced more often, which is costly. Also, hedging programs that require to purchase options become expensive as option prices increase when volatility increases. Thus, as fees are typically a fixed percentage of the fund value, the VA issuers' income becomes smaller when it is most needed.

In this paper, we study a new product design that allows to better align the guarantees' market value and the corresponding hedging costs when the market environment is changing. To do so, we propose to link the issuer's income to the volatility index VIX. We show that it is good for insurers as it allows for a better match in periods of high volatility between the hedging cost and the guarantees' values. We also discuss whether it can also be advantageous to policyholders. The answer is not so clear. On one hand, they will pay lower fees during long periods of low volatility markets. On the other hand, the design is more complicated and harder for policyholders to understand. We also note that by accepting to share the hedging difficulties of the insurer when markets are volatile, they contribute to reducing their risk and thus the probability that their benefits will ultimately be paid, which is a positive outcome of this new fee design.

Our motivation to propose this fee design comes directly from recent VAs that have been offered in the industry. We cite for instance the SunAmerica GLWB issued in 2011, with 8% rollup and ratchet and rider fees linked to the Volatility Index (VIX) reported by the Chicago Board Options Exchange, and the latest variable annuity contracts offered by American General Life in 2014 (see the prospectus dated May 1, 2014 of the "Polaris Variable Annuities Choice IV").[1] In this latter contract, the fee rate adjustment is also tied to changes in VIX. We propose to model and study the pros and cons of charging state-dependent fees in VAs that depend on the volatility level. We expect that the fees will be lower on average for the policyholder and that they will provide better matching for the insurer between the actual value of the guarantee and the premium collected by the insurer. We thus expect to improve the hedging program of VAs for the insurer using state-dependent fees. Such an approach challenges the existing VA modeling: see Coleman, Li, and Patron (2006), Milevsky and Salisbury (2006), Bauer, Kling, and Russ (2008), Chen, Vetzal, and Forsyth (2008), Dai, Kuen Kwok, and Zong (2008), Lin et al. (2009), Hürlimann (2010), and references therein. Almost all the research to date has focused on a fixed fee rate, i.e. the policyholder has to pay a constant percentage of the fund. The pricing and hedging of VAs with state-dependent fees linked to volatility, however, are realistic and attractive in the insurance industry.

The proposed VA design has a state-dependent fee as it depends on the value of the traded VIX index over time. It is in the same spirit of the recent work of Bernard, Hardy, and MacKay (2013); MacKay, Augustyniak, Bernard, and Hardy (2016); Bernard and MacKay (2014); Delong (2014); Moenig and Zhu (2016) who study fees linked to the fund value in another context. In these papers, fees are paid only when the fund goes above some threshold value. However, this type of fee payment is subject to moral issues as the insurer is often also the manager of the fund. A state-dependent fee that decreases when the fund goes up reduces his incentives to increase the fund value. In particular, if the fund value is close to the threshold, the insurer has incentives to misreport the fund value or create a last minute loss to be just below the threshold. Our proposed new design does not suffer from the same issues as the VA fee is now linked to a market index and thus a variable that is not subject to manipulation risk.

In Section 1, we present an example of a Variable Annuity with fees linked to VIX that was issued in the US industry in 2014 (Polaris Choice IV contract). In Section 2,

[1] Fee-determination systems and methods relating to insurance products that are designed to incorporate a measure of volatility into the fees are subject to a U.S. Patent and Trademark Office (Publication number: US 2010/0325063 A1, Stone (2010)).

we develop a toy model motivated by market data from 2004 to 2015 for S&P 500 and VIX. In Section 3, we study on simplified examples the advantages of a VA with fees tied to VIX. In Section 4, we provide a rigorous treatment of the pricing and hedging of the state-dependent fee annuity. We illustrate the results on the Polaris Choice IV contract that motivated the study of fees linked to the VIX index in Section 5. Section 6 concludes.

1 VAs with fees tied to VIX

This section is inspired by the CBOE white paper on that topic (Hede and Wiesner (2013)). We first motivate the study with the example of the Polaris Choice IV contract. We describe this contract and isolate two features from the contract and simplify them. We then use market data to study the VA contract if it were issued on December 31st, 2004 with an accumulation phase of 10 years.

1.1 Example: Polaris Choice IV Contract

We first introduce the Polaris Choice IV contract. It is a variable annuity launched by American General Life in May 2014. In the Accumulation Phase, it builds assets on a tax-deferred basis. In the Income Phase, it provides the policyholder with guaranteed income through annuity income payments. The Polaris Choice IV allows the policyholder to invest in "Variable Portfolios," which like mutual funds, have various investment objectives and performance. The amount of money that accumulates in the contract then depends on the performance of the selected Variable Portfolios by the policyholder.

To grow and secure a maturity income or lifetime annuity income, the policyholder elect to pay for optional Living Benefits for an additional fee. These Living Benefits offer protection in the event that the contract value declines due to unfavorable investment performance. The two typical living benefits from the Polaris Choice IV are called Polaris Income Plus and Polaris Income Builder. Even though the Polaris Choice IV allows subsequent purchase payments with a fee adjustment, we ignore this feature for the ease of presentation. The policy we consider throughout this paper is thus a single premium VA.[2]

The fee for Polaris Income Plus and Polaris Income Builder is assessed against the

[2]Multiple premiums may add lots of complexity in the computation of the fair fee. See Bernard, Cui, and Vanduffel (2016).

Income Base and deducted from the contract value at the end of each benefit quarter. The Initial Annual Fee Rate is guaranteed not to change for the first benefit year. Subsequently, the fee rate may change quarterly subject to changes in the market parameters that are identified in the contract. Specifically, fee adjustments are based on a non-discretionary formula tied to the change in the Volatility Index (VIX), an index of market volatility reported by the Chicago Board Options Exchange. In general, as the average value of the VIX decreases or increases, the fee rate will decrease or increase accordingly, subject to the maximum and the minimum described in the contract. The non-discretionary formula used in the calculation of the Annual Fee Rate applicable after the first Benefit Year is of the form

$$\text{Initial Annual Fee Rate} + 0.05\% \times (\text{Average Value of the VIX} - 20)\,, \tag{1}$$

where the initial annual fee rate is set to 1.1% for a policyholder aged from 65 to 85 by contract in the Polaris Choice IV contract.

1.2 Simplified Contract

To understand the impact of such a state-dependent fee, we first study toy contracts that are simplified versions of the Polaris Choice IV contract. The first one contains a GMAB option. The second one contains a simplified GMWB option.

Option 1: GMAB. The guaranteed minimum accumulation benefit (GMAB) rider pays the greater of the fund value F_T and the guarantee amount K_T at the maturity date T. We can write the policy benefit $\max[F_T, K_T]$ in two ways:

$$\max[F_T, K_T] = F_T + \max[K_T - F_T, 0] = K_T + \max[F_T - K_T, 0]\,. \tag{2}$$

The guarantee amount K_T in the above GMAB rider is determined at inception.

The put option approach as shown in the first part of the equation (2) treats the liability of a GMAB as equal to the value of the underlying asset plus an additional put option. This put option plays the role of the guarantee that future payments will never fall below a certain minimum value K_T. In the second part of the equation, the call option approach decomposes the liability into the value of guaranteed contractual benefits plus an option value reflecting the participation in the performance of the fund value. Funds can be be saved in a risk-free account to hedge the guaranteed amount. However, dynamic investment in the underlying fund is needed to hedge the call option. In this paper, we use the put option approach, which is also the usual presentation in the VA issuer's balance sheet (see Dullaway and Needleman (2003)).

Thus, there are two components in the liability of the contract. First, the liability associated with the investment fund F_T is straightforward to value using the performance of the market index over time adjusted by the fees collected from the fund F_T. Specifically, F_0 is the initial amount invested in the fund and then fully invested in the traded market index at the inception of the contract. To provide the GMAB guarantee, the insurer charges a fee rate c_t, $t \in [0, T]$, from this fund. In particular, a flat average rate, i.e. $c_t = c$, is usually determined by the insurer at the inception of the contract.

We now focus on the valuation of the put option. Based on the put option approach, at maturity T, the liability of the GMAB can be hedged as follows. The amount F_0 is invested in the market index S at inception of the contract and the fees that are continuously collected from the fund between 0 and T must be invested according to a replication strategy of the put option payoff at maturity T

$$\underbrace{F_T\left((c_t)_{t\in[0,T]}\right) + P_T\left((c_t)_{t\in[0,T]}\right)}_{\text{liability of a GMAB}} \qquad \longleftrightarrow \qquad n_T\left((c_t)_{t\in[0,T]}\right) S_T + \quad \underbrace{\text{fees}\left((c_t)_{t\in[0,T]}; T\right)}_{\text{fees deducted from the fund in}[0,T]}, \tag{3}$$

where $(c_t)_{t\in[0,T]}$ denotes the fee rate charged at time t from the fund value between $[0, T]$, P_T denotes the payoff of the put option at T written on the underlying fund value F_T and thus depending on the fees collected. fees $\left((c_t)_{t\in[0,T]}; T\right)$ denotes the accumulated value at time T of the fees collected over the entire period $[0, T]$ and n_T is the number of shares of the risky asset S. We assume that the fund value is fully invested in a market index, thus F_T is hedged by the position $n_T S_T$. On the right hand side of (3), the fees charged during $[0, T]$ are used to hedge the liability of the put option at the maturity of the contract.

To study whether the liability of the GMAB (left-hand side of (3)) can be well hedged by the right-hand side of (3), we compute the respective market values of both sides at intermediary dates. In most VAs issued in the industry the market value of the fees collected over time does not follow the market value of the guarantees embedded in the VA contract. As mentioned already in the introduction, this mismatch is due to the fact that when the put option is very valuable, i.e., when the underlying fund is low, the fees collected using a constant percentage of this underlying fund are typically small. Our goal is to address this mismatch and facilitate the hedging of the put option that appears on the left-hand side of (3) with the fees collected over the life of the contract.

First, observe that the cost of the guarantee put option depends on the market states, such as the fund value and the volatility level. Thus, charging a flat fee rate may not be a good approximate to the true cost of the liability. Instead, we expect that a state-dependent fee structure depending on the market state, such as the volatility level, can provide a better match between the fees collected and the true cost of the liability and can at the same time reduce the hedging risk for the issuer.

As already mentioned earlier, our study is inspired from a contract (Polaris Choice IV) that has been issued in the US variable annuity industry and that is linked to the VIX index. However, for the ease of exposition, we voluntarily simplify the design of the contract and consider a toy contract with the following assumptions on its features:

- A 10-year variable annuity with a GMAB rider;
- with a one-time premium collected at the beginning that is 100% invested in a market index;
- the guaranteed amount K_T^{GMAB} is contracted at inception: we use the value of the initial premium;
- at the end of the 10th year, the policyholder can take out the greater amount of the fund value and the guaranteed amount.

This toy contract will be useful to understand the implications of a state-dependent fee linked to the volatility. These assumptions will then be further discussed towards the end of the paper.

Option 2: Simplified GMWB. As a second feature, we also consider a simplified guaranteed minimum withdrawal benefit (GMWB) rider to illustrate the performance of matching the liability with the state-dependent fee. It guarantees the policyholder's ability to get the premium back by making periodic withdrawals regardless of the impact of poor market performance on the account value. The maximum annual withdrawal amount is specified to support the benefit period in the contract. The GMWB resembles a put option in providing downside protection but is much more complex to value as the withdrawal times are random in practice and the amounts can also be chosen by the policyholder (Liu (2010)). To simplify the product, we look at a GMWB rider with the following features:

- A 10-year variable annuity with a GMWB rider;

- No withdrawals allowed in the first five years;
- At the end of the remaining 5 years, the guaranteed withdrawals are taken, equal to 20% per annum of the initial deposit at the beginning of the 6th year.

The simplified GMWB can be evaluated as a sum of put options and thus a similar approach as for the GMAB option above can be adopted.

1.3 Methodology

As a proxy for the value of a put option in the GMAB option or in the GMWB option at some intermediary date, we use the Black Scholes model with volatility and interest rates parameters that are changing over time (the VIX index value at time t is used for the volatility parameter, the 3 month treasury bill is used for the risk-free rate). We can compute the fund value from the history of the S&P500 value. The fee rate is used as dividend rate in the Black-Scholes formula as it decreases the underlying fund value.

We also compute the expected fee deducted from the market fund in $[t, T]$ to compensate for the put option. It can be computed at t as

$$\xi(t) := \mathbb{E}^Q \left[\int_t^T e^{-r(s-t)} c_s F_s ds | \mathcal{F}_t \right]. \tag{4}$$

We also define the value at time t of the fees already collected between 0 to t, for some $t \in [0, T]$, as

$$\text{fees}(0, t) := \int_0^t e^{r(t-s)} c_s F_s ds. \tag{5}$$

We then compare

$$\text{fees}(0, t) + \xi(t). \tag{6}$$

with the value of the liability at time t.

2 Modelling the joint distribution between the fund and the VIX index

To help us better understand the implications of linking VA fees to the VIX index such as in (1), we assume that the fund is fully invested in the S&P500 index and thus it amounts to study the joint distribution between VIX monthly returns and S&P 500 monthly returns.

2.1 Raw data for the couple (VIX,S&P500)

In Figure 1, we represent raw data to illustrate the negative correlation between the two processes between 1990 and 2010.

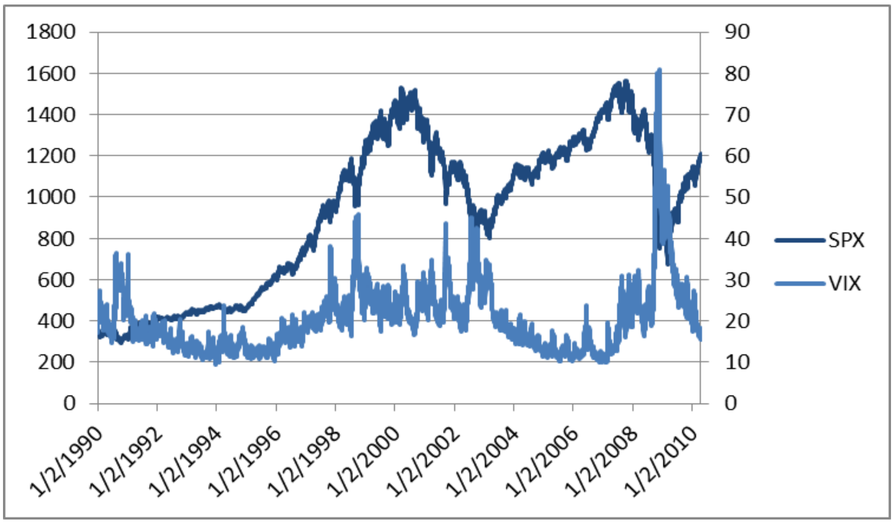

Figure 1: S&P500 and implied volatility (VIX) daily prices (CBOE white paper, Hede and Wiesner (2013))

2.2 Joint Distribution of (VIX, S&P500)

We then look at monthly returns on VIX and on S&P 500 and study the joint distribution of the couple over time for non overlapping months (130 observations) between December 31st, 2004 and June 30, 2015. Let us denote them by (x_i, y_i) for $i = 1, \cdots, 130$.

Fitting the dependence between monthly returns on VIX and on S&P 500. To identify the dependence among them, we transform the data first to rank data (by applying the empirical distributions to the original data) and then apply the quantile function of the standard normal distribution. As a result, the transformed data $\left(\Phi^{-1}(\hat{F}_X(x_i)), \Phi^{-1}(\hat{F}_Y(y_i))\right)$ have standard normal distribution. We plot them in Figure 2. The transformation of the data as in Figure 2 is useful to determine the dependence between the variables (or copula).

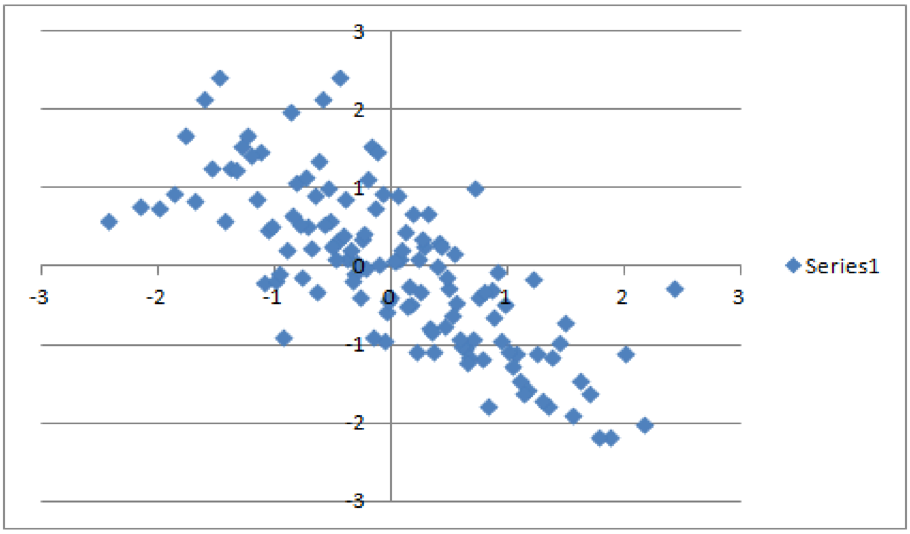

Figure 2: Couples $\left(\Phi^{-1}(\hat{F}_X(x_i)), \Phi^{-1}(\hat{F}_Y(y_i))\right)$ where $\hat{F}_X$ and $\hat{F}_Y$ are the empirical cdfs and where (x_i, y_i) are the couples of observations.

The hypothesis that the copula is a Gaussian copula with a correlation parameter estimated at -0.75 has been tested with the following proposition by Malevergne and Sornette (2003).

Proposition 2.1. *Assuming that the N-dimensional random vector $x = (x_1, \cdots, x_N)$ with distribution function F and marginals F_i, satisfies the null hypothesis H_0, then the variable*

$$z^2 = \sum_{j,i=1}^{N} \Phi^{-1}\left(F_i(x_i)\right)\left(\rho^{-1}\right)_{ij} \Phi^{-1}\left(F_j(x_j)\right),$$

where the matrix ρ is

$$\rho_{ij} = cov\left[\Phi^{-1}\left(F_i(x_i)\right), \Phi^{-1}\left(F_j(x_j)\right)\right],$$

follows a χ^2-distribution with N degrees of freedom.

Fitting the marginal distributions of the monthly returns on VIX and on S&P 500. To determine the marginal distributions of VIX and S&P500 returns, we study the original data. For example, for the marginal distribution of the VIX monthly returns, we consider log returns $\ln(y_i/y_{i-1})$ and use a t-distribution to capture the fact that the tails are fatter than a standard normal distribution. An illustration is given in Figure 3.

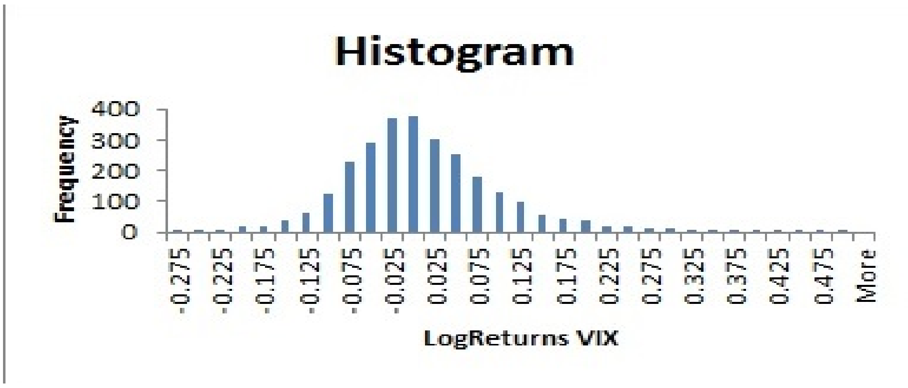

Figure 3: Couples $(\Phi^{-1}(F_X(x_i)), \Phi^{-1}(F_Y(y_i)))$ where $\hat{F}_X$ and $\hat{F}_Y$ are the empirical cdfs and where (x_i, y_i) are the couples of observations.

2.3 Toy model for the distribution of the couple (VIX,S&P500)

To gain intuition on the impact of linking fees to the VIX index, we will make use of simulations. We propose to simulate from the following toy model that is motivated by the empirical observations discussed above. First, the VIX index is initialized as its value in the market at the issuance date, then VIX will be sampled such that the monthly logreturn $\ln(v_{t+1/12}/v_t)$ follows a Student distribution with mean 0.00016, standard deviation 0.096 and 11 degrees of freedom. From the simulated value v_t, we transform the data to Normal distribution and simulate the standard $N(0,1)$ that has correlation -0.75. These increments will be useful for the monthly increments of the S&P500 returns. Furthermore, we compute the log returns on S&P with mean $r/12$ and standard deviation $0.00016\sqrt{\frac{1}{12}}$.

We assume $r = 0.013$ as interest rates have been consistently low for a long period of time (average rate of the 3-month US T bill between 2004 and 2015). This model fits well our dataset from 2004 to 2015 and is sufficiently simple to explain pros and cons of offering VAs with fees tied to the VIX index.

3 Illustrative example

The fund is assumed to be replicating the S&P 500 index S_t and a percentage fee c is paid every month for 10 years where $F_0 = \$10^9$ is the initial unique premium invested in this VA. Parameters for the Black-Scholes model for the fund value change over time according to the empirical data for the closing prices of the S&P 500 (for the current value of S_t), the VIX (for the volatility parameter σ), and the 3-month treasury bill (for the risk free rate r).

We know that the expected liability on new business should match with the fee collected over the life of contract, and it varies with changing market conditions. Our pricing for the expected liability is based on Monte Carlo simulations using our toy model.

Figure 4 illustrates that the state-dependent fees trace the expected liability much better than the constant fees. To produce Figure 4, we have the following steps:

- To value the put option at time t, we use the Black-Scholes formula, in which F_t is the fund value and v_t is the volatility level on that day. Note that v_t is set to be the monthly VIX level from 1/2/1990 to 1/2/2012. The dividend yield is the current fee.
- We then use simulation for the volatility v_t using our toy model. Given a and b, we then obtain a simulation for the fee rate process $c_t = a + bv_t$. The fee c_t is then continuously deducted from the fund value F_t, $t \in [0, T]$. In particular, $M = 50,000$ simulated paths are used to estimate the expected present value of future fees.
- For the constant fee case, $c_t = c$. The average fee rate that would have been used between 1990 and 2012 can be computed using historical values for VIX and average the state-dependent fee defined by (1) over 10 years. We find that it would have been on average equal to $c = 0.6\%$ using the parameters of the Polaris IV contract.

To motivate the need for state dependent fees, we study the implication to use a constant fee rate $c = 0.6\%$ from December 31st, 2004 to December 31st, 2014 by looking at the expected value of future claims and the expected value of future income as a function of the market variables. As it is for the purpose of illustration only, and to provide some intuition, we have used the Black Scholes model to evaluate put option prices as a liability of the GMAB and the simplified GMWB on a monthly basis. Both market values are computed as the expected present value of future cash flows under the risk neutral probability using 3-month T-bill rates.

According to the CBOE paper, Figure 5 shows the market-consistent valuation by replicating the market prices of a simplified GMWB rider to within an acceptable tolerance. They present the valuation of the liability of the simplified GMWB rider in its present value, and both the flat average fees and the variable fees charged for the liability in the contract life are presented in the present values as well. They match the present values of the fees and the liability as the same at the first day of the contract by selecting proper parameters under the specified fee rate structures.

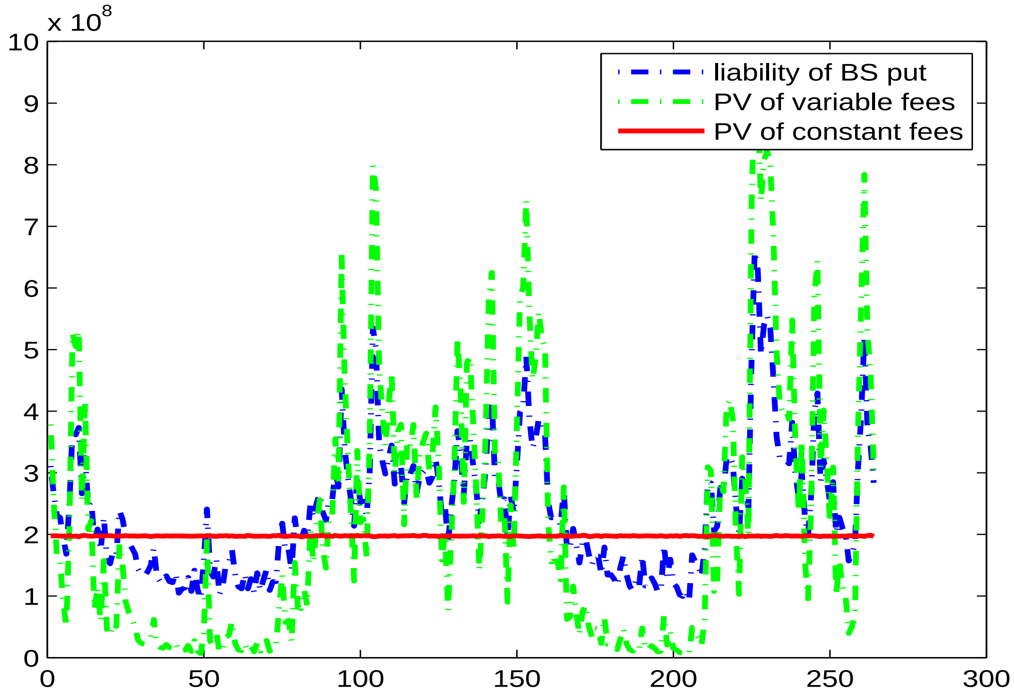

Figure 4: Present values (pricing) of the constant fees and the variable fees charged for the liability of a put using data from 1/2/1990 to 1/2/2012.

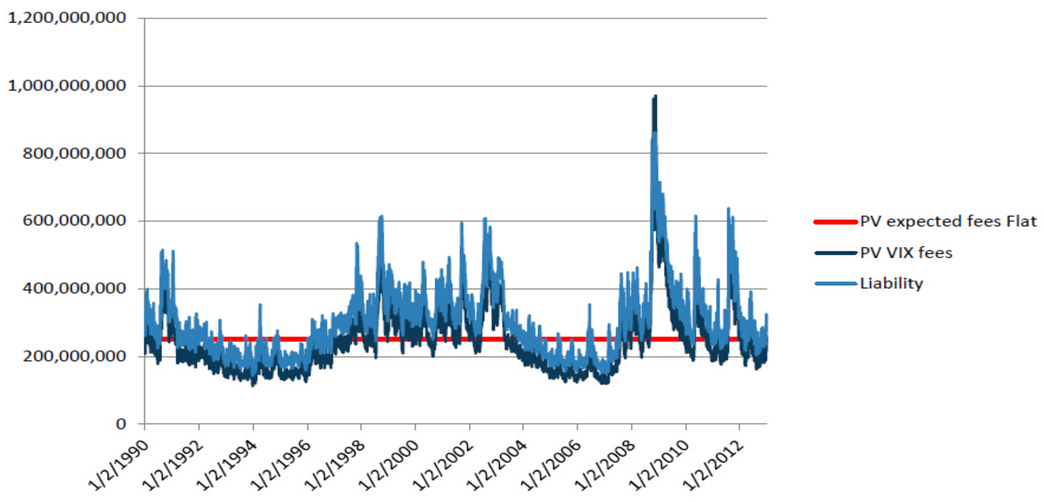

Figure 5: Present values of the flat average fees and the variable fees charged for the liability of a put in present and where the present time is the first day of the contract, i.e. 1/2/1990.

4 Fair fee

This section is the theoretical part of the paper.

4.1 Specificity of Variable Annuities

Guarantees in variable annuities are similar to financial derivatives, with notable differences. If we consider a simplified GMAB with payoff at maturity T

$$\max(G, F_T) = F_T + (G - F_T)^+$$

(i.e., the fund value and a put option with strike G). Then it is tempting to interpret it as a financial derivative, and decompose it as

$$F_0 + \text{price of a put at 0 on F, with maturity T and strike G.}$$

In such a financial derivative, the premium is typically paid upfront.

However, in the insurance world or mutual funds with guarantees (variable annuities), *management fees* and the cost for the *put* are paid for via a **constant fee rate** throughout the life of the contract (constant percentage of the underlying fund value). This particular way of paying for the put option has important consequences. It decreases the return on the VA account (which is also the underlying asset of the put) and therefore, it impacts directly the value of the put option. This is a real challenge for issuers to hedge guarantees by collecting periodic fees from the fund.

Typically, fees are taken as a constant percentage rate of the fund, so that in the Black Scholes setting, we can write the dynamics of the fund as follows

- The premium is fully invested in the index S_t, which follows

$$\frac{dS_t}{S_t} = r\ dt + \sigma\ dW_t$$

- **Standard fee structure:** Management fees paid at rate m and guarantee fees paid at rate c until maturity.

$$\frac{dF_t}{F_t} = (r - m - c)\ \ dt + \sigma\ dW_t, \quad 0 \leqslant t \leqslant T$$

  where F_0 is the initial premium (fair contract). Without loss of generality, we further assume that $m = 0$. Equivalently, monthly payments of c% of the fund value are taken away and given to the insurer to pay for the replication of the terminal guarantees offered in the VA.

If the VA were a typical financial derivative, we would have the following decomposition

$$S_0 + P_0 = E_Q[e^{-rT} S_T] + E_Q[e^{-rT}(G - S_T)^+]$$

where P_0 is the price of a put at 0 on S, with maturity T and strike G. However, fees are paid regularly as a **fixed** percentage c of the fund. In a fair contract, we can write

$$F_0 = E_Q[e^{-rT} F_T(c)] + E_Q[e^{-rT}(G - F_T(c))^+]$$

where F_0 is the value at 0 to get F_T at T with the value at 0 of fees collected in $[0, T]$. On **average** (under Q), collected fees equate market value of guarantee ("market-consistent valuation").

4.2 Fees linked to VIX

Using a market consistent valuation over the life of the contract, we have observed a mismatch between liabilities and the corresponding income coming from the collected fees. Specifically, amounts of fees are large when the fund is high and guarantees' values (liabilities) are high when the fund is low. Volatility and equity are typically negatively correlated, and options prices are increasing in volatility. The hedging program of VAs involves put options and thus it becomes expensive

- when the fund value drops,
- when amounts of fees collected by VA issuers are small.

To address this issue, we propose a new fee structure, which consists of modifying the standard fee structure where fees are paid at rate c until maturity.

$$\frac{dF_t}{F_t} = (r - c)\ dt + \sigma_t\ dW_t, \quad 0 \leqslant t \leqslant T$$

where F_0 is the initial premium **(fair contract)**.

The new fee structure has varying fees: fees are paid at rate c_t and the dynamics of the fund in the risk-neutral world are given by

$$\frac{dF_t}{F_t} = (r - c_t)\ dt + \sigma_t\ dW_t, \quad 0 \leqslant t \leqslant T$$

where F_0 is the initial premium (fair contract) and c_t is an increasing function of VIX index at t. Specifically, fees are linked to the VIX index in a linear way. The goal is to determine the parameters a and b that are driving the fees

$$c_t = a + b\left(\sigma_t - \bar{\sigma}\right).$$

To do so, we study the following two alternatives, whether we choose a better match at the initial date or at a future date.

(C1) "cost of protection at the inception = expected fee collected during the life of the contract" (fair at 0).

$$P^*(0, F_0, \sigma_0; a, b) = E^Q\left[\int_0^T e^{-rs}(a + b\sigma_s)F_s ds\right],$$

where P^* denotes the price of the embedded put price at the beginning.

(C2) Minimize over Δ the distance between "cost of protection at time Δ ($\Delta > 0$) $\approx$ expected fee collected during the life of the contract starting from Δ"

$$(a, b) = \arg\min_{a^*, b^*} E^Q\left(P^*_\Delta - E^Q\left[\int_\Delta^T e^{-rs}(a^* + b^*\sigma_s)F_s ds|\mathcal{F}_\Delta\right]\right)^2$$

where the reduced liability
$P^*_\Delta := P^*(\Delta, F_\Delta, \sigma_\Delta; a^*, b^*)$ – accumulated fees at Δ collected in $[0, \Delta]$.

Within our toy model for the joint distribution of the fund and the VIX index, we will derive the fair parameters a and b that lead to a fair fee

$$a + [b \times (\text{Value of the VIX at t} - 20)], \tag{7}$$

The fees depend on two parameters a and b. In Figure 6, we present a graph with all the couples (a, b) that lead to the fair fee in Variable Annuities with the fee rate structure in the case of the expression (7). Note that this fee is simplified (compared to the original Polaris Choice IV contract) because the dependence is directly in the value of the VIX index at time t and not linked to the average value of the VIX as it is the case in (1). In Figure 6, we determine couples (a, b) by equating the cost of liability put with the expected fees at time 0 in (3). For different a, we can search feasible b to hold this equivalence.

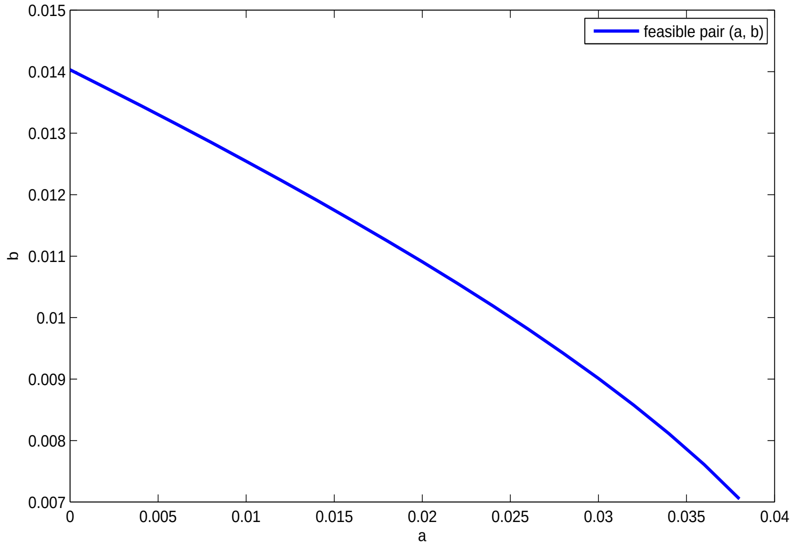

Figure 6: Couples (a, b) when the equality between the cost of liability put of a GMAB and the expected state-dependent fees holds at time 0 using data from 1/2/1990 to 1/2/2012.

5 Example with Polaris Choice

In the last section of this paper, we go back to the Polaris Choice IV contract that motivated our study. Using Monte Carlo simulations, we can keep as many features as possible from the Polaris Choice IV contract. Only the outline of the research is exposed here. Details will be completed in the coming months.

5.1 From VA issuer's perspective

First, we look at the issuer's perspective. We estimate the fair fees a and b in our toy model and compare with the fees in the contract issued in 2014. In particular, we show the advantages of linking the fee to the average VIX instead of linking fees to the value of the VIX at time t. There are pros and cons of using the spot VIX instead: it has a better predictive power, but it is more volatile (fees need to be adjusted more often).

In practice, many insurers use short dated put options to hedge the long-term liability of a VA. The periodic fees, $\text{fees}(t, T, c_t)$ in $[t, T]$, are then collected to compensate for the short dated put options $P(t, T, F_t, K, \sigma_t)$ in $[t, T]$. We thus define the ratio, $\frac{\text{fees}(t,T,c_t)}{P(t,T,F_t,K,\sigma_t)}$, as a measure to the ongoing fees of maintaining the positions. In

Figure 7, we observe that fee income partially matches and better traces the cost of 100%-strike put options when it is tied to VIX.

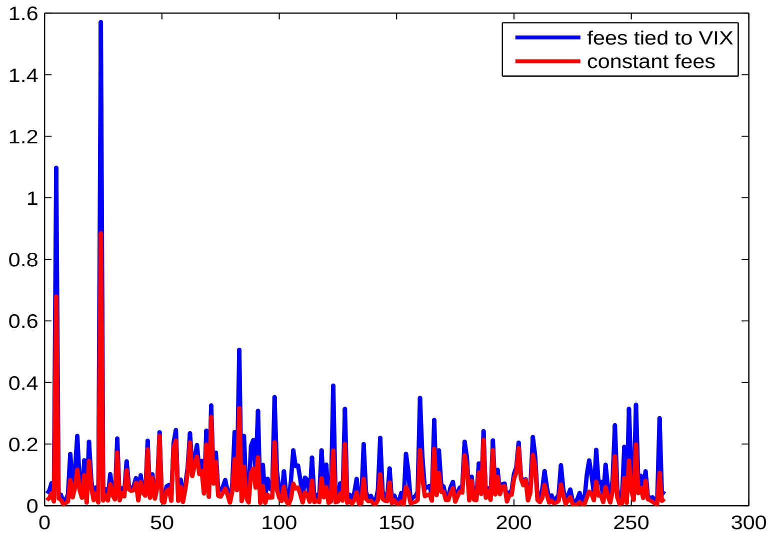

Figure 7: Ratios for the ongoing fees of maintaining the positions by charging variable fees and constant fees respectively by fitting the data from 1/2/1990 to 1/2/2012.

5.2 From policyholders' perspective

Second, we will study the policyholders' perspective and discuss the advantages and disadvantages of charging a fee linked to VIX for policyholders. The CBOE white paper on fees linked to VIX and the description of the US patent is very enthusiastic about the numerous advantages for the issuer but it is not so clear that it is a better design for policyholders.

6 Conclusions and Final Remarks

Our objective in this paper is to explore VIX linked fees as a powerful risk sharing mechanism between policyholders and insurers. We make use of the strong negative correlation between the VIX index and the returns on the S&P500 index to propose an innovative design for the fees paid in variable annuities.

Our study shows important risk management advantages for this new design of state-dependent fees. We show that VAs with fees tied to VIX lead to a better matching between the amount of fees collected and the hedging costs: fees are higher when the volatility is high and thus when the hedging program (e.g., rolling up short term options) is expensive. We also expect that this new design allows to mitigate surrender incentives. Indeed, fees are low when guarantees are less valuable and optimal surrender decision is typically driven by fees that are too large compared to the market value of the guarantees (Bernard, Hardy, and MacKay (2013), MacKay, Augustyniak, Bernard, and Hardy (2016), Moenig and Zhu (2016)). However, fees are large when the fund is low, which may increase the propensity for policyholders to lapse in adverse conditions giving up their maturity guaranteed benefits.

We have developed a toy model to tackle the problem of modeling and pricing VAs with fees tied to VIX. In this paper, we rely a lot on Monte Carlo simulations to gain intuition on the impact of such fees on the risk management of these variable annuities. If such fees become more popular in the industry, the next step will be to extend the model to more advanced market models and develop a stochastic volatility model.

Finally, we should also insist on the fact that using fees linked to VIX facilitate the risk management of the guarantees but does not replace a hedging program. It allows to design products that automatically "re-price" themselves over time. This is an important advantage and will allow to avoid recent stress of insurers that constantly update the guarantees they offer and corresponding fees as market conditions change.

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